

PREFACE

This monograph addresses the evening twilight period, *bein ha-shemashot*. Much has been written attempting to reconcile the *gemara's* description of astronomical conditions around twilight in the Middle East with the opinion of Rabbeinu Tam, and to a significantly lesser extent with the opinion of the *Gaon* of Vilna. I can reconcile neither opinion with the *gemara's* description of astronomical conditions during the period of *bein ha-shemashot*. Both

- a period of *bein ha-shemashot* occurring approximately one hour after sunset in accordance with the view of Rabbeinu Tam, and
- to a lesser degree, a period of *bein ha-shemashot* beginning precisely at sunset in accordance with the view of the *Gaon*,

face observational challenges. Instead, I propose a hybrid position, separating the beginning of the *bein ha-shemashot* period, (slightly) from sunset, a position rarely found in the contemporary literature. That position on when the period of *bein ha-shemashot* occurs is much closer to that of the *Gaon* than to that of Rabbeinu Tam. I demonstrate that it is consistent with the *gemara's* description of astronomical conditions and supported by various authorities. As well, this approach partially justifies centuries of practice.

After completing an initial version of this monograph, I read two important works on *bein ha-shemashot*, *Ohr Ha-meir* by R. Meir Posen

and *Munaḥ Yoma* by R. Emanuel Gettinger, the former suggested to me by R. Mordechai Willig, and the latter just recently published. Both are to varying degrees supportive of the approach suggested in this monograph. While I have added a few brief footnotes relating to their works to the main body of the monograph, it seemed appropriate to add some summary comments to this preface as well.

Using a different methodology from the one employed in this monograph, R. Posen in *Ohr Ha-meir* argues that a position akin to the hybrid position I suggest, was, in fact, the position of the *geonim*, whose position he differentiates from that of the *Gaon*. The position he ascribes to the *geonim* separates the beginning of the *bein ha-shemashot* period slightly further from sunset than what I propose, something with which I do not agree in detail. More importantly, however, his arguments align the *geonim* with the hybrid position I develop, adding significant credence to what is suggested in this monograph.

In a recently published work, *Munaḥ Yoma*, R. Gettinger attempts to explain Rabbeinu Tam in a radically different manner, similar, in some respects, to the hybrid position I develop. Though not consequential to this monograph, I disagree with his interpretation of Rabbeinu Tam. More importantly, however, his understanding of two critical sections of the *gemara* in *Shabbat* is identical, albeit differently argued, to what I will attempt to demonstrate:

- First, the *gemara* does not dispute the time that *Shabbat* (and the *bein ha-shemashot* period) ends; the *gemara* only disputes the time at which the *bein ha-shemashot* period begins.
- Second, when quantitatively specifying the duration of the *bein ha-shemashot* period, the *gemara* is counting back from the end of *Shabbat*, not forward from its beginning.

As should become evident, this reading of the *gemara* is not in any way linked to nor does it depend on the dispute between the *geonim* and Rabbeinu Tam.

Neither R. Posen nor R. Gettinger made use of the actual times at which stars appear during various seasons in the Middle East, something that I found very useful in reading the *gemara* accurately.

My purpose is not to suggest a change in standard practice. What I do suggest is that treating sunset proper as the start of the *bein ha-shemashot* period may be recommended practice as opposed to strict *halakhah*. As such, a decisor might choose to use the hypotheses developed in deciding cases involving various extenuating circumstances. In any case, my primary goal is to clarify an area with a significant and complex history.

I have benefitted from an extensive literature. In particular, R. Yechial Michal Tukitzinsky's work entitled *Bein Ha-shemashot*, combining a deep knowledge of the *halakhic* literature and meticulous observation

with a profound awareness of astronomy, provided a solid foundation for my studies. Lastly, I thank my family and friends who have tolerated my obsession with this and related topics where mathematical reasoning and the observation of nature enhance the understanding of *halakhah* and its development.

PROLOGUE

Time is a critical delimiter for many commandments, including observance of the holy days, the obligation to pray, adherence to the laws of *niddah* and the performance of a *brit milah*. Three subjects

1. the beginning of the daytime period, *alot ha-shaḥar*,
2. the point of transition between days¹ of the week, (*ḥashekhah*, and the period of *bein ha-shemashot* that precedes it) and
3. alternatives to calculate various times during the daytime period

are the primary areas of dispute involving multiple *halakhic* issues.²

Because of the centrality of *Shabbat*, its twilight period, the period of *bein ha-shemashot*, occupies much of the literature. That focus and its relationship to other aspects of *zmanim* will occupy most of our attention as well.³

¹ Note that both the terms “day” and “*yom*” refer to either the daytime period, as in “during the day,” and the day of the week, as in “*yom ha-shishi*.”

² The relationship between these three areas is addressed at various points in the monograph. While I will argue for their logical independence, others have asserted both logical and practical interdependencies.

³ While *posekim* might apply some of our findings, it is not our purpose to reach definitive conclusions even with respect to our primary area of focus, the *bein ha-shemashot* period. Of course, a period of *bein ha-shemashot* occurs daily preceding the transition between days of the week. The relevance of different times during the twilight period to areas of *halakhah* other than *Shabbat* is not covered in any detail. For example, it is not our intent to suggest any definitive *pesak* from the few instances that the day of a *brit milah* is referenced; on the other hand, the viewpoint developed may have relevance beyond those areas addressed.

Rambam in *hilkhot Shabbat* 5: 4 refers to the period between sunset and the appearance of three stars as “*bein ha-shemashot*.” His language, “*hu ha-nikrah bein ha-shemashot*,” “that is **called** *bein ha-shemashot*,” might imply that Rambam is providing practical guidance instead of a precise definition.⁴ In any case, assuming that sunset, as meant by Rambam, is identical to what we colloquially call sunset, we would conclude that presently common practice is exactly as Rambam suggests:

- *Shabbat* starts Friday evening at sunset and ends Saturday night with the appearance of three stars.⁵

Awaiting three stars before ending *Shabbat* represents common practice both before and after Rambam’s formulation, albeit in later times with a variety of stringencies, i. e., three small stars and stars relatively adjacent to each other.⁶ However, starting *Shabbat* precisely at sunset, while now completely accepted, was certainly not common practice for much of European Jewry over many centuries.

⁴ Whether “*hu ha-nikrah*” is an approximation referring only to the beginning of the *bein ha-shemashot* period at sunset, only to its end at the appearance of three stars or to both its beginning and its end is debatable. In my view, it is likely that both are intended as suggested practice, as opposed to either being a precise *halakhic* delimiter. Rambam’s perspective is addressed at length at the end of section 5.

⁵ R. Avraham ben Ha-Rambam defined sunset, as the ball of the sun descending below the horizon, identically to the way the term is defined in standard secular usage. R. Kapach disputes that even this passage of Rambam refers precisely to sunset, a critical point on which I will focus.

⁶ Perhaps as well, as I will explain, stars appearing towards the western horizon, a requirement that better accommodates the opinion of Rabbeinu Tam.

The normative opinion of the *gemara* in *Shabbat*⁷ states that the duration of the *bein ha-shemashot* period equals⁸ the time needed to walk $\frac{3}{4}$ of a *mil*. One would therefore expect to see three stars, depending on the assumed time it takes to walk a *mil*, between 13.5 and 18 minutes after the beginning of the *bein ha-shemashot* period. Rabbeinu Tam, an older contemporary of Rambam living in France, was, in all likelihood, unaware of the impact of latitude on the appearance of stars.⁹ As such, if sunset precisely defines the beginning of *bein ha-shemashot*, Rabbeinu Tam would not have been able to correlate the appearance of three stars with so short a duration of time. For that matter, even those living in the Middle East can rarely see three stars 15 to 18 minutes after sunset, and then only under the most ideal conditions. Rabbeinu Tam, especially during the summer months, living at a more northern latitude in France, might not see what he considered three medium stars until approximately an hour or more after sunset, depending on his interpretation of “medium.” However, a second *gemara* in *Pesaḥim*,¹⁰ in apparent contradiction to the *gemara* in *Shabbat*, but perhaps closer to what Rabbeinu Tam perceived, discusses the appearance of stars at the time needed to walk 4 *milin*, 72 or 90 minutes, after sunset. For reasons to be explained later, Rabbeinu Tam chose to equate the endpoints of these two *sugyot*. Thus, Rabbeinu Tam postulated that the sunset referred to by the *gemara* in *Pesaḥim* is

⁷ *Shabbat* 34b.

⁸ It is more likely that this is only an approximation, and as is argued, a conservative one.

⁹ An early reader of this manuscript, Yitzḥak Radner, pointed out that a discussion in *tosefot Gittin* 8a, s.v. Rabi Yehudah might imply Rabbeinu Tam was not entirely aware of how far north his locale was relative to Israel.

¹⁰ *Pesaḥim* 94a.

(approximately) what we today call sunset. The *gemara* in *Shabbat*, Rabbeinu Tam explained, is referring not to our normal definition of “sunset,” the sun descending below the horizon, but to a **second** sunset defined by the light of the sun being almost minimal, as would occur well after sunset.¹¹ It is at that point, Rabbeinu Tam asserted, at this **second** sunset, that the *bein ha-shemashot* period begins Friday night. On Saturday night, the *bein ha-shemashot* period ends 13.5 to 18 minutes, the time needed to walk $\frac{3}{4}$ of a *mil*, after that point. This theory of Rabbeinu Tam provided the conceptual basis for many Jewish communities in Europe that began *Shabbat* well after sunset.

Perhaps, had Rabbeinu Tam and his contemporaries been more aware of their location relative to the Middle East and / or of the influence of latitude, they would have understood that the appearance of three stars signifying the end of *Shabbat*, occurs later after sunset in their location, at a latitude further from the equator, than in the Middle East. More importantly, some might assert, they would never have postulated a *halakhic* approach that started *Shabbat* so much later. Simply put, Rabbeinu Tam’s assumed lack of geographic / scientific knowledge appears to have created an opportunity for the violation of the laws of *Shabbat* for almost an hour every Friday afternoon. Of course, one could

¹¹ Rabbeinu Tam’s actual language was based on his assumptions about astronomy and is not consequential; reformulating Rabbeinu Tam’s opinions using modern terminology is done throughout. In addition, there are conceptual differences between the actual opinion of Rabbeinu Tam and the way it was formulated by Ramban and many who embraced his approach. These differences are not significant practically. Despite these differences, Ramban’s detailed explanation of Rabbeinu Tam is referred to as the opinion of Rabbeinu Tam throughout this monograph.

argue that since Jews observed *tosefet Shabbat*, a voluntary lengthening of *Shabbat*, the violation was only theoretical. However, this attempt at rationalization is likely incorrect. Centuries later, even after many *posekim* understood the impacts of latitude, the practice of starting *Shabbat* after sunset remained widespread. In the 19th century, R. Moshe Sofer, in his famous *teshuvah* on this subject, plainly states that in his community people normally work well after sunset on Friday.¹² As the opinion of Rabbeinu Tam became the accepted *halakhah*, recorded by both R. Yosef Caro and Rama, it is widely acknowledged that many communities in Europe worked well after sunset on Friday.¹³

Frankly, I am not bothered by how inadequate knowledge of science might have contributed to a questionable *halakhic* approach. Rabbeinu Tam and those who followed his approach marshaled considerable logical and textual support for their point of view. While the *gemara* in

¹² *Teshuvah* 80 is covered in more detail in subsequent sections. On that basis, R. Sofer ruled that a baby born 27 minutes after sunset on Saturday evening should have his *brit* on the following *Shabbat*. See R. Haim Benish, *Ha-zemanim Be-halakhah*, Bnai Brak, 5756 / 1995, vol. 2, page 242.

¹³ Chapters 46 and 51 in *Ha-zemanim Be-halakhah*, contain dozens of references with some remnants of this practice continuing until the Second World War. In a recent work, http://www.math.harvard.edu/~shlomo/docs/bein_ha-shemashot.pdf, Prof. Sternberg argues that prior to the (widespread) appearance of clocks, Rabbeinu Tam was only a theoretical opinion. I avoid drawing any precise historical conclusions. While I strongly agree that the widespread growth of clocks beginning in the 15th century had substantial impact, I find it difficult to conclude that prior to the era of clocks, *Shabbat* always started precisely at or before sunset. If the start of *Shabbat* in the Middle East was some small number of minutes after sunset, migration to areas further from the equator might naturally result in a variety of approaches that might further postpone the onset of *Shabbat*. More than likely, prior to the existence of clocks, observation lessened the practical impact of questionable concepts on *halakhic* observance. As clocks reduced the role of observation, it is likely that the impact of problematic concepts increased, an area addressed more extensively in the epilogue.

Shabbat refers to the appearance of three stars and the *gemara* in *Pesaḥim* refers to *tzait ha-kokhavim*, the *Yerushalmi* in the beginning of *Berakhot* equates *tzait ha-kokhavim* with the appearance of three stars. While many centuries later, the Vilna *Gaon* demonstrated convincingly that the term *tzait ha-kokhavim* as used in *Pesaḥim*, is different from its use in *Berakhot*, it is hardly surprising that *rishonim* might assume similar semantics to the term *tzait ha-kokhavim* across its usage in all *sugyot*. We should also remember that Ramban, who agreed in principle with Rabbeinu Tam, lived in Spain and later in Israel.

Even were one to prove that Rabbeinu Tam would never have formulated his approach had he only been more aware of geography / astronomy, something that is by its nature plausible but hardly provable, the opinions of Rabbeinu Tam and his followers present a conceptually elegant *halakhic* position. Furthermore, a number of subsequent authorities, starting with R. Avraham Pimental, the 17th century author of *Minḥat Kohen*, the first comprehensive analysis of this topic, and in modern times both R. Moshe Feinstein as well as R. Joseph Soloveitchik, **all aware of the impact of latitude**, side with Rabbeinu Tam, at least partially. Although each makes adjustments to the commonly assumed 72-minute opinion of Rabbeinu Tam based on latitude, Rabbeinu Tam's approach to the ending of *Shabbat* is considered normative *halakhah* in the case of R. Pimental and R. Feinstein, and a personally practiced stringency for R. Soloveitchik. While each of the approaches of the above authorities to the opinion of Rabbeinu Tam faces almost intractable challenges, it would be

erroneous (in addition to arrogant) to dismiss Rabbeinu Tam's approach to the period of *bein ha-shemashot* in its entirety, as simply the result of inadequate understanding of astronomy. To the contrary, this monograph marshals support for Rabbeinu Tam's reading of critical parts of the text of the *gemara* in *Shabbat*.

Rabbeinu Tam's opinion remains primarily both a conceptual approach and a stringency that extends the end of *Shabbat*. What is troubling, however, is the start of *Shabbat*. If one rejects the opinion of Rabbeinu Tam, must one then conclude that (inadvertent) violation of *Shabbat*, perhaps even at a biblical level, was commonplace every Friday afternoon for a long period of our history? Despite multiple conceptual and textual difficulties that the opinion of Rabbeinu Tam must address, **my goal is to explain and defend *halakhic* practice independent of the *halakhic* theory its adherents may have assumed.** As Abayeh or R. Yosef told Raba bar R. Ḥanan: "Go forth and see how the public are accustomed to act."¹⁴ I will outline alternatives based on previous *halakhic* approaches that support different starting times for the beginning of *Shabbat*. Thus, some or perhaps most of the practice of Jews throughout the ages is defended, using *halakhic* concepts partially based on what Rabbeinu Tam suggested. Of course, any such defense depends on just how late various communities actually started *Shabbat*. Without reliance on the entire opinion of Rabbeinu Tam and avoiding its more challenging elements, I hope to defend those who started *Shabbat*

¹⁴ *Berakhot* 45a, Soncino translation.

after sunset but not as late as the conceptual view of Rabbeinu Tam might have allowed.¹⁵

While some communities keep *Shabbat* later on Saturday night, largely as a stringency based on the opinion of Rabbeinu Tam, we all begin *Shabbat* exactly as Rambam suggested, at what we refer to as sunset. Under normal circumstances, no one today would allow starting *Shabbat* after sunset. Ironically, what follows from the approach that is developed is that the stringency based on Rabbeinu Tam's approach is what is problematic given the *gemara's* description of the end of the *bein ha-shemashot* period. Up to some point after sunset, the leniency that follows from Rabbeinu Tam's approach, similar to the assumed practice of European Jewry over the centuries, (but only when coupled with reasonable adjustments based on latitude and season), represents a plausible, if not a preferred, reading of the *gemara's* assumed beginning of the *bein ha-shemashot* period on Friday evening.

Beyond the period of *bein ha-shemashot* discussed above, there are other complex topics associated with the entire area of *zemanim*,

¹⁵ We simply do not have much of an historical record on precisely when *Shabbat* was started. As a stringency and perhaps to better align with Rabbeinu Tam's conceptualization of the time at which *Shabbat* ends, the conclusion of *Shabbat* was lengthened from three *medium* stars, stated in the *gemara*, to three *smaller and adjacent* stars. That stringency, based on the opinion of Rabbeinu Yonah (and perhaps a *Yerushalmi*) and meant to include *tosefet Shabbat* as well, was codified in the *Shulhan Arukh*. The very same logic that would apply a stringency extending the end of *Shabbat* should not have delayed the start of *Shabbat*, at least not past the appearance of three *medium* stars. I doubt that even clocks would have made (most) people comfortable starting *Shabbat* as late as both the theory would support and the time many today pray the *minḥah* service.

independent to some extent, though often interrelated. The area is difficult even absent any issues of inadequate observation or errors in arithmetic obscuring *halakhic* principles. A number of factors including reticence to

- criticize major authorities and articulate potential errors clearly,
- acknowledge changes in practice or
- consider the impact of evolving knowledge of science

have not helped to promote clarity in an already complex area. I will be as clear as possible without any of the above hesitations. In doing so, I can defend more accurately what I consider more important, namely both the integrity of traditional Jewish practice as well as the deep *halakhic* insights of generations of *posekim*. It is particularly critical that we identify any misconceptions related to indisputable observation, logic or arithmetic so that one can separate the *halakhic* approach of various authorities from (any errors in) their assumed fact base in order to (re)apply their *halakhic* insights in practice.¹⁶

¹⁶ Even reading the more recent *halakhic* literature or listening to some popular lectures, with all of the necessary science and basic arithmetic already well known, one still finds no shortage of confusion and / or misstatements. Despite a preference for clarity, even in the epilogue, where I am willing to conjecture more freely, I avoid excessive concentration on the origin, cause and history of errors. Even when a particular error is prevalent, I typically only mention a few examples. It suffices to recognize that not all authorities recognized the impact of latitude or season or were immune from errors that are more basic; this is compounded by a tendency to rely on texts without sufficient observation. On occasion, poorly reasoned attempts to defend a difficult point of view end up amplifying the problem. Nonetheless, when a particular viewpoint is plagued by (what appears to be) an obvious error, it behooves us to uncover the circumstances that contributed to its occurrence. Often that enables an appreciation for an underlying *halakhic* principle

While these laws are clearly unique, they, perhaps more so even than the topics surrounding *shiurim*, clearly stress the importance of a mimetic tradition and force one to seriously confront the nature of how *halakhah* might be conceptualized and how its development occurs. I leave it to others to draw whatever philosophic implications they care to make about the relationship between *halakhic* concepts and *minhag Yisroel saba*, the mimetic tradition. My purpose is to render this area as clear as I can, and consequently explain, consistent with both the primary *sugyot* and observable reality, alternative positions on critical *halakhic* topics and centuries of *halakhic* practice.

There is an extensive *halakhic* literature dealing in depth with diverse opinions covering all of the multiple facets of this area. *Ha-zemanim Be-halakhah*, a contemporary two-volume work by R. Haim Benish¹⁷ provides a comprehensive summary. This essay does not attempt to be comprehensive or to reiterate and analyze in-depth a wealth of primary sources.¹⁸ Our focus is primarily the period of *bein ha-shemashot*; I

obscured by the error. For example, there are *teshuvot* that differentiate between different types of activity and then address their permissibility at a particular time during the *bein ha-shemashot* period. Inaccuracy in specifying a *zeman* ought not obscure an important concept in differentiating between various activities.

¹⁷ *Ha-zemanim Be-halakhah* and the numerous primary sources it references provide a great deal of the material needed for further study. This monograph should make the reading of those sources easier.

¹⁸ Some fascinating areas adjacent to our main focus that are covered only partially include

- the period after sunset in Rambam *hilkhot Shabbat*, *Kiddush Ha-ḥodesh* and other areas of *Mishnah Torah*,
- a plausible approach to the enigmatic opinion of R. Nosson Adler, quoted by R. Sofer,

cover other topics insofar as they are critical to the period of *bein ha-shemashot* according to at least some opinions. After a lengthy introduction and followed by an explicit list of assumptions, I outline nine major categories that can act as a checklist¹⁹ when reading the vast *halakhic* literature, and cover

- major positions within each of the categories,
- arguments supporting and challenging each position,
- interrelationships between these categories,
- elements within each category that are problematic in terms of observation, arithmetic, or logic that complicate the reading of this vast *halakhic* literature and
- an approach with which to analyze each category.

The monograph concludes with an epilogue highlighting areas of innovation and providing summary conclusions for the nine categories discussed in detail.

-
- options for calculating the opinion of Magen Avraham and
 - reconciling the multiple *teshuvot* of R. Israel Isserlein in *Trumat Ha-deshen* and the opinions of R. Yosef b. Moshe, his student and the author of *Leket Yosher*.

These, as well as numerous other technical aspects of *zemanim*, have not been addressed adequately; all are areas that I hope to address more fully in the future.

¹⁹ I have found that when reading the vast literature, the first **eight** topics are a helpful checklist for classifying viewpoints, identifying assumptions as well as addressing any assumed dependencies between the categories. (As I attempt to demonstrate, I do not believe there are as many dependencies between these categories as many claim). The ninth category is personally fascinating and rarely addressed despite its fundamental nature; it is a clear example of the modern *Brisker*, conceptual approach.

To that end, the main sections of this essay deal objectively with each of the underlying *halakhic* categories. Only in the epilogue will I return to a more intuitive discussion, including some hypotheses of how practice and its rationale may have evolved.

Both this prologue and the epilogue contain opinion and conjecture that readers may find plausible. I have no way of proving, for example, what knowledge (of geography or science) Rabbeinu Tam possessed. I can only conjecture, based on what he wrote. I cannot even be certain of Jewish practice for beginning and ending *Shabbat* from the 13th through much of the 18th centuries. I hope that historians may one day fill in some more detail; theologians can then deal with implications for the *halakhic* process.

Allow me one somewhat repetitive point. An approbation from R. Yitzhak Hutner gently cautions Prof. Levi²⁰ and others against attempting to decide between the “giants of previous generations.” My goal is not to do that; in fact, adjusted for their factual inaccuracies, **almost all of the major conceptual approaches of *posekim* are supportable**, as I hope this monograph will make clear. To that end, this monograph facilitates separating the *halakhic* framework and concepts from any factual errors that may have been made. To my

²⁰ See the approbations at the beginning of his book, *Halakhic Times*, originally published in Jerusalem in 5727 / 1967. R. Hutner was likely reacting to Prof. Levi’s strong opposition to the widely assumed time required to walk a *mil* of 18 minutes as well as, perhaps, his clear bias in favor of the opinion of the *geonim*.

mind, doing otherwise is both dishonest and complicates study of the entire area.

However, in the interest of both completeness and candor, I must admit that I wonder whether some *halakhic* insights are innovative solutions to problems created by incomplete knowledge. It is difficult not to wonder what changes might have been made by Rabbeinu Tam had he been aware that it turns dark and stars appear much longer after sunset in the north of France than in Israel. I remind the reader that, even in science, great insight may result from flawed data. Knowledge of Rabbeinu Tam's position and its extensive support from *sugyot* in the Talmud are important, if only as issues to be addressed by those who oppose his viewpoint. Similarly, understanding the approach of generations of *halakhic* authorities apart from any errors they may have made, allows a clearer articulation of their opinions and insights. To do otherwise, would continue to obscure a critical area of *halakhah*.

INTRODUCTION: Some current positions

Accessing *zemanim* on an older version of the Orthodox Union's website provided a variety of *halakhic* information, comporting with widespread current practice. Included were two differently calculated times for the end of *Shabbat* at locations around the world. In the New York area, the first alternative, approximately 40 to 50 minutes after what is commonly referred to as sunset, is presented as the opinion of the *geonim*, according to whom *Shabbat* ends the time it takes to walk $\frac{3}{4}$ of a *mil* after the beginning of the *bein ha-shemashot* period. The second alternative, 72 minutes, is presented as the opinion of Rabbeinu Tam, according to whom *Shabbat* ends the time it takes to walk 4 *milin* after sunset. Of course, this is immediately problematic. If the time needed to walk 4 *milin* is 72 minutes, then the time needed to walk one *mil* is 18 minutes and the time needed to walk $\frac{3}{4}$ of a *mil* equals 13.5, not 40 to 50 minutes. Part (but certainly not all) of this discrepancy can be attributed to the fact that the opinion of the *geonim* is normally adjusted based on (location /) latitude and often further adjusted based on season of the year. However, those two adjustments alone applied to 13.5 minutes do not yield a range of approximately 40 to 50 minutes in the New York area. Other factors include

1. the extension of the end of *Shabbat* from the appearance of medium stars to the appearance of small stars,
2. the impact of doubt as applied to a prohibition as serious as the observance of *Shabbat*,

3. the impact of *tosefet Shabbat*,
4. the length of the time needed to walk a *mil* and
5. the beginning of the *bein ha-shemashot* period.

The opinion of Rabbeinu Tam, however, is assumed to be fixed at 72 minutes, at any latitude and during all seasons of the year. (Currently, for reasons I could not even begin to justify, the OU website no longer adjusts the opinion of the *geonim* for the end of *Shabbat*, leaving it at a fixed 42 minutes. Most websites and printed calendars adjust the end of *Shabbat* according to the *geonim*).

Halakhic principles, logical consistency and theory notwithstanding, the vast majority of current practice follows one of these two opinions. Unquestionably, common practice adjusts the length of the period of *bein ha-shemashot* based on latitude (and even season on occasion) when following the opinion of the *geonim*, while leaving the length of the *bein ha-shemashot* period constant when following the opinion of Rabbeinu Tam. This is not surprising. R. Elijah of Vilna and R. Shneur Zalman of Liadi, the first Lubavitcher Rebbe,²¹ both explicitly require these adjustments in applying the approach of the *geonim*. They were the two most prominent authorities that battled to reinstate the practice of the *geonim* against what had become the practice of an overwhelming majority of European Jewry who had adopted the opinion of Rabbeinu Tam. That majority included many if not most subsequent *rishonim*,

²¹ In his *siddur*, R. Shneur Zalman of Liadi abandons the position he took in *Shulḥan Arukh Ha-rav* that supports Rabbeinu Tam.

both R. Yosef Caro and Rama, and most early commentators on the *Shulḥan Arukh*. For those who followed the opinion of Rabbeinu Tam, there is no conclusive evidence²² in either *halakhic* literature or practice, that Rabbeinu Tam or anyone else until **after** the time of the *Shulḥan Arukh* ever adjusted the start or the end of the *bein ha-shemashot* period with explicit reference to either latitude or season.²³ Similar to the time needed to walk 4 *milin*, it was assumed to be invariant.

Comporting with practice and consistent with the opinion of the major authorities whose opinion those practices appear to follow, would not normally warrant further analysis. However, as I will quickly illustrate, almost every aspect of *zemanim* involves a challenging mosaic of major and minor *halakhic* issues that have to be resolved and then applied, consistent with observable astronomical facts.

Consider two other contemporary opinions on this topic, Rabbi Feinstein,²⁴ and Rabbi Willig.²⁵ R. Feinstein cites two alternatives to be followed in the New York metropolitan area: 72 minutes and approximately 50 minutes. Clearly, the practice recommended on many websites is rather consistent with R. Feinstein's ruling. However, R.

²² *Minḥat Kohen*, written in the 17th century, provides a comprehensive review of *zemanim*, and when suggesting latitude-based adjustments to the opinion of Rabbeinu Tam, mentions no earlier source supporting that approach.

²³ This disparity in approach, adjusting one measure and not another, has the effect of reducing the period of dispute around the end of *Shabbat* in northern European countries.

²⁴ *Igrot Moshe*, O. H. 4:62.

²⁵ *Am Mordechai on Masekhet Berakhot*, New York, 1992, chapter 2.

Feinstein's logic is radically different. First, R. Feinstein argues that latitude is commonly accepted as a determinant in the length of the *bein ha-shemashot* period and is thus to be applied to the opinion of Rabbeinu Tam as well. Second, R. Feinstein observes that while **he waited 72 minutes in Lithuania, in accordance with Rabbeinu Tam**, the sky is equivalently dark and starry in New York after only 50 minutes. Therefore, R. Feinstein adjusts Rabbeinu Tam's opinion based on latitude, and concludes that there is an adequate *halakhic* basis for those living in the New York area and following Rabbeinu Tam to wait only 50 minutes.²⁶ This opinion is similar to one given by Rabbi Pimental, in *Minḥat Kohen*, in applying Rabbeinu Tam's opinion.²⁷ R. Pimental suggested that the Jews living in the low altitude²⁸ of the Netherlands need only wait 48 minutes after sunset (in the spring) to end *Shabbat* according to Rabbeinu Tam. In reality however, the length of time from sunset to darkness (however defined) increases as one moves further away from the equator traversing the latitudes of the Middle East (~30 degrees), the Northern United States (~40 degrees), the Netherlands (~50 degrees) and Lithuania (~55 degrees). Assuming adjustments based on latitude, the end of *Shabbat* is later after sunset in the New York area or the Netherlands than in Israel. Problematically, R. Feinstein's end to *Shabbat* is earlier in the New York area (similar to the Netherlands in the opinion of R. Pimental) than in the Middle East

²⁶ 72 minutes is preferred by R. Feinstein, but only as a recommended stringency.

²⁷ *Maamar sheni*, chapters 4 and 5.

²⁸ R. Pimental was perplexed by the expected impact of latitude and explicitly questions why the twilight period in the Netherlands is not longer than 72 minutes given that the Netherlands is further from the equator than the Middle East. In an attempt to explain 48 versus 72 minutes, R. Pimental significantly overestimated the effect of elevation and its impact on the twilight period in low-lying Holland.

according to Rabbeinu Tam's understanding of the *sugyot*. Despite the widespread acceptance and application of this opinion of R. Feinstein in many popular and scholarly contexts, analysis is often muted or absent entirely.²⁹

While many websites and calendars align with R. Feinstein's *pesak*,³⁰ rationales differ entirely.³¹ When one waits approximately 50 minutes, almost all websites assume one is following the *geonim*, while R. Feinstein assumes that one is still following Rabbeinu Tam. Unlike R. Feinstein's ruling, most websites do not adjust Rabbeinu Tam's opinion for latitude or season. Of course, one could argue that Rabbeinu Tam's measurements of twilight and nightfall apply to France and not the Middle East. This would imply that the *sugyot* that Rabbeinu Tam is analyzing defined *zemanim* for Europe and not the Middle East, hardly a plausible position. On the other hand, both historical practice and Rabbeinu Tam's own position made no explicit latitudinal adjustments between the Middle East and France. One can only assume that Rabbeinu Tam assumed his *zemanim* applied uniformly to both the Middle East and France. In that regard, the OU and most other websites

²⁹ Both R. Willig *Am Mordechai, Berakhot* chapter 2, in the last section, and R. Dovid Heber in *Shaarei Zemanim*, page 90, raise this fundamental issue with R. Feinstein's position.

³⁰ There are differences, as the *zemanim* provided by many websites adjust for seasonality as well. R. Belsky argues that R. Feinstein would have agreed to that change. See his approbation on the www.myzemanim.com website.

³¹ As we will see on other issues, practice can on occasion be explained to comport with entirely different conceptual positions. Thus, opinions on ending *Shabbat*, while radically divergent in theory, might vary less significantly in practice.

that keep 72 minutes invariant are consistent with both Rabbeinu Tam's viewpoint and how it has been practiced.³²

In his *sefer, Am Mordechai*, R. Willig, like R. Feinstein, takes as a given that latitude must be a determinant in applying the approach of Rabbeinu Tam as well. However, R. Willig follows that to its logical conclusion:

- The 72-minute practice in Lithuania was not, as R. Feinstein (and probably the vast majority of Orthodox Jewry) assumed, the correct practice of Rabbeinu Tam, but rather just the practice of the *geonim* applied at a location / latitude more than 50 degrees from the equator.

In R. Willig's approach, the logic of Rabbeinu Tam's opinion, regardless of the practice actually followed by Rabbeinu Tam or those who adhered to his position, must be coupled with the laws of astronomy as currently understood. Thus, the time needed to walk 4 *milin* that Rabbeinu Tam rules as the end of *Shabbat*, only applies around the spring and fall equinox and only at the latitude of Israel and Babylonia (approximately 30 degrees north latitude). Rabbeinu Tam's locale in France, as well as New York and Lithuania would require a significantly later end to *Shabbat*. In fact, R. Willig provides both a slightly shorter

³² While normally no precise latitude based adjustments were made to Rabbeinu Tam's position, approaches like that of R. Pimental, which use the appearance of stars, regardless of the length of time since sunset, to define the end of *Shabbat*, effectively do incorporate both season and latitude based adjustments.

(for those following the *geonim*) and a significantly longer (for those following Rabbeinu Tam) end-time to *Shabbat* consistent with the opinion and (private) practice of his mentor Rabbi Joseph Soloveitchik.³³ While their logic and application of basic astronomy to *zemanim* is impeccable, it hardly comports with practice. Interestingly, both R. Soloveitchik's personal stringency and R. Feinstein's ruling for the entire community follow Rabbeinu Tam. In addition, both agree that adjustments for latitude must be made. However, their conclusions are radically different. R. Feinstein ends up with a *pesak* that comports with tradition, albeit, employing logic that is, at least to this author, puzzling. R. Soloveitchik, on the other hand, followed (and refined) a family (*Brisker*) practice that is almost unprecedented, coupling a precise understanding of astronomy with a *halakhic* formulation rooted in Rabbeinu Tam's approach. As a result, R. Soloveitchik ended up waiting considerably longer for the end of *Shabbat* than either Rabbeinu Tam or the vast majority of his prior adherents who did not apply latitudinal adjustments to increase the length of time after sunset for Rabbeinu Tam's end to the period of *bein ha-shemashot*.³⁴

³³ R. Soloveitchik, as will be explained later, considered the time needed to walk 4 *milin* to be 90 minutes (the *Brisker achtel*, an eighth of a day in Yiddish) as opposed to the 72 minutes used by most *posekim*, and then adjusted 90 minutes by latitude and season. As a result, R. Soloveitchik maintained *Shabbat* with respect to Biblical restrictions until almost 2 and ½ hours after sunset in Boston around the summer solstice.

³⁴ In a *yarzeit shiur*, (*Shiurim Le-zekher Avi Mori*, volume 1, *yom ve-lailah*) R. Soloveitchik outlined a compelling, albeit highly non-traditional, approach that reworked Rabbeinu Tam to a position, consistent with astronomical observation. However, even this approach to Rabbeinu Tam must deal with the textual issues in reading the *gemara* in *Shabbat* that are discussed in section 7, a topic that R. Soloveitchik did not address.

INTRODUCTION: The primary *sugyot*, *Shabbat* 34b - 35a and *Pesaḥim* 94a, and their interrelationship; the basic opinions of the *geonim* and Rabbeinu Tam, and some of the fundamental challenges each position must address.

The *gemara* in *Pesaḥim* discusses whether the period between *alot ha-shaḥar* and sunrise (and the equivalent period from sunset to *tzait ha-kokhavim*) is the time needed to walk 4 or 5 *milin*.³⁵ The *gemara* in *Shabbat* describes the period of *bein ha-shemashot* in a variety of ways to be outlined below, and analyzed in detail in sections 5 – 8. However, when quantified using time, the *gemara* in *Shabbat* sets the length of the period of *bein ha-shemashot* at either the time needed to walk $\frac{2}{3}$ or $\frac{3}{4}$ of a *mil*. Although it is the *tanna* Rabi³⁶ Yehudah whose opinion is quoted in both *sugyot*,³⁷ the contradiction could be resolved, as some have suggested, by assuming that perhaps Rabi Yehudah changed positions. However, given the significant discrepancy, almost all commentators attempt to resolve the inconsistency by postulating that the two *sugyot* are addressing differing intervals.

³⁵ We often use the phrase “the time needed to walk X *milin*.” Undoubtedly, a person walking for an entire day will cover fewer *milin* per hour than one who walks for a shorter period. That adjustment is not how this system of specifying time-periods is defined. Instead, if one walks 32 *milin* in a daytime period of 12 hours, the time needed to walk a single *mil* is $(12 * 60 / 32 =)$ 22.5 minutes, despite the fact that walking only one *mil* takes significantly less than 22.5 minutes. Similarly, the number of *milin* covered in 90 minutes is $(90 / 720 =)$ $\frac{1}{8} * 32 \text{ milin} = 4 \text{ milin}$.

³⁶ I use “Rabi” to denote a *tanna* and “R.” for an *amora*.

³⁷ It is Rabi Yehudah quoted explicitly in *Pesaḥim* specifying that the period of *bein ha-shemashot* is $1/10^{\text{th}}$ of a day; in *Shabbat*, there is a dispute between Rabbah and R. Yosef concerning the length of Rabi Yehudah’s *bein ha-shemashot* period.

To ground this introduction, assume that the interval between *alot ha-shaḥar* and sunrise is the time it takes to walk 4 *milin*, and that the time needed to walk each *mil* is 22.5 minutes.³⁸ On a canonical day, around the spring and fall equinox, sunrise and sunset are at 6:00AM and 6:00PM respectively. Under these assumptions, according to the *gemara* in *Pesaḥim*, *alot ha-shaḥar* is (4 * 22.5 =) 90 minutes before sunrise, at 4:30AM, and *tzait ha-kokhavim* is 90 minutes after sunset, at 7:30PM.³⁹

The conceptual approach of Rabbeinu Tam posits that the endpoints of the *sugyot* in *Shabbat* and *Pesaḥim* are identical.⁴⁰ The end of *Shabbat* and the *bein ha-shemashot* period in *masekhet Shabbat* is at 7:30PM, 90 minutes after sunset. However, the beginnings of the periods differ between the *sugyot*. The *gemara* in *Shabbat* refers not to sunset, as we commonly refer to it, but a **secondary** sunset that occurs later, not just when the sun is no longer visible, but rather when almost all of its light is no longer visible as well. That occurs at 7:13:7.5PM, more than an hour after what we colloquially call sunset, and at that time, the period

³⁸ Both the length of time to walk a *mil* and the number of *milin* in the twilight interval are covered in detail in sections 1 and 2. Choosing either 22.5 minutes or the more typical 18 minutes is not consequential. I chose 22.5 minutes as the time needed to walk a *mil* because it is a much more broadly based opinion than most assume currently.

³⁹ A careful reading of Rabbeinu Tam (and other *posekim*) might align 6:00PM with the sun beginning to go below the horizon, a few minutes prior to the sun having gone completely below the horizon, which is our usual definition of sunset. Note that this makes the day at the equinox exactly 12 hour, while *ḥatzot* is a few minutes early. I have disregarded this and other minute differences.

⁴⁰ Rabbeinu Tam is likely to have been influenced by the *Yerushalmi* in the beginning of *Berakhot* and other *sugyot* that define *tzait ha-kokhavim*, the term used in *Pesaḥim*, as the appearance of three (medium) stars; in the *sugyah* in *Shabbat*, the appearance of three medium stars indicates the end of the day.

of *bein ha-shemashot* begins. Until that time, the day continues, and on Friday night, work is permitted.

The *geonim*⁴¹ take exactly the opposite position. In their formulation, the beginning points of the two *sugyot* are (almost and according to many commentators exactly) identical. Thus, the period of *bein ha-shemashot* begins at sunset.⁴² Shortly thereafter, at 6:16:52.5PM, *Shabbat* ends and work is again permitted. At 7:30PM, after the full 4-*milin* period of the *sugyah* in *Pesaḥim*, **all the stars** appear,⁴³ not just the three medium stars that signify the end of *Shabbat*.

Practice has at various times and for a variety of reasons somewhat softened both positions. Nevertheless, it is critical to recognize that the disagreement is significant. In many European communities, Jews who followed Rabbeinu Tam worked well after what we refer to as sunset on Friday evening; many Jews living in Israel and points closer to the equator followed the *geonim* and ended *Shabbat* within 30 minutes after sunset. At least in terms of *halakhic* theory, a period of approximately an hour, defining both the beginning and end of *Shabbat*, is in dispute.

⁴¹ The position of the *geonim* and the *Gaon* in this area are often viewed as identical. While the *Gaon* clearly identified sunset as the beginning of the period of *bein ha-shemashot*, the opinion of the *geonim* is less clear. See footnote 46.

⁴² I do not deal with the isolated opinion of the R. Eliezer mi-Mitz, the author of the *Yeraim* that posits that the interval in *masekhet Shabbat* begins approximately 15 minutes **before** sunset.

⁴³ The *Gaon* in *O. H.* 459 clarifies this by adding the word “*kol*” to *tzait ha-kokhavim*. *Tzait (kol) ha-kokhavim* in *Pesaḥim* is not the appearance of just three stars, but the much later appearance of all the stars. The potential *halakhic* consequences, if any, of the appearance of all the stars is discussed in section 9.

The approach of the *geonim* differentiates the *sugyah* in *masekhet Shabbat*, which equates the end of *Shabbat* with the time at which three stars appear, from the *tzait ha-kokhavim* of *masekhet Pesahim*, the time when all the stars appear. It is therefore intuitively obvious why according to the approach of the *geonim*, the period from *alot ha-shaḥar* to sunrise is identical in length to the period from sunset and the appearance of all the stars. **All the stars appear when no remaining light from the sun impair their visibility; equivalently *alot ha-shaḥar* is coincident with the first rays of light in the morning.** In the morning, as more light from the sun becomes visible, the number of stars that remain visible decreases; in the evening, the reverse occurs and as illumination from the sun disappears completely, all the stars (that can possibly be seen) become visible. An equal length of time between sunset and when light (however defined) disappears and again between the time light reappears and sunrise is a consequence of this symmetric definition. However, the approach of Rabbeinu Tam, which equates the *tzait ha-kokhavim* of *masekhet Shabbat* and the *tzait ha-kokhavim* of *masekhet Pesahim*, must deal with this issue of asymmetry. How can one equate

- the length of time between *alot ha-shaḥar*, when (almost) all of the stars are still visible, and sunrise, with
- the length of time from sunset to the appearance of only three stars?⁴⁴

⁴⁴ This issue is most fundamental. As will become clear, it is impossible to address the issue fully without radically changing some fundamental element of Rabbeinu Tam's conceptual opinion and / or how it is practiced. R. Soloveitchik did exactly

While the approaches of the *geonim* and Rabbeinu Tam are cited almost exclusively, it is critical to realize that other points of view are possible. The *gemara* in *Pesaḥim* defines a longer interval of 90 minutes. As normally assumed, the short interval of approximately a quarter of an hour that is defined by the *gemara* in *Shabbat* is mapped by the *geonim* to the beginning of the 90-minute interval, while according to Rabbeinu Tam, it maps to the end of the 90-minute interval. However, one must also consider **a hybrid or intermediate position where the interval in *masekhet Shabbat* begins somewhere between the points suggested by the *geonim* and Rabbeinu Tam, beginning after sunset at 6:00PM but before 7:13:7.5PM.**⁴⁵ The positions of Rabbeinu Tam, who equates the endpoints of the intervals in *Shabbat* and *Pesaḥim*, and the *geonim*, who equate the initial points, are two extremes. An intermediate position would place the smaller interval of the *gemara* in

that both in (personal) practice and in the theory developed in his *yarzeit shiur*. Absent so major a change in Rabbeinu Tam's approach, attempts to deal with this issue will introduce other complications. One representative issue is *ḥatzot*, when defined as the midpoint between *alot ha-shaḥar* and the appearance of only three stars. *Ḥatzot* will occur too early, not at the precise point when the sun is directly overhead. Many, as illustrated by R. Benish, simply disregard the issue or assume that *ḥatzot* need not be precise. *Ha-zemanim Be-halakhah* cites many examples of calendars from major Jewish communities, ostensibly endorsed by their rabbinic leaders, which would calculate *ḥatzot* assuming the appearance of three stars and dawn are symmetric endpoints. In a somewhat similar fashion, R. Feinstein's *teshuvah* allowing early morning prayers 90 minutes before sunrise (*Igrot Moshe O. Ḥ. 4:6*) and his *teshuvah* on nighttime *zemanim* (*Igrot Moshe O. Ḥ. 4:62*) that established 50 or 72 minutes after sunset as the end of *Shabbat*, are inconsistent and not cross-referenced. This fundamental issue is addressed in multiple contexts in this monograph.

⁴⁵ Alternatively, at 6:58:30PM, $\frac{3}{4}$ of the time needed to walk a *mil*, 13.5 minutes before *tzait ha-kokhavim*, if the interval is the more customary (4 * 18 =) 72 minutes, ending at 7:12PM.

Shabbat, the time needed to walk $\frac{3}{4}$ of a *mil*, somewhere within the much longer interval in *Pesaḥim*, the time needed to walk 4 *milin*. I raise this not just as a theoretical possibility. Rather, as will become evident, the strongest arguments that have been adduced in favor of the position of Rabbeinu Tam within the classical literature as well as those to be developed in this monograph, imply that the day continues after sunset delaying the **start** to the *bein ha-shemashot* period until sometime after sunset. Conversely, the greatest challenges to the opinion of Rabbeinu Tam result from his position that the **end** to the *bein ha-shemashot* period extends as late as the time needed to walk 4 *milin* after sunset. First, this requires that the astronomic conditions described in *masekhet Shabbat* that define the end of the *bein ha-shemashot* period are observable around the time it takes to walk 4 *milin* after sunset. Second, those conditions must be parallel to conditions that exist around *alot ha-shaḥar*. Apparently, neither is true.

Therefore, for purpose of analysis, I will separate the argument between the *geonim*⁴⁶ and Rabbeinu Tam into two parts:

- When does the period of *bein ha-shemashot* begin?
- When does the period of *bein ha-shemashot* end?

⁴⁶ What I call, and is commonly assumed to be, the position of the *geonim*, is more aptly the position of the *Gaon*. R. Posen throughout *Ohr Ha-meir* argues that while the *Gaon* himself considered sunset proper as the beginning of the *bein ha-shemashot* period, the *geonim* maintained a version of the intermediate position developed in this monograph. While a thorough analysis of R. Posen's position is beyond our scope, I concur with his assertion that the start of the *bein ha-shemashot* period according to the *geonim* begins after sunset proper. However, I do not believe it is quite as late as R. Posen maintains.

The length of the period (in the Middle East) is, according to all opinions, (perhaps as most currently assume only around the spring equinox) the time needed to walk $\frac{2}{3}$ or $\frac{3}{4}$ of a *mil*.

The text of the *gemara* in *Shabbat* contains three sections that must be analyzed carefully. Unlike Rabi Yosi, who considers the length of the *bein ha-shemashot* period as instantaneous, the blink of an eye, Rabi Yehudah, in a statement that the *gemara* rules as normative for the beginning of *Shabbat*, defines the period of *bein ha-shemashot* in terms of the appearance of the horizon. Rabi Yehudah's precise wording is challenged by the *gemara* as being inconsistent. It contains three initial phrases

- one referring to the setting sun,
- one referring to the sun's illumination and
- one referring to the darkening horizon,

and then a fourth phrase that defines the end of the *bein ha-shemashot* period. Rabbah, who approximates the period of *bein ha-shemashot* as the time needed to walk $\frac{3}{4}$ of a *mil*, assumes that all three initial phrases apply to the *bein ha-shemashot* period. The difficulty with this approach is the repetitive nature of the description, requiring Rabbah to explain that the phrases refer to the beginning and two intermediate points within the *bein ha-shemashot* period. The need for describing two

intermediate points is somewhat forced.⁴⁷ On the other hand, R. Yosef adds a word – “**daytime**” – and he explains that

- the initial phrase describes an interval after sunset that is still daytime,
- the second phrase describes a level of illumination just prior to the beginning of the *bein ha-shemashot* period and
- the third phrase describes a point during the period of *bein ha-shemashot*.

The difficulty with this approach is that the word “**daytime**” has to be added and must be assumed to be understood implicitly in Rabi Yehudah’s original formulation. This slightly shorter period for *bein ha-shemashot* aligns with R. Yosef’s position that maintains the length of the period is the time needed to walk only $\frac{2}{3}$ of a *mil*, $\frac{1}{12}$ th of a *mil* shorter than Rabbah’s $\frac{3}{4}$ of a *mil* interval.

The *gemara* then records a discussion between Abaye and Ravah debating whether one should be looking at the western or eastern horizon; this passage is critical and must be explained according to the interpretations of both the *geonim* and Rabbeinu Tam.⁴⁸

⁴⁷ Perhaps, as is subsequently debated by Ravah and Abaye, the two intermediate points are separately describing conditions in the eastern and western sky or perhaps earlier and later points during the interval of *bein ha-shemashot*.

⁴⁸ This debate between Abaye and Ravah is covered in section 7.

Both Rabbah and R. Yosef attribute their opinion of the position of the *tanna* Rabi Yehudah to the opinion of the *amora* R. Yehudah in the name of Shmuel. Later, the *sugyah*, without any suggestion of disagreement, quotes another statement of R. Yehudah in the name of Shmuel concerning 1, 2 and 3 stars. I address alternative interpretations of this statement and its relationship to the remainder of the *sugyah*, at length. That analysis is central to the viewpoint developed in this monograph.⁴⁹

With respect to the argument of Rabbeinu Tam and the *geonim*, a number of other *sugyot* are referenced in support of the *geonim*. One that is representative is the *gemara* in *Zevaḥim*, which asserts that (sacrificial) blood is disqualified at sunset, seemingly in opposition to the opinion of Rabbeinu Tam.⁵⁰ In such instances, one of two arguments can be invoked:

- The *halakhah* that specifies *precisely* sunset only applies to a unique situation, as in the case of *korbanot*.
- The language is less clear than one might assume, and sunset refers not its precise astronomical definition but to some later point when the sun's impact is diminished.

⁴⁹ This statement is analyzed in Section 5.

⁵⁰ *Tosefot's* answers in *Zevaḥim* 56a s.v. *menayin le-dam* and the discussion in *Minḥat Kohen, maamar rishon* chapters 4, 10, 11, 12 and 14 are prototypical.

As these *sugyot* have been extensively quoted and debated,⁵¹ it is unnecessary, except on rare occasion, to review them in this monograph. Instead, I concentrate on analysis of the basic *sugyot* mentioned.

One additional source that bears on these two *sugyot* is the lengthy discussion in the *Yerushalmi*, beginning in *Berakhot* 2b,⁵² that considers the verse in Nehemiah 4:15,

➤ *Ve-anaḥnu osim be-melaḥah.....mei-alot ha-shaḥar ad tzait ha-kokhavim,*

as defining the daytime period from *alot ha-shaḥar* until the appearance of three stars. Rabbeinu Tam is entirely consistent with this approach; in fact, the verse provides an explicit *sugyah* that links the appearance of three stars in the evening as the endpoint corresponding to *alot ha-shaḥar* in the morning. How the asymmetry is to be dealt with is very different, however, from the overall problem of asymmetry that Rabbeinu Tam must address. Unlike the *gemara* in *Pesaḥim*, nothing in the *Yerushalmi*, as well as the other texts that quote the verse in Nehemiah (even remotely) implies symmetry. Those *sugyot* that quote the verse in Nehemiah appear to support a definition of the daytime period that is asymmetric relative to both *ḥatzot* and the time from *alot*

⁵¹ *Minḥat Kohen* organizes and comprehensively covers the major *sugyot* on this issue.

⁵² Discussion of the verse occurs as well in more abridged form in the *Bavli*, *Megillah* 20a and *Berakhot* 2b.

ha-shaḥar to sunrise and sunset to *tzait ha-kokhavim*. While this asymmetry may be surprising, it is not, in and of itself, an issue. On the other hand, the *gemara* in *Pesaḥim* explicitly introduces symmetry by asserting equal intervals bracketing sunrise and sunset, each of the same duration, the time needed to walk either 4 or 5 *milin*. The opinion of Rabbeinu Tam further extends the symmetry of the *gemara* in *Pesaḥim* by equating the endpoints of the *sugyot* in *Shabbat* and *Pesaḥim*. By implication, Rabbeinu Tam equates the endpoint in the *Yerushalmi* as well. **For the *geonim*, however, the *gemara* in *Shabbat* (and the *Yerushalmi* in *Berakhot*) is addressing a different endpoint than the (later endpoint referenced in the) *gemara* in *Pesaḥim*.**

There are two approaches to this later point, referred to by the *Gaon* as *tzait kol ha-kokhavim*, when **all** the (possible) stars appear. According to the *Gaon*,⁵³ the *gemara* in *Pesaḥim* is theoretical; the evening equivalent of *alot ha-shaḥar*, 4 *milin* after sunset, when all the stars appear, is not ascribed any particular *halakhic* significance.

Alternatively, one can assume that while the appearance of three stars approximates or defines the end of each day of the week, the daytime period extends beyond that point, until all illumination from the sun has disappeared. That later point may apply to specific areas of *halakhah*, where only the daytime period as opposed to the specific day of the week is relevant, a theoretical possibility addressed in section 9.

⁵³ *O. H.* 261.

The *geonim* must deal with a number of questions. First, since it is universally acknowledged that the day starts with *alot ha-shaḥar*, should it not correspondingly end at an equivalent point after sunset, i. e., *tzait (kol) ha-kokhavim*? This question is easily addressed.

Symmetry is required only according to the approach of Rabbeinu Tam; for the *geonim*, given the two distinct definitions of day, the day of the week versus the daytime period, symmetry need not be expected.⁵⁴

Second,⁵⁵ and this question is fundamental, after an interval of the time needed to walk $\frac{3}{4}$ of a *mil* after sunset, in the Middle East, three medium stars are barely visible, if at all.

Rabbeinu Tam must address a number of fundamental questions. First, in the Middle East three medium stars appear well before the time needed to walk 4 *milin* after sunset. Second, the description of the horizon around the end of *Shabbat* as well as the debate between Abaye and Ravah all appear to support the opinion of the *geonim*.⁵⁶ Third, and most fundamentally, how could the time of the appearance of three stars and the time of *alot ha-shaḥar*, when all the stars are still visible, be identically separated (as specified by the *gemara* in *Pesaḥim*) from sunset and sunrise respectively? The analog to three stars becoming

⁵⁴ R. Soloveitchik in his *yartzeit shiur* previously referenced, developed and advanced an argument that clearly affirmed this type of symmetry as he reformulated the opinion of Rabbeinu Tam. However, as will be discussed, the way Rabbeinu Tam's position was practiced, did not always comply with this principle of symmetry.

⁵⁵ It is somewhat surprising that this question was not broadly discussed until the 20th century. Perhaps, consistent with our central thesis, *Sabbath* did not begin precisely at sunset as most now assume, even according to the *geonim*.

⁵⁶ This is covered in more detail in section 7.

visible is not *alot ha-shaḥar* when (almost) all stars are still visible.⁵⁷ This last issue and the debate between Ravah and Abaye present the largest challenges to the opinion of Rabbeinu Tam.

I return to these two *sugyot* while analyzing the primary *halakhic* categories below, attempting to formulate responses to the above questions throughout the monograph.

⁵⁷ As will be discussed further in detail, only approaches as that of R. Soloveitchik tackle fully the fundamental questions raised by symmetry with *alot ha-shaḥar* that is central to Rabbeinu Tam's position.

INTRODUCTION: Primary *halakhic* categories

1. How many *milin* does one walk during the period from sunrise to sunset? What is the time needed to walk a *mil*: 18, 22.5, 24 minutes, etc.?
2. How long is the period from sunset to *tzait (kol) ha-kokhavim* (or equivalently from *alot ha-shaḥar* to sunrise) in the Middle East around the time of the equinox: 72 minutes, 90 minutes, 96 minutes, 120 minutes, etc.?
3. How is the period from *alot ha-shaḥar* to sunrise or its equivalent period from sunset to *tzait (kol) ha-kokhavim* to be adjusted at different locations and during different seasons (if at all)?
4. How are we to define the hours of the day: from sunrise to sunset or from *alot ha-shaḥar* to darkness?
5. Of the three criteria given by the *gemara* in *Shabbat* – time, the appearance of stars and darkness (darkening / appearance of the sky / horizon), which, if any, are the definition of night, and which are just approximations or an indication? How can opinions expressed using these three terms be compared?

6. How is the duration⁵⁸ of the *bein ha-shemashot* period to be adjusted at different locations and during different seasons (if at all)? Might this depend on whether the period of *bein ha-shemashot* is:
- 1) a period of uncertainty that is its own unique *halakhic* category, either
 - a) a combination of both day and night, or perhaps
 - b) a category of its own, or
 - 2) a period with a definitive transition point that we are uncertain how to pinpoint, either
 - a) practically or perhaps
 - b) because of some element of *halakhic* uncertainty, or
 - 3) an example of the Rabbis establishing a fence?
7. When does the period of *bein ha-shemashot* end? How are the criteria specifying the end of the *bein ha-shemashot* period interpreted by various authorities?
8. How does the beginning of the *bein ha-shemashot* period relate to what we call sunset? What alternatives might be considered for the beginning of the *bein ha-shemashot* period?
9. How do the two meanings of day – day as in “day of the week” and day as in “during the daytime” relate? Must the end of the daytime period coincide with the end of a day of the week?

⁵⁸ This issue links to both categories 3 and 7. Placed here, it facilitates the flow of the monograph.

INTRODUCTION: Assumptions

- **Hours as they occur in a *halakhic* context with respect to times during the day are usually *shaot zemaniot*.** Thus, the length of an hour during the day varies with the length of the day calculated either from sunrise to sunset or *alot ha-shaḥar* to *tzait ha-kokhavim*. When the context is unclear, I will refer to standard hours as “clock hours.” Similarly, the term “day” normally is referring to the *halakhic* day of the week; on occasion “day” refers to the interval from sunrise to sunset.⁵⁹
- **Under all circumstances, *ḥatzot* must occur at the (precise) point when the sun is at the highest point in the sky and casts no shadow (the midpoint between what is nominally reported as sunrise and sunset).** This assumption is not universally accepted.⁶⁰ In fact, calendars that use *alot ha-shaḥar* and three stars as the endpoints for defining the daytime period and calculate the hours of the day based on observation of those endpoints always derive a point of *ḥatzot* that is earlier than midday.⁶¹ This deviation from

⁵⁹ Hours in a lunar month or *tekufah* are standard, clock hours. Other exceptions include the time one waits between meals and possibly, time used as *fence* where the *size of the fence* does not vary logically, e.g., *Trumat Ha-deshen* on the two-hour period before *ḥatzot* on *erev Pesah*. See Sacha Stern in *Time and Process in Ancient Judaism* (Oxford, England and Portland, Oregon, 2003), pages 51 – 53 and particularly footnote 23.

⁶⁰ *Minḥat Yitzḥak* 4:53 strongly supports the necessity to calculate *ḥatzot* accurately.

⁶¹ Note that an assumption of asymmetric endpoints is not, in any way excluded. The epilogue, when discussing *shaot zemaniot*, contains a conjecture that maintains a precise definition of *ḥatzot* despite asymmetric endpoints.

midday is apparent even at latitudes around the Middle East, increasing as one moves further from the equator.⁶²

- **The daytime period begins with *alot ha-shaḥar*.**
- ***Alot ha-shaḥar* always precedes *mi-she-yakir*, which occurs between *alot ha-shaḥar* and sunrise.**
- **The interval between *alot ha-shaḥar* and sunrise must exactly equal the time from sunset to *tzait ha-kokhavim* as the *sugyah* in *Pesaḥim* clearly states. If the daytime period begins at *alot ha-shaḥar*, symmetry suggests that, in some sense, it ends at *tzait (kol) ha-kokhavim*.⁶³**
- ***Pelag ha-minḥah* cannot (systematically) occur after sunset at latitudes around the Middle East.**

⁶² If both of these intervals are set to a fixed 72 or 90 minutes, this problem is avoided. However, as I will point out, this raises other issues with regard to *mi-she-yakir* at latitudes further from the equator.

⁶³ This must be reconciled with the verse in Nehemiah. As noted, the verse in Nehemiah 4:15, even as normally interpreted referring to the interval from *alot ha-shaḥar* until the appearance of three stars, makes no claim that those points are equidistant from sunrise and sunset respectively or how *shaot ha-yom* are to be calculated.

Introduction: Observable natural phenomenon⁶⁴

Variation by latitude: The interval between sunset and darkness, the assumed interval of *bein ha-shemashot*, increases as we move away from the equator. As one moves towards the poles, at some point, given varying definitions of darkness, it never turns dark during the summer. More generally, the degree of variation by latitude is a function of the precise level of darkness. Currently, the degree of darkness is measured using a depression angle, the number of degrees the sun must descend below the horizon to produce a particular level of darkness. A larger depression angle equates to a greater degree of darkness. As one goes further from the equator, the time required to reach a larger depression angle increases non-linearly.⁶⁵ In New York versus Jerusalem, for example, it is a longer time after sunset both absolutely and relatively before reaching a greater level of darkness. The reader should examine the table below for various dates and locations to see this explicitly.

⁶⁴ Readers who want a deeper understanding and some intuitive background on the relevant astronomy should consult *Ha-zemanim Be-halakhah* or Halakhic Times. For purposes of this monograph, I assume no familiarity with either astronomy or spherical trigonometry.

⁶⁵ The length of time between sunset and reaching an 8-degree depression angle, versus the time between depression angles of 4 degrees and 8 degrees or 6 degrees and 8 degrees, is less than two and four times as long, respectively. The percentage growth in the shorter intervals (from 4 to 8 or 6 to 8 degrees) is greater than in the longer interval starting from sunset (from 0 to 8 degrees). Concretely, if reaching a depression angle of 9 degrees occurs 54 minutes after sunset at some particular latitude, then the last degree (going from a depression angle of 8 to 9 degrees) will take more than its proportional share of 6 minutes.

| | | sunset | 6 ^o | min.later | 8.5 ^o | min.later | 16.1 ^o | min.later |
|-----|-----------|--------|----------------|-----------|------------------|-----------|-------------------|-----------|
| 20- | | | | | | | | |
| Mar | Jerusalem | 17:50 | 18:14 | 24 | 18:26 | 36 | 19:02 | 72 |
| | New York | 18:07 | 18:34 | 27 | 18:48 | 41 | 19:28 | 81 |
| | Prague | 18:14 | 18:46 | 32 | 19:02 | 48 | 19:50 | 96 |
| 21- | | | | | | | | |
| Jun | Jerusalem | 18:48 | 19:15 | 27 | 19:30 | 42 | 20:15 | 87 |
| | New York | 19:31 | 20:03 | 32 | 20:21 | 50 | 21:20 | 109 |
| | Prague | 20:16 | 21:00 | 44 | 21:25 | 69 | 23:29 | 193 |
| 21- | | | | | | | | |
| Dec | Jerusalem | 16:39 | 17:06 | 27 | 17:19 | 40 | 17:57 | 78 |
| | New York | 16:32 | 17:02 | 30 | 17:17 | 44 | 18:00 | 88 |
| | Prague | 16:02 | 16:40 | 38 | 16:58 | 56 | 17:49 | 107 |

TABLE A

Variation by season: At a given latitude, the interval between sunset and darkness is shortest around (but not exactly at) the spring and fall equinox, longer in the winter and longest in the summer. Some *posekim* inadequately address this variation. Some, perhaps considering the

variation inconsequential, do not explicitly deal with its impact.⁶⁶ Other *posekim* incorrectly assume that the length of the twilight interval varies with the length of sunrise to sunset period.⁶⁷ Thus, the interval is longer in the summer, which is directionally correct, albeit the ratio is incorrect. However, in the winter, the length of the twilight period is longer than in spring or fall. Interestingly, many *posekim* who assert this incorrect variation and assume the twilight period is shorter in the winter, fortunately compensate by only applying it only as a stringency in the summer.⁶⁸ The combined effect of both latitude and season is compounded for larger depression angles. The table illustrates this impact. For example, in Prague, using an 8.5 degrees depression angle to define the end of *Shabbat* equates to 48 minutes after sunset around the spring equinox and grows by around 43% to about 69 minutes around the summer solstice. In Israel, an 8.5 degrees depression angle equates to 36 minutes after sunset at the spring equinox and only grows by about 16% to 42 minutes in the summer. New York, whose latitude is approximately half way between Jerusalem and Prague the difference is approximately 10 minute, or 25%. Given the combination of non-linearities, particularly during the summer and further from the equator, larger depression angles are reached noticeably later, if at all.

⁶⁶ As illustrated in Table A, the impact of season is more consequential at latitudes further from the equator and for stricter definitions of darkness.

⁶⁷ Some have noted that paradoxically this would mean that in order to calculate *shaot zemanot* using the opinion of Magen Avraham, one would have to first calculate using the *Gaon's* endpoints.

⁶⁸ As we will see, this error provides unwarranted support to the position of Ramban and Magen Avraham.

Variation by altitude: At higher altitudes, the sun will appear to rise earlier and set later than at sea level. The impact in Jerusalem is a few minutes. How and if this variation is to be dealt with in *halakhah*, was, at least until recently, neither controversial nor that material.⁶⁹

⁶⁹ See *Igrot Moshe O. H.* 1:97 concerning the start of *Shabbat* in the mountains and the *gemara* in *Shabbat* 118b about Tiberius. If, when and how calendars based on sea level ought to be adjusted and differences between a mountain and an elevated plateau are not addressed. There is also significant literature, particularly in recent times, dealing with views obstructed by mountains at various distances. This topic, often phrased in terms of the ability to see the rising or setting sun, is also not covered.

Section 1. How many *milin*⁷⁰ does one walk during the period from sunrise to sunset? What is the time needed to walk a *mil*:- 18, 22.5, 24 minutes, etc.?

The *gemara* in *Pesaḥim* debates whether the interval between sunset and *tzait ha-kokhavim* or (*alot ha-shaḥar* to sunrise) is the time needed to walk 4 or 5 *milin*. Assuming⁷¹ the oft-quoted statement, “a person walks 40 *milin* in one day,” the majority of *rishonim* divide the 40 *milin* as follows:

- During each of the (equal) periods from *alot ha-shaḥar* to sunrise and sunset to *tzait ha-kokhavim*, one walks 4 or 5 *milin*.
- During the period from sunrise to sunset, one walks 32 or 30 *milin*.

Those who maintain a 5 *milin* interval, are left with a daytime period of the time it takes to walk 30 *milin*, while those who maintain a 4 *milin* interval, are left with a 32-*milin* daytime period. Given that a day around the spring (or fall) equinox is always assumed,⁷² the interval from sunrise to sunset is 12 hours or (12 * 60 =) 720 minutes. Dividing

⁷⁰ The time needed to walk a *mil* arises in multiple contexts including parameters for defining *ḥametz* and salting meat as well as multiple *zemanim* with which we deal. The famous gloss of the *Gaon* on the length of time to walk a *mil* is in *hilkhot Pesaḥ*.

⁷¹ An alternative to this assumption is considered in the next section.

⁷² Since the length of the day varies by season, (more dramatically as one moves further from the equator), it is universally accepted that the *gemara* in *Pesaḥim* must be referring to those days around the spring and fall equinox where the length of the day and night are (approximately) equal.

720 minutes by 30 yields a time to walk a *mil* of 24 minutes, which is the opinion of Rambam; dividing 720 minutes by 32 yields a time to walk a *mil* of 22.5 minutes, which is the opinion of many other *rishonim*.

Our text of the *gemara* would appear to conclude definitively that the interval between *alot ha-shaḥar and sunrise* is the time needed to walk 4 *milin*. However, the Mishnah in *Pesaḥim*⁷³ and the final *pesak* of Rambam, state that the sunrise to midday period equates to an interval of the time in which one walks 15 *milin*; hence the sunrise to sunset interval equates to the time needed to walk 30 *milin*. By implication, the *alot ha-shaḥar* to sunrise period would be the time needed to walk 5 *milin* or 120 minutes. I address Rambam's opinion in the next section.

While 22.5 and 24 minutes are currently mentioned on occasion, in the vast majority of cases, 18 minutes has become the prevailing *halakhic* view as, for example, in the "18 minute *matzah*." Although 18 minute as the time needed to walk a *mil* was adopted by the *Shulḥan Arukh* and most *aḥronim*, that position has only a few isolated references⁷⁴ prior to

⁷³ This assumes the uncontested interpretation of Ullah specifying the distance to Modiim as the time needed to walk 15 *milin*. How that might be reconciled with presently observed distances is not covered in this monograph.

⁷⁴ Rav Ovadiah Yosef, *Yeḥaveh Daat* volume 2, page 38, attempting to find support for the *Shulḥan Arukh* finds only a few early references, including however, R. Saadyah Gaon. Furthermore, those references are often indirect. Instead of a direct reference to 18 minutes, the reference is to 72 minutes normally expressed as 1 and $\frac{1}{5}$ th hours, which is assumed to be derived from 4 intervals of 18 minutes. That assumption, while highly likely, is not conclusive. Two alternatives to an 18 minute time to walk a *mil* as the definitive inference from a 72-minute interval are presented, one in this section and one in the next section. First, similar to the possibility raised by Prof. Levi's reading of *Leket Yosher* to be discussed more fully in this section, 1 and $\frac{1}{5}$ th hours can equate to 90 minutes if the *halakhic* hour in

Trumat Ha-deshen, written by R. Israel Isserlein around the beginning of the fifteenth century. R. Isserlein, in discussing the *halakhot* of *matzah*, describes the time needed to walk a *mil* as $\frac{1}{4}$ of an hour (15 minute) plus $\frac{1}{20}$ th of an hour (3 minutes). To reconcile a time to walk a *mil* of 18 minutes with the *gemara* in *Pesaḥim*, a second element has to be added to the debate. To this point, both sides in the 4 *milin* versus 5 *milin* debate agree that the 40 *milin* that an average person walks in a day occurs between *alot ha-shaḥar* and *tzait ha-kokhavim*. In order to derive 18 minutes, the periods from *alot ha-shaḥar* to sunrise and sunset to *tzait ha-kokhavim*, both lasting the time needed to walk 4 *milin*,⁷⁵ must each occur outside of the 40-*milin* period. Thus, a person walks 4 *milin* between *alot ha-shaḥar* and sunrise, 40 *milin* from sunrise to sunset and 4 *milin* from sunset to *tzait ha-kokhavim*. Dividing 720 minutes by 40 yields our 18-minute *mil*. Under this interpretation, the dispute in the *gemara* is twofold:

question is 75 minutes, counting the hours of the day from *alot ha-shaḥar* until *tzait ha-kokhavim*. Second, in the next section I will cover an interpretation of Rambam suggested by R. Schlesinger that derives 72 minutes independently of an 18 minute time to walk a *mil*. Only if we were able to rule out both of these options, does the implication of an 18 minute time to walk a *mil* automatically follow from a 72-minute interval.

⁷⁵ In addition to 4 *milin*, 5 *milin* is also theoretically possible, though that opinion is rarely found. Thus even if one asserts 5 *milin* is the distance walked from *alot ha-shaḥar* to sunrise, one cannot necessarily assume a time to walk a *mil* of 24 minutes, derived from 30 *milin* walked from sunrise to sunset. Additionally, one must rule out the possibility of a time to walk a *mil* of 18 minutes, if one were to maintain that 40 *milin*, as opposed to 30 *milin*, are walked during the sunrise to sunset period. Thus, in addition to the more usual 90 minutes thought of as 4 intervals of 22.5 minutes, 90 minutes may also be comprised of 5 intervals of 18 minutes. Similarly, as we will discuss in the next section, 72 minutes might also be 3 intervals of 24 minutes, in addition to the more usual 4 intervals of 18 minutes. These rare possibilities are often overlooked.

1. What period does the time needed to walk 40 *milin* refer to, sunrise to sunset or *alot ha-shaḥar* to *tzait ha-kokhavim*?
2. Is the *alot ha-shaḥar* to sunrise period and its equivalent in the evening the time needed to walk 4 or 5 *milin*?

The *Gaon*⁷⁶ strongly attacks *Trumat Ha-deshen*, but not because of the issue of 18 versus 22.5 minutes. Rather, the *Gaon* assumed that the 40 *milin* must mean *alot ha-shaḥar* to *tzait ha-kokhavim* (again limiting the *gemara* to only the second argument). Taking that as a given, he accuses R. Isserlein of calculating based on an assumption that at the spring equinox there are 12 hours from *alot ha-shaḥar* to *tzait ha-kokhavim*. Were that the opinion of R. Isserlein, it would be, as the *Gaon* asserts, unfathomable; during a day around the equinox, there are 12 hours between sunrise and sunset.

The *Gaon*, however, raises an interesting conjecture that provides a strong textual basis for supporting 18 minutes as the time needed to walk a *mil*. When contrasting 5 *milin* to 4 *milin*, the *gemara* uses the fractions $\frac{1}{6}$ th and $\frac{1}{10}$ th of the daytime period. The *gemara* rejects a 5-*milin* interval for *alot ha-shaḥar* to sunrise, which it refers to as $\frac{1}{6}$ th of a 30-*milin* day in favor of an interval corresponding to $\frac{1}{10}$ th of a day. Clearly, $\frac{1}{6}$ th means that the 5 *milin* walked between *alot ha-shaḥar* and sunrise is $\frac{1}{6}$ th of the 30 *milin* walked from sunrise to sunset. Assuming an identical meaning to the numerator and denominator, then $\frac{1}{10}$ th must mean that the 4 *milin* walked between *alot ha-shaḥar* and sunrise

⁷⁶ *Hilkhhot Pesah*, O. H. 459.

is $1/10^{\text{th}}$ of the 40 *milin* walked between sunrise and sunset. Since $1/6^{\text{th}}$ is clearly *mi-le-gav*,⁷⁷ by implication, $1/10^{\text{th}}$ is as well. Many later commentators assume that the *Gaon* himself ends up supporting a time to walk a *mil* of 18 minutes, perhaps influenced, among other factors, by how convincingly the *Gaon* expresses this comment. Others read the *Gaon* differently, i. e., the *Gaon* himself cites this interpretation of 40 *milin* referring to sunrise to sunset period only as the opinion of Rambam, who, in any case, apparently rejected reliance on the *sugyah* as normative.⁷⁸ Thus, despite the *gemara's* conclusion of 4 *milin*, Rambam maintained his 5-*milin* position. The *Gaon's* position on the length of time to walk a *mil* is at best unclear.⁷⁹ His argument, assuming like intervals in the denominator, would be more convincing if it were the

⁷⁷ *Mi-le-gav* (from the inside) is the standard way that fractions are currently expressed; e.g., $1/4$ of 12 = 3. The alternative is to say that $1/5^{\text{th}}$ of 12 is 3, meaning that if you add 3 to 12 and take $1/5^{\text{th}}$ of the sum, 15, the result is 3. Thus, the biblical term “*ve-ḥamishiso*” meaning a fifth is what our standard use would call a fourth; in biblical usage, $1/5^{\text{th}}$ of 10 is 2.5. This alternative is referred to as *mi-le-bar* or from the outside, meaning from the sum.

⁷⁸ See the next section for an alternative interpretation proposing that Rambam accepts both the Mishnah and the fraction $1/10^{\text{th}}$ as (*mi-le-gav* and) normative, and, as a result, rejects another statement in the *gemara*.

⁷⁹ Ironically, R. Willig in *Am Mordechai, Berakhot* chapter 2 assumes that an 18 minute time to walk a *mil* is not just the *Gaon's* position but also fundamental to his viewpoint. Others, including for example R. Feinstein in *O. H.* 4:6, assume that the *Gaon* maintained a 22.5 minutes position. R. Feinstein quotes the *Gaon* as support for his position allowing early morning prayers 90 minutes before sunrise in extraordinary circumstances, relying on a ($4 * 22.5 =$) 90-minute interval before sunrise for *alot ha-shaḥar* as opposed to the normative *pesak* of the *Shulḥan Arukh*. Two indications that the *Gaon's* position is 22.5 minutes are often cited. First, the *Gaon's* gloss to *O. H.* 261 gives the length of the interval from *alot ha-shaḥar* to sunrise, at latitude around the Middle East, as one and one half hours (90 minutes). Second, in *O. H.* 459 he specifies that the period from *alot ha-shaḥar* to *tzait kol ha-kokhavim* is 15 hours, two 90-minute periods added to the 12-hour daytime period. Support for those ascribing to the *Gaon* an 18 minute time to walk a *mil* derives from his commentary on *Berakhot*.

same person associated with both expressions. As that is not the case, and given the otherwise large number of *rishonim* who support 22.5 minutes, coupled with the fact that there is no hint in the *gemara* of a two-faceted disagreement, despite its widespread acceptance, R. Yaakov Reischer, the author of *Chok Yaakov*, and others⁸⁰ question 18 minutes as the time needed to walk a *mil*. Note however, that those who maintain 22.5 minutes as the time needed to walk a *mil*, must interpret 1/10th as a variant of *mi-le-bar*,⁸¹ 1/10th of 900, adding 90 not once but twice, corresponding to the two 90-minute intervals bracketing the 720 daytime minutes, in both the morning and the evening.

One additional area bearing on this argument is our text in the *gemara* that apparently contains an added “**vav**.” When bringing proof to Rabi Yehudah’s 1/10th, our text of the *gemara* reads “*teidah...“u”-me-alot ha-shaḥar*” in the second such phrase addressing the interval between dawn and sunrise. That “**vav**” would seem to imply that the 4 *milin* is **not** included within the 40 *milin*. It would be important to determine the precise text of the majority of *rishonim* who include the 4 *milin* within the 40 *milin*. Some early texts I have seen do not have the “**vav**.” This possible textual emendation might have been made to support or clarify the separate interval of the time needed to walk 4 *milin*, outside of the 40 *milin* walked during the 12-hour daytime period.

⁸⁰ Those include R. Sofer, many *Brisker* adherents of an “*achtel*” and Prof. Levi.

⁸¹ Normally we would say that 90 is 1/10th of 810 *mi-le-bar*, adding 90 minutes to 810, not to 720, and then taking 1/10th of 900. Given that in this case a 90-minute interval must be added twice, representing the morning and the evening intervals, we say that 90 is 1/10th of 720, adding 90 twice to 720.

Prof. Levi, quoting R. Yosef ben Moshe, a student of R. Isserlein and the author of *Leket Yosher*,⁸² claims that even R. Isserlein did not mean 18 minutes.⁸³ As discussed below, R. Isserlein measured the day from *alot ha-shaḥar* to *tzait ha-kokhavim*.⁸⁴ Arguing that R. Isserlein also agreed to 22.5 minutes as the time needed to walk a *mil*, he claims that the hour is not 60 minutes but a *halakhic* hour of 75 minutes. Adding 2 intervals of $(4 * 22.5 =) 90$ minutes for the *alot ha-shaḥar* to sunrise and sunset to *tzait ha-kokhavim* periods to the 720 minutes from sunrise to sunset, yields a daytime period of 900 minutes. In a 900-minute day, $1/12^{\text{th}}$ of the day is $(900 \text{ minutes} / 12 =) 75$ minutes. In a 75 minute hour, $1/4$ of an hour + $1/20^{\text{th}}$ of an hour is $(1/4 \text{ of } 75 =) 18.75 \text{ minutes} + (1/20 * 75 =) 3.75 \text{ minutes} = 22.5 \text{ minutes}$.⁸⁵ Accepting this interpretation of *Trumat Ha-deshen*, Prof. Levi concludes that the time needed to walk a *mil* of 18

⁸² Prof. Levi, *Halakhic Times*, Hebrew section, page 39, quotes *Leket Yosher*, in an attempt to demonstrate complete opposition to 18 minutes as the time needed to walk a *mil*.

⁸³ An almost identical argument is made in *Hok Yaakov O. H.* 459. However, *Leket Yosher*, page 79, is slightly less clear and could be read to assert that R. Isserlein assumed there are 12 clock hours from *alot ha-shaḥar* to *tzait ha-kokhavim*. In *O. H.* 459, The *Gaon* attacks any such possibility as being completely at variance with reality; I have a hard time believing anyone could have held such a view.

⁸⁴ Ironically, this is also debated as noted in *Ha-zemanim Be-halakhah* chapter 13, footnote 1, again quoting *Leket Yosher*, maintaining that R. Isserlein calculated the hours of the day from sunrise to sunset, as will be discussed further in section 4. It is tacitly assumed, that R. Isserlein's position is to begin counting from *alot ha-shaḥar*, as stated implicitly in the *Shulḥan Arukh* and explicitly in *Minḥat Kohen*.

⁸⁵ To formally complete the argument one solves for $(720 + 8X) / 12 * (1/4 + 1/20) = X$; the unique solution is $X = 22.5$. For example, if one attempts the same calculation with $X = 18$: $(720 + 8 * 18) / 12 * (1/4 + 1/20) = 864 / 12 * (1/4 + 1/20) = 72 * (1/4 + 1/20) = (18 + 3.6)$, not 18. Note that if one were to do the parallel calculation with 5 *milin*, with the time needed to walk a *mil* of 24 minutes, the sum of $(1/4 + 1/20)$ of an 80 minute hour, $((720 + 2 * 120 =) 960 \text{ minutes} / 12 = 80 \text{ minutes})$, would also equal 24 minutes.

minutes has no support among *rishonim*.⁸⁶ One can strongly question if R. Isserlein indeed meant an hour as 75 minutes. Though not conclusive as to its use in this context, the first *teshuvah* in *Trumat Ha-deshen* deals with starting *Shabbat* very early. In that responsa and others the term “hour” occurs repeatedly and clearly means a clock hour of 60 minutes.⁸⁷

In any case, the *Shulḥan Arukh* quoting *Trumat Ha-deshen* accepted 18 minutes as the length of time needed to walk a *mil*, and it is clear that during the 15th and 16th centuries 18 minutes became the accepted norm.⁸⁸

It should be noted that support from many *rishonim* for 22.5 minutes as the time needed to walk a *mil*, while conclusive, is indirect. Ideally, we would like to have 22.5 minutes expressed as $\frac{3}{8}$ of an hour (or equivalently as $\frac{1}{4}$ of an hour + $\frac{1}{8}$ of an hour) or by explicitly stating that one walks 32 *milin* in the daytime period from sunrise to sunset, from which we could derive 22.5 minutes by dividing 12 hours or 720 minutes by 32. Instead, starting with Ramban and with many

⁸⁶ Regardless of Prof. Levi’s radical position, it is unquestionable that 22.5 minutes enjoyed significant support throughout much of the 13th and 14th century among most *ḥakhmai Sefarad*. Prof. Levi would have to argue that each of R. Yosef’s examples of *geonim* and *rishonim* who he claims held 18 minute as the time needed to walk a *mil* could be differently interpreted, as noted previously.

⁸⁷ On the other hand, in *teshuvah* 121, concerning the latest time for *ḥametz* on *erev Pesah*, it is more likely that hour means $\frac{1}{12}$ th of the day from *alot ha-shaḥar* to *tzait ha-kokhavim*. Reconciling the multiple *teshuvot* of R. Isserlein either according to how he was understood by R. Yosef b. Moshe or in the *Shulḥan Arukh* is beyond the scope of this monograph. In a subsequent paper, I hope to demonstrate the correctness of the standard interpretation.

⁸⁸ Speculation on possible reasons for this change is addressed in the epilogue.

subsequent *ḥakhmai Sefarad* we can derive the length of the *mil* only indirectly, from their specification of the time of *pelag ha-minḥah*. These *rishonim*⁸⁹ assert that *pelag ha-minḥah* occurs at the time needed to walk $\frac{1}{6}$ th of a *mil* before sunset.⁹⁰ Since *pelag ha-minḥah* occurs 1 and $\frac{1}{4}$ hours before the end of the day, they must all define the hours of the day from *alot ha-shaḥar* to *tzait ha-kokhavim* (as opposed to sunrise to sunset).

Now consider a canonical 12-hour day around the spring equinox. If Ramban maintained a time to walk a *mil* of 18 minute, that would add 72 minutes to both ends of the day. *Alot ha-shaḥar* would be at 4:48AM and *tzait ha-kokhavim* at 7:12PM. A day would be 864 minutes long, and hence an hour would be 72 minutes. *Pelag ha-minḥah* occurs 1 and $\frac{1}{4}$ hours, $(1.25 * 72 =)$ 90 minutes, before 7:12PM at 5:42PM, exactly the time needed to walk one *mil* before sunset. Were one to agree with Ramban on calculating the hours of the day from *alot ha-shaḥar* to *tzait ha-kokhavim*, and thus calculate *pelag ha-minḥah* from *tzait ha-kokhavim* as opposed to sunset, while maintaining 18 minutes as the time needed to walk a *mil*, the above calculation would be entirely correct.⁹¹ However, since Ramban and others maintain that *pelag ha-minḥah* occurs the time needed to walk $\frac{1}{6}$ th of a *mil* before sunset, this cannot be their viewpoint; some parameter above must be wrong. The

⁸⁹ These *rishonim* include Ramban, Reah, Ritva, Rashba, etc.

⁹⁰ In an era before clocks, having two *halakhic* times defined so close to each other may be viewed as problematic. However, to these *rishonim*, sunset is not a particularly important *halakhic* time.

⁹¹ This is the assumed position of R. Isserlein, the *Shulḥan Arukh*, Magen Avraham, etc.

only parameter chosen was the length of time to walk a *mil*, so let us repeat the calculation with a time to walk a *mil* of 22.5 minutes. Now *alot ha-shaḥar* is at 4:30AM and *tzait ha-kokhavim* is at 7:30PM. The length of the day is 900 minutes and a *halakhic* hour is 75 minutes, and 1 and $\frac{1}{4}$ hours is $(1.25 * 75 =)$ 93 minutes and 45 seconds. Thus, *pelag ha-minḥah* occurs at 5:56:15 or 3 and $\frac{3}{4}$ minutes before sunset at 6:00PM. While not the expected 3 minutes that is $\frac{1}{6}$ th of 18 minutes, 3 and $\frac{3}{4}$ minutes is exactly $\frac{1}{6}$ th of 22.5 minutes. This represents conclusive proof that all *rishonim* who set *pelag ha-minḥah* at the time needed to walk $\frac{1}{6}$ th of a *mil* before sunset must maintain a time to walk a *mil* of 22.5 minutes. I will refer to the former and latter approaches as approaches 1 and 2 respectively.⁹²

Despite conclusive proof, due to an inadvertent arithmetic error, some⁹³ have tried to reconcile Ramban's assertion about *pelag ha-minḥah* occurring the time needed to walk $\frac{1}{6}$ th of a *mil* before sunset and 18 minutes as the time needed to walk a *mil*. They performed the above calculation using a third approach. Assuming that the length of time required to walk a *mil* is 18 minutes, *tzait ha-kokhavim* is, as before, at 7:12PM. *Pelag ha-minḥah* is 1 and $\frac{1}{4}$ hours⁹⁴ or 75 minutes before *tzait ha-kokhavim*, occurring at 5:57PM, three minutes, $\frac{1}{6}$ th of 18 minutes, before sunset.

⁹² For a formal proof that only a time to walk a *mil* of 22.5 minutes results in *pelag ha-minḥah* occurring $\frac{1}{6}$ th of the time needed to walk a *mil* before sunset, note that only $X = 22.5$ is a solution to the equation: $(720 + 8X) / 12 * 1.25 = 4X + \frac{1}{6} X$.

⁹³ *Orot Haim* and others.

⁹⁴ Seventy-five minutes is $(1 + \frac{1}{4}) * 60$ (clock) minutes.

R. Willig points out that this approach contradicts a *gemara*⁹⁵ and is, as a result, incorrect. The *gemara* maintains that there is a three-hour period between *minḥah gedolah* and *minḥah ketanah*. Using approach 2, an hour is 75 minutes and on our canonical day, *minḥah gedolah* is at 12:37:30 PM ($\frac{1}{2}$ of 75 minutes after noon). *Minḥah ketanah*, 2 and $\frac{1}{2}$ hours before *tzait ha-kokhavim* or 1 and $\frac{1}{4}$ hour before *pelag ha-minḥah* is at 4:22:30PM. The difference between *minḥah gedolah* and *minḥah ketanah* is 3 hours and 45 minutes (225 minutes) or exactly the length of three hours, each 75 minutes.

Similarly, using approach 1, *pelag ha-minḥah* occurs at 5:42PM and *minḥah ketanah* is at 4:12PM. In this approach, an hour is 72 minutes and *minḥah gedolah* is at 12:36PM. Thus, three hours, each 72 minutes, or 216 minutes again separates *minḥah gedolah* and *minḥah ketanah*.⁹⁶ However, as R. Willig points out, if we apply approach 3, subtracting 75 minutes from *tzait ha-kokhavim* to arrive at *pelag ha-minḥah* at 5:57PM, and then subtracting another 75 minutes to arrive at *minḥah ketanah* at 4:42PM, the difference from *ḥatzot* at 12:30PM is not 3 hours, but 4 hours and 12 minutes.

⁹⁵ See *Am Mordechai, Berakhot*, chapter 2. The *gemara* in *Berakhot* 26b specifies that the times of *minḥah gedolah* and *minḥah ketanah* are at six and half and nine and half hour respectively, a three hour difference.

⁹⁶ This also demonstrates that the time of *minḥah gedolah*, 30 minutes after *ḥatzot*, is not calculated using clock hours, at least for those who calculate *halakhic* hours from *alot ha-shaḥar* to *tzait ha-kokhavim*. It is less clear whether to employ a *shaah zemanit* according to those who calculate *halakhic* hours from sunrise to sunset; on a twelve-hour day, clock hours and *shaot zemaniot* coincide. Note that if the three hour period between *minḥah gedolah* and *minḥah ketanah* is meant to apply not just on a twelve hour day but all year round, it would imply that the use of *shaot zemaniot* for the calculation of *minḥah gedolah* is required even according to those who calculate from sunrise to sunset.

R. Willig's demonstrated inconsistency with a *gemara* is only the beginning of the problem. Not only does approach 3 contradict a *gemara*, it is untenable to count the hours of the day from *alot ha-shaḥar* to *tzait ha-kokhavim* at the spring equinox, and still assume a 60-minute hour. That would imply that the period from *alot ha-shaḥar* to *tzait ha-kokhavim* is 12 hours. This is precisely the blatant error that the *Gaon* attacks⁹⁷ when he assumed that it was the position of *Trumat Ha-deshen*. To illustrate one consequence of this approach, consider the simple task of calculating *ḥatzot*. You would expect that if we count back 6 hours from *tzait ha-kokhavim* or alternatively count forward 6 hours from *alot ha-shaḥar*, in both cases we would arrive at the identical point of *ḥatzot*. Note that both approaches 1 and 2 calculate *ḥatzot* at exactly 12:00 noon. Using approach 3, if we count back from *tzait ha-kokhavim*, we arrive at *ḥatzot* at 1:12PM, and if we count forward from *alot ha-shaḥar*, we arrive at *ḥatzot* at 10:48AM. The missing 144 minutes (72 minutes on both sides of 12 noon) is exactly the 12 minutes from each of the twelve hours of the day that this calculation omitted incorrectly. Approach 3 does not need to contradict a *gemara* to be wrong; it is both fundamentally illogical and provably incorrect, nothing more than an inadvertent arithmetic sleight of hand. To define the day from *alot ha-shaḥar* to *tzait ha-kokhavim* means **that you divide the length of time in that interval by twelve** to derive an hour. When you define the day from *alot ha-shaḥar* to *tzait ha-kokhavim* and assume a time to walk a *mil* of 18 minutes, you must calculate using approach 1

⁹⁷ *O. H.* 459.

above. Approach 3 that asserts both Ramban's position that *pelag ha-minḥah* occurs the time needed to walk $\frac{1}{6}$ th of a *mil* before sunset as well as 18 minutes as the time needed to walk a *mil*, proposed in multiple places, is fundamentally in error.⁹⁸

R. Pimental, despite his encyclopedic work covering almost all aspects of *zemanim*, never engages in this entire issue. He simply applies $1/10$ th to the daytime period of 720 minutes and derives a 72-minute twilight period and a time to walk a *mil* of 18 minutes.⁹⁹

In summary, there is definitive support for each of the three opinions. However, as a number of *aḥronim*¹⁰⁰ and more recently Prof. Levi, argue, a time to walk a *mil* of 18 minutes, though now the prevalent opinion, has limited direct support prior to R. Isserlein.¹⁰¹

⁹⁸ Another significant issue influencing the calculation of *ḥatzot* will be addressed when we cover defining the hours of the day. R. Feinstein (*Igrot Moshe O. H. 2:20*) insists that we do not calculate *ḥatzot*; it is the same time every day. R. Adler and others maintained (see *Ha-zemanim Be-halakhah* vol. 1, chapter 13, footnote 14) that the calculation of *ḥatzot* need not be astronomically precise. While this mistaken approach would result in an inaccurate calculation of *ḥatzot*, this rather grievous error is unlikely to have contributed to the above positions. Other potential rationales for those positions are addressed in the section on calculating the hours of the day, and more speculatively, in that section of the epilogue.

⁹⁹ One can only speculate about his lack of discussion of this issue, particularly because he quotes the position of a number of *ḥakhmai Sefarad* as supportive of the position of *Trumat Ha-deshen*. He does not draw the implication that they held that the time needed to walk a *mil* was 22.5 minutes from their position on *pelag ha-minḥah*.

¹⁰⁰ Among *aḥronim*, this appears to have begun most notably with R. Yaakov Reischer in his commentary *Chok Yaakov, O. H. 459*.

¹⁰¹ Interestingly, *Ḥatam Sofer*, who finds the arguments of R. Yaakov Reischer conclusive both in his commentary on *Shabbat* 35a as well as in *teshuvah* 80, points out at the end of his commentary on *Shabbat* 35a that some geographic distances described in the Talmud seem to support a time to walk a *mil* of 18 minutes.

Section 2. How long is the period from sunset to *tzait (kol) ha-kokhavim* (or equivalently from *alot ha-shaḥar* to sunrise) in the Middle East around the time of the equinox: 72 minutes, 90 minutes, 96 minutes, 120 minutes, etc.?

Given the prior discussion on the time it takes to walk a *mil*, it is normally assumed that for each alternative length of time to walk a *mil* there is a corresponding interval defined for the periods of *alot ha-shaḥar* to sunrise and sunset to *tzait ha-kokhavim*:

- A time to walk a *mil* of 18 minutes associates with a 72-minute interval.
- A time to walk a *mil* of 22.5 minutes associates with a 90-minute interval.

A 96-minute interval, 4 times the time needed to walk a *mil* of 24 minutes, is mentioned on occasion.¹⁰² However, it is unsupportable since to derive a time to walk a *mil* of $(720 / 30 =)$ 24 minutes requires an interval of the time needed to walk 5 and not 4 *milin* between *alot ha-shaḥar* and sunrise, in order for there to be an interval of the time needed to walk 30 *milin* during the sunrise to sunset period. Those who, in opposition to the conclusion of our text in the *gemara*, insist that the interval between *alot ha-shaḥar* and sunrise is the time needed to walk 5 *milin*, must maintain an interval of 120, not 96 minutes.

¹⁰² See *Igrot Moshe O. H. 4:62* and *Melamaid Le-hoil 30*.

Interestingly,¹⁰³ in Israel at the spring equinox, scientists have determined that the sun produces no measurable light approximately 80 minutes after sunset, corresponding to a depression angle of approximately 18 degrees. Since the *halakhah* often disregards minuscule, non-visible quantities, this provides observational support for the classic 72-minute period (and by implication for the time needed to walk a *mil* of 18 minutes). In any case, 72 and 90 minutes are both reasonably close approximations to this scientific observation.¹⁰⁴

However, taking for granted Rambam's astute observation of the natural order, and given his position of only 30 *milin* travelled between sunrise and sunset, his assumed 120-minute period from *alot ha-shaḥar* to sunrise is puzzling. It is difficult to imagine that Rambam could have added a full 40 minutes prior to the earliest point of any possible light being visible in the Middle East. Unexpectedly, rather than maintain a 120-minute interval, Rambam in *Peirush Ha-mishnayot*¹⁰⁵ asserts a 72-minute interval.

This apparent contradiction has not been resolved definitively. It is highly unlikely¹⁰⁶ that Rambam simply changed his mind about the time needed to walk a *mil*, since he maintains the 24-minute interval both in

¹⁰³ *Ha-zemanim Be-halakhah*, chapter 16, provides necessary background.

¹⁰⁴ R. Boorstyn, in *Zemanim Ke-hilkhatam*, suggested that the 90-minute period should not be used where it would create a leniency, since it is counter-indicated by current scientific observation that cannot discern any light at so early a point. While I might be accused of apologetics, it may be the case, that at 90 minutes before sunrise people begin their day in anticipation of visible light about 20 minutes later..

¹⁰⁵ See the first Mishnah in *Berakhot*.

¹⁰⁶ The assumption of R. Karelitz; see section 13 of *Ḥazon Ish O. Ḥ*.

his commentary on the Mishnah as well as in *Mishnah Torah*. Some of the efforts to resolve this apparent contradiction are themselves instructive.

First, the *Gaon*¹⁰⁷ in a different context suggests that Rambam was giving *shiurim* that were to apply at the equator. The *Gaon* uses this possibility to explain a smaller discrepancy. In our context, this would remain problematic as 72 minutes at the equator adjusts to less than 90 minutes in the Middle East, and certainly not 120 minutes.

Second, R. Sofer¹⁰⁸ suggests that perhaps when Rambam wrote 72 minutes he meant the interval from a slightly later point to sunrise and not from *alot ha-shaḥar*, and that later point occurs the time needed to walk $\frac{3}{4}$ of a *mil* after *alot ha-shaḥar*.¹⁰⁹ This approach also only addresses the use of 72 minutes versus 90 minutes; a 90-minute interval that R. Sofer ascribes to Rambam, is rarely attributed to Rambam. Furthermore, R. Sofer's approach faces two other issues. First, this explanation assumes that Rambam's terminology was imprecise writing *alot ha-shaḥar* while meaning later point. Second, even using 90 minutes¹¹⁰ and a 22.5-minute¹¹¹ time to walk a *mil*,

¹⁰⁷ *O. H.* 261 and *Y. D.* 262.

¹⁰⁸ *Hiddushim al Shas, Shabbat* 34a.

¹⁰⁹ Perhaps, R. Sofer was trying to establish a morning analogue to *bein ha-shemashot*, something that R. Weiss objects to in *Minḥat Yitzḥok* 4:53, paragraph 18, and attributes to R. Pimental in *Minḥat Kohen*.

¹¹⁰ While I have never seen 90 minutes and 120 minutes interchanged, I have seen $\frac{3}{8}$ th of an hour or 22.5 minutes and $\frac{2}{5}$ th of an hour or 24 minutes used as if they were identical. While this is not critical to the issues discussed, readers should be aware of this occasional occurrence.

subtracting $\frac{3}{4}$ of 22.5 from 90 leaves 73 and $\frac{1}{8}$ th minutes, to which 72 minutes is only an approximation, a point R. Sofer makes. Ascribing imprecision in both language and arithmetic to Rambam, as well as a change in his overall position, is less than convincing.

R. Y. Schlesinger¹¹² suggested a third, rather novel approach. In addition to explaining the 72-minute interval, it also explains Rambam's rejection of the *sugyah* in *Pesahim*. R. Schlesinger suggests that the *gemara* could not possibly mean to question the distance from Modiim to Jerusalem; it was a known distance, not subject to debate.¹¹³ Hence, Rambam took it as a non-debatable statement of R. Akiva in a Mishnah that the length of time from sunrise to *ḥatzot* was the time needed to walk 15 *milin*. Rambam also took as the final view of the *gemara* that the ratio of the *alot ha-shaḥar* to sunrise period to the entire day was 1:10, based on the statement of another *tanna* Rabi Yehudah. **If one assumes that both of these assertions of *tannaim* are normative, then approximating the length of the day as the time needed to walk 40 *milin* must be discarded.** The distance that a person walks in one day would then be the 30 *milin* during the sunrise to sunset period in addition to two intervals of the time needed to walk 3 *milin* ($1/10$ th of

¹¹¹ 90 minutes equates to 5 times 18 minutes, a position rarely seen, or 4 times 22.5 minutes, the very common position of many *rishonim*. A time to walk a *mil* of 24 minutes, which would make subtracting ($\frac{3}{4} * 24 =$) 18 minutes from 90 minutes precise, is incompatible with 90 minutes.

¹¹² *Yeshuran* 5695.

¹¹³ Ironically, the distance from Jerusalem to Modiim (if one assumes the Arab town of Al Midya) is approximately 16 miles. Depending on how one defined a *mil*, it would favor either of the other two opinions on the distance walked in 6 hours. These geographic issues are not addressed in this monograph.

30) - one in the period between *alot ha-shaḥar* and sunrise and a second between sunset and *tzait ha-kokhavim*. Prior to R. Schlesinger's article, I know of no one who ever suggested disagreement with the commonly assumed assertion that the average person walks 40 *milin* in a day. Perhaps more troubling than its originality, is Rambam's own ruling in *Hilkhot Aveilut* 7:4 that *derekh reḥokah*, a distant place that that cannot be reached in one day's walk, is 40 *milin*. Not to be so easily defeated, R. Schlesinger suggests that while for standard purposes Rambam uses a 36 *milin* distance for what one can typically walk in a day, Rambam's use of 40 *milin* is just an application of the principle of deciding leniently with respect to the laws of *aveilut*.^{114 115} Note that this approach, while rejecting both the conclusion of the *gemara* that the twilight period is the time needed to walk 4 *milin*, as well as the normally assumed distance traveled in a day of 40 *milin*, restores the similarity of the fractions $\frac{1}{6}$ th and $\frac{1}{10}$ th, noted by the *Gaon*.¹¹⁶

Other than the difficult position of R. Karelitz, which claims that Rambam changed his mind, I know of no other way to explain

¹¹⁴ It is also possible that a person hurrying to join the other *aveilim* will travel more than 30 *milin* in a day.

¹¹⁵ R. Willig cites a proof from the *Orḥot Haim* that purports to prove the 18 minute time to walk a *mil* conclusively from a similar *pesak* by the Rif concerning *aveilut*. The fact that Rambam in all likelihood did not maintain an 18 minute time to walk a *mil* and nonetheless records a similar position to that of Rif, makes the proof less conclusive. An alternative interpretation of both Rambam and Rif is discussed in section 9.

¹¹⁶ As mentioned at the end of the last section, generations prior to R. Schlesinger's approach, R. Pimental simply applies $\frac{1}{10}$ th to the daytime period of 720 minutes and derives a 72-minute twilight interval.

Rambam's 72 minutes.¹¹⁷ It is ironic that Ramban and other *ḥakhmai Sefarad* who support Rabbeinu Tam and are often associated with a 72-minute interval maintained a 90-minute interval. Thinking of 72 minutes as 4 intervals of 18 minutes and / or as 1/10th of the day is almost universally accepted from at least the time of the *Shulḥan Arukh*; thinking of 72 minutes as three intervals of 24 minutes was to my knowledge never mentioned prior to R. Schlesinger.

¹¹⁷ In practice, a 120-minute period has few definite followers other than R. Ovadiah mi'Bartenra. Some have suggested R. Shneur Zalman of Liadi, who suggests 3 hours before sunrise as the beginning of *asarah be'tevet* and *ḥatzot ha-lailah* as time of *alot ha-shaḥar*, after which *sefirah* should not be counted *with a berakhah* closer to the summer. However, neither *zeman* implies a 120-minute period. His use of *ḥatzot ha-lailah* as the time for *alot ha-shaḥar* is likely just a latitudinal adjustment of 72 minutes during the summer at his very northern location, approximately 53 degrees north latitude. At that latitude, *alot ha-shaḥar* is never less than 100 minutes before sunrise. His use of 3 hours before sunrise as the beginning of the fast of *asarah be'tevet*, is too short an interval if he were adjusting a 120-minute interval. R. Menahem Mendel Schneerson and others argue that a different *zeman* for the start of the fast is being used. See *Ha-zemanim Be-halakhah*, chapter 21, footnotes 39-43, where the position of R. Shneur Zalman of Liadi is examined in detail.

Section 3. How is the period from *alot ha-shaḥar* to sunrise or its equivalent from sunset to *tzait (kol) ha-kokhavim* to be adjusted at different locations and during different seasons (if at all)?

Despite our habitual focus on clocks and time, it is important to stipulate that both intervals are likely defined by physical events to which a period of time is associated, rather than by time itself.

To apply these *zemanim* at other latitudes and during other seasons of the year, we can utilize depression angles, by first establishing the number of degrees below the horizon, that the sun is located 72, 90 and 120 minutes after sunset (or equivalently before sunrise) in the Middle East, around the spring (and fall) equinox.¹¹⁸ The shorter 72-minute interval corresponds to the sun being approximately 16 degrees below the horizon, while the longer 90-minute interval corresponds to a depression angle of approximately 20 degrees.

As noted earlier, we normally adjust times associated with the opinion of the *geonim's* end to *Shabbat* to correlate to observed physical events.

There are five possible approaches for calculating *alot ha-shaḥar* as well as its nighttime equivalent, which is of interest at least according to the opinion of Rabbeinu Tam:

¹¹⁸ The location chosen is Jerusalem, although it should not be assumed that statements in the Talmud always refer to Jerusalem. It would be of minimal consequence if instead another location in Babylonia were assumed.

1. Uniformly adjust using depression angles to correlate to observed physical events.
2. Make no adjustments and use the intervals of the Middle East (Jerusalem) at the spring and fall equinox uniformly on a global basis.
3. Adjust the times based on season in the Middle East and then apply globally.
4. Adjust based on latitude but assume the interval is (treated as) constant, independent of seasons.
5. Adjust the twilight interval under the assumption that it varies linearly with the length of the day from sunrise to sunset.

Despite the fact that approach 3 appears preferable to approach 2, I know of no instance where it was ever used or even suggested.

Approach 4 results either from a *posek's* desire to simplify approach 1 and use a fixed, conservative interval uniformly throughout the year,¹¹⁹ or approach 4 might be the result of a *posek* not realizing the impact of seasonal variation. In latitudes as far north as New York, the variation is only about 15 minutes. I deal with approach 5 later in this section; it is significantly at variance with observation.

With minor exception, only approaches 1 and 2 are used in practice. Approach 2, which treats a number of minutes as defining as opposed to only providing an approximation, is implausible and is in all likelihood a

¹¹⁹ A number of rabbis, well versed in this area, have expressed the need to “keep practical *pesak* simple.”

result of the popularity of clocks in modern times. Both the *sugyah* in *Pesahim*, which uses the term *oveyo shel rakia*, and all of *masekhet Tamid*, which refers only to the appearance of varying levels of illumination, argue strongly against a “time to walk 4 or 5 *milin*” as definitional.¹²⁰ The *Gaon’s* clear support for adjustment, given his assertion that *alot ha-shaḥar* and its evening equivalent do not occur during the summer at very northern latitudes, is rarely quoted. Regardless, in every case it is critical to avoid two pitfalls:

1. Making some adjustments based on physical observation and leaving other times fixed. Invariably this will lead to some anomaly, often an observable challenge to the assumptions that were stipulated.
2. While attempting to make adjustments to correlate to observed physical events, inadvertently do so incorrectly, as for example in approach 5 above.

Some calendars and *posekim* have not avoided one or the other of these pitfalls.

Despite a strong preference for *zemanim* that equate to physical observations, many still argue in favor of an unadjusted interval; perhaps we simply use intervals around sunrise and sunset that equate to the duration of those intervals in Israel (and only in the spring and

¹²⁰ One might argue that the use of clocks and their ostensible accuracy contributed to this type of anachronistic reasoning.

fall). Some defend this practice either based on the eventual breakdown of any methodology as one approaches the north and south poles, or by an appeal to tradition that in many cases made no adjustments. Thus, at least in theory, one could not begin the day prior to 72 or 90 minutes before sunrise, regardless of the amount of light that is present; similarly, *Shabbat* might end once a fixed 72 or 90-minute interval has passed, regardless of the number of stars visible. Again, I find such an approach entirely implausible; you might ask how such an approach would have been practiced prior to the widespread use of clocks?

For *tzait ha-kokhavim*, calendars commonly provide Rabbeinu Tam's normally assumed 72-minute interval, rarely publishing latitudinal and seasonal adjustments. However, when calendars publish the times for *alot ha-shaḥar*, often a seasonal and latitudinal adjustment of 72 minutes is (also) given. This inconsistent treatment is required to avoid *mi-she-yakir* occurring before *alot ha-shaḥar*. For standard definitions of *mi-she-yakir*, in areas further north of the equator, *mi-she-yakir* will occur more than 72 minutes before sunrise.¹²¹ As an example of the consequence of making neither adjustment, leaving both *alot ha-shaḥar* and *tzait ha-kokhavim* fixed at 72 minutes, consider the calendar entry taken from the OU website, a number of years ago, for London on June 22, 2007:

¹²¹ I do not cover the precise definition of *mi-she-yakir*, but it typically equates to a depression angle of approximately 11.5 degrees, with *posekim* in northern Europe typically implying a smaller angle (more light) and some *posekim* in the Middle East implying a larger angle. Whether *mi-she-yakir* is considered more than simply a *harḥakah*, or how its multiple physical definitions relate, is not covered.

| | |
|-------------------------|------------------------|
| Alos | 3:35 A |
| Talis | 3:01 A |
| Sunrise | 4:47 A |
| Sof Zman Shema | 8:56 A (GRA) |
| | 8:20 A (MA) |
| Sof Zman Tefila | 10:19 A (GRA) |
| | 9:55 A (MA) |
| Chatzot (Midday) | 1:05 P |
| Mincha Gedolah | 1:46 P |
| Mincha Ketana | 5:55 P |
| Plag Mincha | 7:39 P |
| Sunset | 9:23 P |
| Tzeis Hakochavim | 10:36 P (Gaonim) |
| | 10:34 P (Rabbeinu Tam) |

As a result, a common, but highly questionable practice is using an unadjusted 72 minutes before sunrise as the start of fast days at latitudes farther from the equator than Israel, and especially for fast days like the 17th of Tammuz, occurring in the summer. In the New York area, the use of 72 minutes without adjustment allows eating when it is, according to some opinions, approximately the point of *mi-she-yakir*.¹²²

¹²² Of course, this depends on the precise definition of *mi-she-yakir*.

Alot ha-shaḥar is always a critical *zeman* as it is relevant according to all opinions. Its analogue in the evening is relevant only for those who follow the Rabbeinu Tam. While equating both *alot ha-shaḥar* and *tzait ha-kokhavim* to depression angles has many adherents, practice in this area remains inconsistent. **Adjusting *alot ha-shaḥar* for latitude and season but not *tzait (kol) ha-kokhavim* has no conceptual rationale.** Despite that, many calendars¹²³ abandon logical consistency, and give the morning times adjusted while leaving the nighttime a constant 72 minutes. The OU website makes neither adjustment, and ends up with *mi-she-yakir* (denoted by the line “*talis*”) occurring before *alot ha-shaḥar*. In particular, those who calculate the hours of the day from *alot ha-shaḥar* to *tzait ha-kokhavim*¹²⁴ often make neither adjustment and simply add / subtract 72 (or 90) minutes to / from sunset and sunrise.

Some, who acknowledge the need to make adjustments to both periods consistently, have made technical errors in calculating the adjustments. Two errors are most common. First, some assume that the period of *alot ha-shaḥar* to *tzait ha-kokhavim* varies with the length of the day from sunrise to sunset.¹²⁵ Second, some calculate using *alot ha-shaḥar*

¹²³ A calendar commonly posted in many synagogues by R. Mordechai Premock is representative. On the other hand, www.myzmanim.com uses depression angles consistently.

¹²⁴ This is discussed further in the next section.

¹²⁵ This position is widespread; see for example R. Schachter’s notes on R. Soloveitchik’s *shiurim* on *masekhet Pesachim*, pages 14 – 15, where this position is attributed to R. Soloveitchik’s grandfather. R. Haim Soloveitchik also interpreted Rambam as specifying the twilight period as 120 minutes and acted that way as a personal *ḥumrah*. This opinion, referred to as a *szekstel*, a sixth (120 / 720) of a day in Yiddish, was supplemented a generation later by the *Brisker achtel*, an eighth of a

and three stars as equivalent endpoints. Both of these positions can be traced at least as far back as the 17th century to R. Pimental, in *Minḥat Kohen*.¹²⁶

This first error adjusts the interval from sunset to *tzait ha-kokhavim*, and equivalently the interval from *alot ha-shaḥar* to sunrise, proportionally to the length of the day between sunrise and sunset. A number of *aḥronim* adopt this approach.¹²⁷ The 72 or 90 minutes that, by observation, is longer in the winter, is actually shortened in the case of an 8-hour day by 8/12 or 2/3. As a result, 72 minutes becomes 48 minutes and 90 minutes becomes 72 minutes. A time to walk a *mil* of 22.5 or 18 minutes is similarly “reduced” by 2/3 to 15 minutes and 12

day (90 / 720). I assume that R. Soloveitchik differed with his grandfather on two counts, first, using the *achtel* and second, using adjustments based on depression angles not *shaot zemaniot* measured from sunrise to sunset.

¹²⁶ Two problematic opinions of R. Pimental are referenced in this monograph. His masterful *sefer* was the first comprehensive treatment of the overall subject of *zemanim*. To the best of my knowledge, he was the first rabbinic author to discuss the impact of latitude and season, albeit with errors. It is a testament to his influence, that his concepts became widely accepted, unfortunately including the few problematic opinions he held.

¹²⁷ See *Minḥat Kohen* and R. Pimental’s precise calculation based on this approach for the end of *Shabbat* in the summer in the Netherlands. R. Pimental assumed that on a 12-hour day, three small stars appeared in the Netherlands 48 minutes, 1/15th of the (720-minute) day, after sunset. Similarly, on a 16 and 1/2 hour summer day (990 minutes) he observed that three small stars appeared 66 minutes after sunset, again 1/15th of a day. Unfortunately, he seems to have generalized (to a linear equation, correlating 1/15th of the daytime period between sunset with the appearance of three stars) based on these observations, apparently without verifying during the winter months. As is the case with almost all who make this error, he did not apply this approach as a leniency in the winter months. R. David Tzvi Hoffman, (*Melamid Le-hoil* 30), is among the first to have questioned this mistaken approach explicitly.

minutes, respectively.¹²⁸ When this adjustment is used the previous times for both approaches 1 and 2 for calculating *pelag ha-minḥah* are exactly maintained.¹²⁹ Thus, the 90-minute interval from sunset to *tzait ha-kokhavim* reduces to 60 minutes and the *alot ha-shaḥar* to *tzait ha-kokhavim* period is exactly 600 minutes, occurring at 7:00AM and 5:00PM respectively. Continuing with a 50-minute hour, (the time from *alot ha-shaḥar* to *tzait ha-kokhavim* divided by 12), *pelag ha-minḥah* occurs at 3:57:30PM, 2.5 minutes before sunset, which is exactly $\frac{1}{6}$ th of an “adjusted” ($\frac{2}{3} * 22.5 =$) 15 minute *mil*. Likewise, 72 minutes is reduced by $\frac{1}{3}$ rd to 48 minutes; *pelag ha-minḥah* is at 3:48PM, exactly the time needed to walk one “adjusted” *mil* of ($\frac{2}{3} * 18 =$) 12 minutes before sunset. This creates *zemanim* that do not correlate with reality. Specifically, when applied in the morning, *alot ha-shaḥar* occurs well after *mi-she-yakir*.¹³⁰

A second problematic approach uses *alot ha-shaḥar* and *tzait ha-kokhavim* to calculate the hours of the day. Violating two of our assumptions above, using this approach sets *ḥatzot* to a point before

¹²⁸ Of course, the time needed to walk a *mil* is not adjusted; the interval is shortened and referred to imprecisely as an “adjusted” *mil*. Actually shortening the time needed to walk a *mil* and then applying a shortened time to walk a *mil*, has also been suggested; such an approach is completely unwarranted, as will be discussed. Our language, imprecisely referring to a “shortened” *mil*, reflects what is sometimes found in the literature and is misleading; hence, the use of quotation marks.

¹²⁹ This type of apparently confirming arithmetic may well have provided further support for a factually untenable approach. Equivalence resulting from nothing more than basic arithmetic is mistakenly taken as confirmation of a *halakhic* position.

¹³⁰ The period from sunset to *tzait ha-kokhavim* is also too short. However, since this approach was never used to create a leniency and as discussed below, many assumed that three stars are defining in the evening, the error had no real consequence.

midday, since *alot ha-shaḥar* occurs longer before sunrise than *tzait ha-kokhavim* occurs after sunset. As well, an interval between *alot ha-shaḥar* and sunrise that is longer than the interval between sunset and *tzait ha-kokhavim* conflicts with the *sugyah* in *Pesaḥim*.¹³¹ R. Pimental, who adjusts Rabbeinu Tam to when three stars are actually observed, provides implicit support for this approach. As indicated, this approach is untenable as *Shabbat* ends earlier in the Netherlands than in the Middle East. Since Rabbeinu Tam equates the endpoints of the *sugyot* in *Pesaḥim* and *Shabbat*, its adherents then used an observed *alot ha-shaḥar* as the other endpoint to three stars for calculating the times of the day. This approach was widespread throughout Europe. R. Tukitzinsky tells the history of how the calendar in Jerusalem was calculated this way until it was changed.¹³²

¹³¹ Independent of the argument of the *geonim* and Rabbeinu Tam, and the strong implication from the *gemara* in *Pesaḥim* that asserts equivalent intervals for the morning and evening twilight period, the *Yerushalmi* in *Berakhot* mentioned earlier, that discussed *alot ha-shaḥar* and the appearance of three stars, is often incorrectly cited as justification for this approach. As I have noted, while the *Yerushalmi* delimits a day by those endpoints, in no sense does it declare them equidistant from either *ḥatzot* or from sunrise and sunset.

¹³² *Ha-zemanim Be-halakhah* quotes a number of accounts varying in minor details concerning the circumstances around this change. The Jerusalem calendar used during the last century uses *tzait kol ha-kokhavim* as opposed to just three stars. See *Minḥat Yitzḥak* 4:53 as well, for a lengthy discussion where he concludes that any approach that does not calculate *ḥatzot* accurately is, by definition, flawed. The approaches that used *alot ha-shaḥar* and three stars shortened slightly the *halakhic* hour, and thereby created a minor (unwarranted) *ḥumrah* for morning *zemanim*, in addition to an early point of *ḥatzot*. Footnote 51 in the epilogue elaborates on this point in detail. This and his other more famous dispute over the dateline (and perhaps his work on the *heter mekhirah*) may have motivated some to quibble with R. Tukitzinsky over minutiae in the calendar he established for Jerusalem. More than any contemporary *posek*, his insights and observations were fundamental, allowing many, myself included, to understand the laws of *zemanim* consistent with astronomic observations.

In summary, making no adjustments for both *alot ha-shaḥar* and *tzait ha-kokhavim*, though hardly justifiable, is in widespread use and often creates anomalies, as is apparent in the OU calendar. Using depression angles, as has become more prevalent in recent years, is consistent with both the physical events described in the various *sugyot* and, more than likely, with practice in the generations before clocks.

Section 4. How are we to define the hours of the day: from sunrise to sunset or from *alot ha-shahar* to darkness?

Most daily calendars provide two alternative methods for defining the hours of the day:

- The method ascribed to the Vilna *Gaon* calculates the hours of the day from sunrise to sunset.
- The method ascribed to Magen Avraham calculates the hours of the day beginning at *alot ha-shahar* until *tzait (kol) ha-kokhavim*.¹³³

In reality, this disagreement traces back to Ramban, and perhaps, as some maintain, Rambam.

Ramban in *Torat Ha-adam*¹³⁴ strongly supports the position of Rabbeinu Tam. Ramban argues that *tosefet Shabbat* could only begin after sunset during an interval that is still considered completely part of the daytime period; *tosefet Shabbat* prior to sunset is not meaningful, equated by Ramban to the value of illumination from a candle during daylight. Further Ramban asserts that *pelag ha-minḥah* occurs at the time that it takes to walk $\frac{1}{6}$ th of a *mil* prior to sunset. This has two obvious and important consequences. First, Ramban is clearly calculating the hours

¹³³ A century prior to the *Gaon*, R. Pimental, a contemporary of Magen Avraham, cites this disagreement as between the *Levushim*, authored by R. Mordekhai Yaffe, and R. Isserlein in *Trumat Ha-deshen*.

¹³⁴ Pages 251 - 252, Chavel edition, Mossad Ha-rav Kook.

of the day from *alot ha-shaḥar* to *tzait ha-kokhavim*. Second, as was discussed previously, Ramban maintained that the time needed to walk a *mil* is 22.5 minutes.

Calculations based on this approach of Ramban, even computed correctly, encounter an issue at latitudes further from the equator. Consider a place where the daytime period is only eight hours from 8:00AM to 4:00PM. Assume that two unadjusted 72-minute intervals are added to the daylight period of (8 hours =) 480 minutes. Note that adjusting for latitude or season or both, and / or using a 90-minute interval that I demonstrated to be the position of Ramban, would each increase 72 minutes; using 72 minutes demonstrates the issue *a fortiori*.

If the daytime period is only eight hours, then the *halakhic* day is $(480 + 2 * 72 =)$ 624 minutes and the *halakhic* hour is $(624 / 12 =)$ 52 minutes. *Hatzot* is at noon, *tzait ha-kokhavim* is at 5:12PM and *pelag ha-minḥah* occurs $((1 + \frac{1}{4}) * 52 =)$ 65 minutes before *tzait ha-kokhavim* at 4:07PM, **7 minutes after sunset**. Clearly, on its own this is not entirely problematic. While the approach that measures the hours of the day from sunrise to sunset cannot encounter this problem, we can simply assert that this is another example of some laws of *zemanim* requiring different treatment as we move further from the equator.¹³⁵ A *posek* can

¹³⁵ Like the end of *Shabbat* defined by the sun going 16 degrees below the horizon, which does not occur during the summer at latitudes in northern Europe, the calculation of *pelag ha-minḥah* has to be different during other months when the daytime period is short. For the end of *Shabbat*, some suggest (*Minḥat Yitzḥak* 4:53, for example) midnight. To my knowledge, *posekim* have not addressed how to define *pelag ha-minḥah* under these circumstances. Perhaps these types of

then choose to allow *pelag ha-minḥah* to occur after sunset,¹³⁶ consider *pelag ha-minḥah* to be undefined or artificially set *pelag ha-minḥah* to either sunset or $\frac{1}{6}$ th of a *mil* prior to sunset, during certain months of the year at latitudes further from the equator.^{137 138}

While it is indisputable that Ramban is an early source for the position of Magen Avraham, some assert that Rambam calculated the hours of the day via the method later prescribed by R. Mordekhai Yaffe and the *Gaon*.¹³⁹ Surprisingly, beyond this (disputed) source in *Mishnah Torah*,

complexities may have contributed to the overall preference for the approach of the *Gaon* in northern Europe.

¹³⁶ Consistent with our overall position, sunset may not be that critical a *zeman* particularly for Magen Avraham. Perhaps, *pelag ha-minḥah* falling after sunset is not that disturbing; implications to the contrary in the *gemara* might only apply to the Middle East.

¹³⁷ Hypothetically, based on Ramban, one could canonically set the twilight period to $1 + \frac{1}{4} - (\frac{1}{6} * 22.5 / 75)$ or 1 and $\frac{1}{5}$ hours and calculate the length of remaining 9 and $\frac{3}{5}$ hours from sunrise to sunset. Hours during the sunrise to sunset period would be shorter than the hours during twilight. In the epilogue on this section, an alternative method to calculate the approach of Magen Avraham is outlined. It provides a mechanism that solves this issue for almost all inhabited regions. That approach will be covered in depth in a planned monograph on *Shaot Ha-yom*.

¹³⁸ Yet more problematic is using 90 versus 72 minutes when calculating according to Magen Avraham even at latitudes similar to that of Jerusalem. Note that for those who maintain a 90-minute interval, *pelag ha-minḥah* occurs 3.75 minutes before sunset in the spring. However, in the winter, when the *halakhic* hour is approximately 10 minutes shorter, *pelag ha-minḥah* occurs at least ten minutes later and a few minutes after sunset. Were 90 minutes to be adjusted by season, to approximately 96 minutes in the winter in Jerusalem, as would be normally assumed, *pelag ha-minḥah* would occur yet another 5 minutes later. However, readers can verify that even in the winter, when the *halakhic* hour is 10 minutes shorter, using an adjusted 72 minutes, approximately 78 minutes, still leaves *pelag ha-minḥah* occurring just prior to sunset. Again, the conjecture in the epilogue outlining a different method to calculate the approach of Magen Avraham places *pelag ha-minḥah* prior to sunset in Jerusalem even assuming an adjusted 90-minute interval.

¹³⁹ See, for example, R. Schachter's argument in *Mi-pininei Ha-rav*, page 31, ascribed by R. Soloveitchik to his grandfather, which draws an implication from Rambam's

this approach does not have much if any explicit support among *rishonim*. Since *mitzvot* restricted to the daytime period can be performed from *alot ha-shaḥar* (at least *be-di-avad*), this approach must address why we do not calculate the hours of the day from that point. Similar to R. Yaffe, the *Gaon* answers¹⁴⁰ that the intrinsic definition of hours relates to the sun and is therefore defined only during the sunrise to sunset period. The *Gaon* and his school would likely argue that such a definitional notion was inherently understood in previous generations.

Despite attempted attribution of the approach of the *Gaon* to Rambam and support claimed from other earlier authorities, the approach of Rambam and others gained such widespread acceptance, that it is possible that R. Isserlein was not even aware of an alternative viewpoint. The first *teshuvah* of *Trumat Ha-deshen* deals with starting *Shabbat* in the summer 3 to 4 hours before *tzait ha-kokhavim*. R. Isserlein offers no basis for this practice other than mentioning that it was the practice of notable Rabbis of his time. While 4 hours is always problematic, a calculation of *pelag ha-minḥah* from sunset (as opposed to *tzait ha-kokhavim*) on a 17-hour day places *pelag ha-minḥah* 106.25 minutes before sunset and approximately 3 hours prior to *tzait ha-*

calculation of *derekh reḥokah* for *korban Pesah* only beginning at sunrise, that Rambam agrees with the position ascribed to the *Gaon* and R. M. Yaffe. Rambam is clearly assuming one calculates leaving at sunrise, and while certainly indicative of how the hours of the day might be calculated, it is not compelling and strongly disputed by R. Kapach throughout his commentary on *Mishnah Torah*. R. Kapach maintains that Rambam calculated the hours of the day from *alot ha-shaḥar*, claiming support as well from the ancient customs of *Teiman*. Reconciling Rambam *teshuvah* 51 concerning *shaot zemaniot* to R. Kapach's position is challenging.

¹⁴⁰ *O. H.* 261.

kokhavim. Since R. Isserlein does not offer this as a possible justification, one might conjecture that he may not even have been aware of this viewpoint. In *Sefer Levushim O. H.* 267, R. Yaffe questions why R. Isserlein did not proffer this explanation.

Ironically, R. Benish notes an attempt, based on R. Isserlein's student, R. Yosef ben Moshe, the author of *Leket Yosher*, to demonstrate from two *teshuvot* in *Trumat Ha-deshen* that R. Isserlein also calculated from sunrise to sunset.¹⁴¹ One *teshuvah* considers whether eating *ḥametz* until the end of the 4th hour must be measured using *shaot zemaniot*, or is it sufficient to stop eating *ḥametz* two clock hours before midday. As was outlined in the *teshuvah*, this became relevant in practice during a leap year, when *erev Pesah* occurred in late April.¹⁴² That year, at the end of the 4th hour, there were still approximately 3 hours until *ḥatzot*. R. Isserlein adopts the more lenient approach and allows eating *ḥametz* until 2 hours before *ḥatzot*. Since sunrise and *ḥatzot* were at approximately 5:00AM and 12 o'clock noon respectively, it would imply that R. Isserlein is calculating seven hours from sunrise to *ḥatzot*. This purported "proof" is rather problematic. First, as R. Benish himself asks,

¹⁴¹ See *Ha-zemanim Be-halakhah* volume 1, page 113. Paradoxically, as noted in section 1, in the paragraph adjacent to footnote 83, it is also claimed that R. Yosef ben Moshe disputes the view that R. Isserlein maintained that the time needed to walk a *mil* was 18 minutes. That argument in favor of 22.5 over 18 minutes is entirely dependent on the fact that R. Isserlein counted the hours of the day from *alot ha-shaḥar* until *tzait ha-kokhavim*. Thus, in an entirely contradictory manner, based on *Leket Yosher*, both positions normally associated with R. Isserlein have been disputed,

- counting hours of the day from *alot ha-shaḥar* to *tzait ha-kokhavim*, and
- maintaining 18 minutes as the time needed to walk a *mil*.

¹⁴² At his northern location, the daytime period was already approximately 14 hours.

when the *teshuvah* is discussed, *Leket Yosher* writes explicitly that it was 4 hours after *amud ha-shaḥar*. It is much more likely that *amud ha-shaḥar* refers *alot ha-shaḥar* than to sunrise. Second, in order to create the issue and calculate from sunrise at 5:00AM, it appears you must use unadjusted clock hours. Using clock hours, an unlikely assumption, the end of the 4th hour is at 9:00AM, 3 hours before *ḥatzot*. Third, were you to calculate instead from sunrise at 5:00AM until sunset at 7:00PM, using *shaot zemaniot* as must be assumed, the end of the 4th hour, the time to stop eating *ḥametz*, is at 9:40AM. Would one refer to 2 hours and 20 minutes as “approximately” 3 hours before noon? I doubt the question would ever have been raised. However, if we calculate using the practice that R. Isserlein is normally assumed to have followed, then *alot ha-shaḥar* was at 3:48AM (even assuming an unadjusted 72 minutes) and the end of the 4th hour of the day was at 9:16AM, approximately 3 hours (2 hours and 44 minutes) before *ḥatzot*.¹⁴³ This is much more likely and leaves the *teshuvah* consistent with the standard assumption of R. Isserlein’s position of calculating from *alot ha-shaḥar*. A second *teshuvah*¹⁴⁴ on reading the *Megillah* early in a special situation presents no direct indication of R. Isserlein’s position.

¹⁴³ Though highly unlikely, if R. Isserlein, in opposition to the opinion of his view by the *Shulḥan Arukh*, supported a 22.5 minute time to walk a *mil* then the approximation to 3 hours is even closer. What is more plausible is that R. Isserlein used an observed *alot ha-shaḥar* that at his location also occurred approximately 90 minutes before sunrise. In that case, three hours is almost exact. The day begins at 3:30AM, and the end of the 4th hour, at 9:10AM, is approximately 3 hours before *ḥatzot*.

¹⁴⁴ *Trumat Ha-deshen*, *teshuvah* 109. This *teshuvah* in particular, is critical to the approach referenced in footnote 137 and will be analyzed in detail in a planned monograph on *Shaot Ha-yom*. The monograph supports the *Shulḥan Arukh*’s view of R. Isserlein’s positions.

However, an account of the circumstances in *Leket Yosher*¹⁴⁵ is assumed to support the view that R. Isserlein would calculate *pelag ha-minḥah* from sunset, at least in this particular case of need. That would indicate that R. Isserlein was aware of the position, in which case the first *teshuvah* must have dealt with a situation where *Shabbat* was begun so early that even calculating *pelag ha-minḥah* from sunset would not be sufficient to allow so early a start to *Shabbat*. In any case, the opinion normally ascribed to R. Isserlein was widely followed.¹⁴⁶

Some have argued that anyone who calculates from *alot ha-shaḥar* until *tzait (kol) ha-kokhavim* must follow the opinion of Rabbeinu Tam as opposed to the *geonim*.¹⁴⁷ While I am not aware of any major *halakhic* authority who maintains that the hours of the day be calculated from *alot ha-shaḥar* until *tzait ha-kokhavim* and also followed the *geonim*, many communities in Jerusalem, who end *Shabbat* according to the view of the *geonim*, observe the times of Magen Avraham for morning prayers. Conceptually, this combination of views is easily justified as follows: Since almost all *mitzvot* of the day can be performed from *alot ha-shaḥar*, it would seem at least plausible to count the hours of the day

¹⁴⁵ *Leket Yosher*, page 157 - 158, where the time of *pelag ha-minḥah* is quoted as being a few minutes before 5:00PM, certainly at variance with the assumed position of R. Isserlein, if he was calculating *pelag ha-minḥah* from a late point of *tzait (kol) ha-kokhavim*. See *Ha-zemanim Be-halakhah*, volume 1, page 113.

¹⁴⁶ The alternative for the how the hours of the day may have been calculated, mentioned in footnote 137, addresses this issue as well, even assuming R. Yosef ben Moshe's assertion that *pelag ha-minḥah* occurred on that day slightly before 5:00PM.

¹⁴⁷ R. Kotler, R. Willig and others have argued for this position. R. Kotler raised the issue of the practice of Jerusalem noted in the next sentence, questioning his own approach.

from that point.¹⁴⁸ In order to maintain a precise calculation of *ḥatzot*, one might use *tzait kol ha-kokhavim* as the other endpoint for purposes of calculation. However, that need not imply that *tzait kol ha-kokhavim* is the endpoint for any particular *halakhah*; the end of the day and the endpoint for various *mitzvot*, may still occur earlier with the appearance of three stars or even before that time. As mentioned earlier, this asymmetric day is strongly suggested by the *Yerushalmi* in *Berakhot* 2b based on the verse in Nehemiah. One might also choose to distinguish between the definition of the daytime period, from *alot ha-shaḥar* until *tzait kol ha-kokhavim*, from the definition of the day of the week that

¹⁴⁸ In *Am Mordechai*, R. Willig goes further:

1. R. Willig draws the implication in the other direction as well, i. e., that those who follow Rabbeinu Tam must define the hours of the day from *alot ha-shaḥar* to *tzait ha-kokhavim*.
2. He argues that the dispute over 18 versus 22.5 minutes as the time needed to walk a *mil* also links to the basic argument of the *geonim* and Rabbeinu Tam.

Both of those claims have known counter-examples. As R. Schachter points out R. Soloveitchik maintained a personal *ḥumrah* based on Rabbeinu Tam, but he did not concern himself with the opinion of Magen Avraham, as was the *minhag* of much of Lithuania. In his *yarzeit shiur*, (see the section in the *shiur* adjacent to footnote 8), R. Soloveitchik argues strongly in favor of the *Gaon* versus Magen Avraham and indicates that Rabbeinu Tam and his followers might also calculate from sunrise to sunset, as R. M. Yaffe actually maintained in practice. Regardless of whether one agrees with R. Soloveitchik's view, it would make it difficult to maintain a logical implication in direct opposition. The second implication again has R. Yaffe as a counter-example. It may also be contrary to the *Gaon* himself who, according to many sources, also maintained 22.5 minute as the time needed to walk a *mil*. More critically, the 18 versus 22.5 minute argument is likely completely independent of the argument around the end of *Shabbat*, perhaps largely independent of *halakhic* theory as well and centered only on the length of time that is associated with particular phenomena. In the new edition of *Am Mordechai*, R. Willig modifies his position significantly.

begins and ends at or around the time of the appearance of three stars, as I will explore further in section 9.¹⁴⁹

Despite its apparent logical necessity, one cannot assume that *tzait kol ha-kokhavim* was (commonly) used prior to the last century to calculate the hours of the day. Many calendars, including the calendar of Jerusalem mentioned in a previous footnote,¹⁵⁰ used *alot ha-shaḥar* and three stars violating our assumption about the necessity to calculate *ḥatzot* accurately.^{151 152}

As noted earlier, R. Schachter quotes an opinion of R. Soloveitchik that Rambam calculated *halakhic* hours from sunrise. Interestingly, if one were to maintain that Rambam considered the *alot ha-shaḥar* to sunrise and the sunset to *tzait ha-kokhavim* period as 120 minutes, a viewpoint that some maintain, then it is clear that Rambam could not count the hours of the day from *alot ha-shaḥar* to *tzait ha-kokhavim*. Assume a time to walk 5 *milin* of 120 minutes on a canonical 12-hour day; *alot ha-shaḥar* is at 4:00AM and *tzait ha-kokhavim* is at 8:00PM. An hour is $((720 + 240) / 12 =)$ 80 minutes and *pelag ha-minḥah*, 1 and $\frac{1}{4}$ hours

¹⁴⁹ This viewpoint is raised by R. Isser Zalman Meltzer quoted at length by R. Tukitzinsky in *Bein Ha-shemashot*, pages 37 - 41, albeit differently than will be addressed in sections 5 and 9. As already noted, when the *Gaon* introduced the notion of *tzait kol ha-kokhavim*, he did not propose any specific *halakhot* that might depend on that *zeman*.

¹⁵⁰ See footnote 132, as well as footnote 51 in the epilogue.

¹⁵¹ It is left as an exercise to the reader to demonstrate that such an approach does not violate the observation that *minḥah gedolah* and *minḥah ketanah* are separated by exactly three hours.

¹⁵² As already noted in footnote 137, the epilogue contains a conjecture of a practice that used asymmetric endpoints while calculating *ḥatzot* accurately, from which this approach may have evolved.

(100 minutes) before 8:00PM, is at 6:20PM, **20 minutes after sunset even on a 12-hour day.** While there is no logical dependency between how hours are calculated and either a 72 or 90 minute *alot ha-shaḥar* to sunrise interval, the 120 minute alternative apparently¹⁵³ forces calculating the hours of the day from sunrise to sunset.

There is scant evidence from the Talmud supporting either position and what evidence exists is not unassailable.¹⁵⁴ Two *sugyot*, one relating to an early *maariv* one afternoon after which the clouds lifted and the sun reappeared and a second tolerating inaccuracy by witnesses on the second versus the third hour of the day, are somewhat less forced when one assumes the position of R. Yaffe. Interestingly, rather than reliance on either of these *sugyot*, instead, early opinions make fundamental arguments, either,

- the day begins at *alot ha-shaḥar* and hours should be calculated from the *halakhic* beginning of the day, or
- hours are, by definition, defined by the rising and setting sun.¹⁵⁵

¹⁵³ The conjecture referenced in the previous footnote creates the possibility of defining *alot ha-shaḥar* at 120 minutes before sunrise, counting the hours of the day from *alot ha-shaḥar* but using an asymmetric point in the evening at, for example, 40 minutes after sunset. This avoids *pelag ha-minḥah* occurring after sunset, and it can be implemented preserving the accuracy of *ḥatzot*.

¹⁵⁴ See *Minḥat Kohen, maamar sheni*, chapters 6 to 9, who carefully examines the relevant *sugyot*; his conclusion is that the position of the *Levushim* is to be preferred.

¹⁵⁵ R. Soloveitchik, in the previously mentioned *yartzeit shiur*, argues for the position of the *Gaon*, without supporting sources.

While the method of R. Yaffe has only one primary method of calculation, the method of R. Isserlein has spawned multiple methods of calculation along three quasi-independent variables:

1. Is the interval between *alot ha-shaḥar* and sunrise in the Middle East around the spring and fall equinox 72, 90 or some other number of minutes?
2. As one changes location or in different seasons, is the interval invariant, or does it vary either in proportion to the length of the day or as defined by depression angles?
3. Is the evening endpoint used for calculation and *alot ha-shaḥar* equidistant from sunset and sunrise respectively, or is it earlier, coincident with the appearance of three stars.¹⁵⁶ Implementing an earlier evening endpoint has a number of computational alternatives.¹⁵⁷

¹⁵⁶ Those who would use three stars would still have to determine the exact time to use in practice. Interestingly, the greatest stringency for morning *zemanim* results from the earliest assumed time for three (medium) stars.

¹⁵⁷ Not all of these theoretical alternatives have specific proponents.

Section 5. Of the three criteria given by the gemara in *Shabbat* – time, the appearance of stars and darkness (darkening / appearance of the sky / horizon), which, if any, are the definition of night, and which are just approximations or an indication? How can opinions expressed using these three terms be compared?

We begin by stipulating one critical assumption:

- **Except in highly unusual circumstances, approximations must be more stringent than the defining criteria; otherwise, an approximation would have the effect of being potentially misleading.**¹⁵⁸

Thus, if a specific level of darkness is the defining criteria for the transition between days of the week, and three medium stars were to appear prior to that defined level of darkness, providing three stars as an approximation is of dubious value. Similarly, intervals such as the time it takes to walk $\frac{3}{4}$ of a *mil* should also provide a conservative approximation. This assumption coupled with the need to align the *sugyah* in *Shabbat* with observation, is fundamental to the approach to be developed in this section.

¹⁵⁸ It is possible that one can prefer an approximation that on rare occasion is incorrect. For example, we might examine X because with very high probability it is indicative of Y and since Y is difficult to ascertain, we do not require that Y be checked directly.

Despite its current popularity and widespread use, it should be obvious that time clearly does not define the end of the *bein ha-shemashot* period. In the next section, the possible role of time in specifying the beginning of the *bein ha-shemashot* period is analyzed.

Most commentators consider the appearance of three medium stars the defining criteria. As the Vilna *Gaon* observes, darkness and the appearance of the horizon are to be used either on a cloudy day, when stars would not be visible and / or because the definition of a medium star is difficult for the average person to ascertain.¹⁵⁹ Some treat Nehemiah 4:15, which describes the working day from *alot ha-shaḥar* until *tzait ha-kokhavim*, as providing a definition.

Darkness, characterized by the darkening of the sky and the horizon, is central to the discussion of the *gemara* in *Shabbat*. *Ḥashekhah* is the term used almost exclusively in early *tannaic* literature. The appearance of sky and the horizon correlate precisely to the degree of darkness. The fact that three medium stars as a descriptor for the end of *Shabbat* is first recorded at the end of the *tannaic* period is one of the more compelling arguments in favor of darkness being the defining

¹⁵⁹ It is ironic that the *Gaon* considers the definition of the sky / horizon as clearer for the average person than that of a medium star. Of course, he is assuming his view and that of the *geonim* as opposed to that of Rabbeinu Tam. I have found that on a clear night with an unobstructed view of both the eastern and western horizon, the *sugyah's* description of the horizon and the darkening of the sky at the end of the *bein ha-shemashot* period, according to the opinion of the *geonim*, are rather intuitive. On a clear night around the period of *bein ha-shemashot*, flying due north at a moderate altitude, or either standing on-deck at sea or in a tall building with an unobstructed view of both the eastern and western horizon, provides a vantage point that helps to visualize and clarify the *sugyah*; it is strongly recommended.

criteria.¹⁶⁰ Darkness as the defining characteristic has a number of proponents.¹⁶¹ Further support for this point of view comes from the language of the *gemara*¹⁶² that uses the phrase “*siman le-davar*,” “a sign (or indication) for the thing,” when discussing the appearance of stars.

While there are a number of inferences favoring darkness,¹⁶³ none is compelling. Among the most suggestive is the *gemara*¹⁶⁴ that recommends that we behave like the people of Tiberius for the start of *Shabbat* and like the people of Tzippori for its end. Perhaps by necessity, residents of Tiberius situated in a valley, unable to see the setting sun, started *Shabbat* early. However, assuming the people of Tzippori are looking for stars, why would they see three stars later because of their altitude? Given general atmospheric conditions at a higher altitude, they might even see three stars earlier. Thus, I suspect that they were looking at the sky, and given their altitude, and from their vantage point, looking to the east, the apex of the sky might take slightly longer to appear as dark as what they observe in the eastern most portion of the horizon. As well, the western sky would appear more illuminated from their elevation than from sea level. Does this

¹⁶⁰ Stars are mentioned for the first time in the *tosefta Berakhot* and are then discussed at length by *amoraim* in a number of places. *Ḥashekhah* (darkness) is the dominant term in *tannaic* literature. The Mishnah in *Shabbat* speaks of *ḥashekhah*, while the *gemara* in *Shabbat* revolves around *hiḥsif*. See *Ha-zemanim Be-halakhah*, Chapter 63.

¹⁶¹ See *Meiri Shabbat 34* and a short excerpt from the period of the *geonim* quoted in the appendix to *Ha-zemanim Be-halakhah*, volume 2.

¹⁶² See both the *Bavli* and *Yerushalmi* in the beginning of *Berakhot*.

¹⁶³ A unique argument, given by R. E. Wasserman in the second entry in *Kovetz Shiurim - Pesahim*, argues that stars were only created on the fourth day.

¹⁶⁴ *Shabbat* 118b.

constitute compelling proof that darkness is defining? Not quite, one could still argue that perhaps at their altitude, seeing more illumination in the western sky made them hesitant to end *Shabbat* despite the defining appearance of three stars.

To examine this issue, we analyze in detail the uncontested opinion of R. Yehudah in the name of Shmuel who states “*one star - daytime, two stars - bein ha-shemashot, and three stars - night,*” but only according to the opinion of the *geonim*.¹⁶⁵ To begin I note the opinion of R. Benish,¹⁶⁶ who points out that this statement reads better if the appearance of three stars is merely an indicator. The first phrase cannot be providing a definition; “*one - star day*” is an indicator that it may still be day. In no sense can “*one - star day*” be a definition. Similarly, he argues that it would make more sense if we read “*three - stars night*” as being indicative that night has begun, rather than the appearance of three stars defining the point of transition. What I will demonstrate goes one-step further; the language is entirely uniform if all three phrases are read as indicators:

1. The appearance of one star need not indicate that the period of *bein ha-shemashot* has begun; rather, it may still be day.
2. The appearance of two stars indicates that we are in the period of *bein ha-shemashot*.

¹⁶⁵ The *Yerushalmi* records this and similar statements from multiple sources in conjunction with a variety of viewpoints. I do not address how one might interpret this statement in the *Yerushalmi*; it might support alternative positions and interpretations.

¹⁶⁶ See *Ha-zemanim Be-halakhah*, vol. 2, chapter 48, section 5, p. 498.

3. The appearance of three stars indicates that night has begun, and the transition to the next day has occurred.

While R. Benish draws the implication from the first to the third star, this section suggests a similar interpretation regarding the second star as well.

Before proceeding with an analysis of the Shmuel's summary opinion, some background is required. While stars and darkness both have proponents within *halakhah*, from a scientific perspective, the appearance of stars is clearly a consequence, an observation already made in the era of the *geonim*¹⁶⁷ and repeated in the *halakhic* literature. For stars of a certain magnitude and distance from earth to be visible, the level of darkness must reach a particular point. Conversely, reaching that level of darkness does not cause a star of that magnitude and distance from earth to appear. At a minimum, that part of the earth has to be facing in a specific direction for a particular star to appear. For example, we might see a star situated in the eastern sky earlier than a star of even greater magnitude situated towards the west where illumination from the setting sun still hinders its visibility. Using basic astronomy and spherical trigonometry the *gemara's* description of *ḥashekhah / hiḥsif has* been equated to depression angles. As indicated previously, depression angles quantify how far below the horizon the sun has descended, providing an accurate measurement of the level of

¹⁶⁷ See citations from the *geonim* in the addendum to *Ha-zemanim Be-halakhah*, volume 2.

residual illumination. Using depression angles as the precise formalization of the degree of darkness has become the preferred way to define the end of *Shabbat* and almost all other *zemanim* relating to the twilight periods of both dawn and nightfall.

Regardless of whether stars or darkness is assumed to be defining, the earliest appearance of three stars and the *gemara's* description of the appearance of the sky and the horizon at the end of *Shabbat* are strongly correlated. What are problematic, however, are various attempts to correlate the appearance of one or two stars with the various options for defining the *bein ha-shemashot* period. Before further analysis, we need to clarify the *gemara's* use of the terms small, medium and large star.

To begin, many currently assume that all stars, as opposed to planets, are what the *gemara* refers to as medium and small stars. The *gemara* defines a large star as one that is, on occasion, visible prior to sunset. Those heavenly bodies, referred to as "*kokhavei lekhet*," moving stars, are planets like Venus, Mars and Jupiter that can be seen, on occasion, prior to sunset. As Prof. Levi points out, to be visible prior to sunset, a star would have to appear to be approximately 2500 times brighter than an average small star. Only planets satisfy that characteristic; what the *gemara* refers to as large stars is widely assumed to be planets, the "*stars of the sun*." Except for these large stars, very few stars are bright enough to be visible within the first 15 minutes after sunset. In Israel, only two stars, Sirius and Canopus, which are respectively 1100 and 600

times brighter than a small star, are potentially visible that early, and only during certain periods of the year. However, despite the *gemara's* statement that large stars are visible in the day, it need not necessarily mean that a large star is defined exclusively by its visibility prior to sunset. It is still possible that the two largest stars, Sirius and Canopus, are considered large stars, as their illumination was judged closer to that of a planet, a large star, than that of a medium star. This possibility cannot be dismissed out-of-hand. Were Sirius and Canopus classified as large stars, then the inter-arrival rate of the remaining medium stars would be very rapid, with significant implications for reading Shmuel's statement concerning 1, 2 and 3 stars.

In the Middle East, approximately one dozen stars, 75 to 250 times the illumination of a small star, are visible by about 30 minutes after sunset. Once three stars are visible, many more stars become potentially visible as well. After approximately 30 minutes, even if a particular star is not in position to be visible, other stars become visible shortly thereafter.

Parenthetically, this may provide another intuitive argument against stars, as opposed to darkness, being defining. The *Yerushalmi* in *Berakhot* asks why we require three stars since the appearance of stars signifying the end of a day ought to require only the smallest plural, i. e., two stars. If stars are defining, this question does not appear to have an entirely satisfactory answer.¹⁶⁸ However, assuming both that darkness

¹⁶⁸ The answer given by the *Yerushalmi* is not to count the first star since it can or does appear in daytime (even though occurring after sunset). See the table by Prof. Levi. Though hardly our area of expertise, it would appear that the *Yerushalmi*

is defining and that Sirius and Capanus are classified as medium stars, a clear-cut answer to the *Yerushalmi's* question can be suggested:

- Two stars are visible much too early when both Sirius and Canopus are positioned to be visible in the Middle East, as occurs during the spring.
- It is only after a third, significantly less visible, star appears, that the requisite level of darkness has been reached.

Of course, if Sirius and Canopus were considered large stars, then I have heard nothing more compelling than the answer that the *Yerushalmi* provides.

Note that at both the fall and spring equinox, the sun appears over the equator and you would expect *Shabbat* to begin and end at the identical time. Certainly, regardless of how one measures darkness, it is equivalently dark any number of minutes after sunset at those two times. However, in Jerusalem and other parts of the Middle East, having nothing to do with the degree of darkness, stars appear later in the fall than they do in the spring. To argue that this is problematic would be entirely circular. If the appearance of three stars defines the end of *Shabbat* then that happening later on a particular day than on another day that we label as equivalent clearly depends on what we choose to

spends more time discussing stars and perhaps considers stars as defining, despite use of the phrase *siman le-davar*, as opposed to the *Bavli* where stars appear only at the end of the *sugyah* in *Shabbat*. Were that the case, the *Yerushalmi* could not have answered as is suggested.

call equivalent. Nonetheless, it is still a bit bothersome to maintain stars as defining, particularly because the appearance of the horizon does not suffer from this anomalous detail and equates precisely to gradations of darkness.

As noted, once three stars are visible, many more become visible shortly thereafter. Thus, three stars always appear within a relatively well-defined interval closely linked to the level of darkness. However, the appearance of the first star exhibits significantly more seasonal variation. To appreciate the difference between darkness and stars as definitions, it is instructive to examine carefully the actual appearance of stars that Prof. Levi records for Jerusalem, reproduced in a table below. The three times listed in each cell of the table correspond to how difficult it is to see a star.

- The shortest time noted is when an expert who knows exactly where to look can observe a star.
- The intermediate time noted is when a star can be seen with great difficulty.
- The third time noted is when a careful observer can see a star.

The time when stars are visible to a casual observer, is yet later.

Prof. Levi felt strongly that darkness and the appearance of the horizon, equated in modern times to depression angles, was defining. The times in the table were therefore adjusted to the spring and fall equinox as

follows. Assume that in the winter three stars are visible to a careful observer 28 minutes after sunset. At that point, assume the sun is 6.5 degrees below the horizon. The time given in the table is not 28 minutes, but rather, the number of minutes after sunset at the spring equinox when the sun is 6.5 degrees below the horizon, a smaller number, 26 minutes in the above chart. The chart shows the greater variability in the time that the first star appears relative to a level of darkness. Given Prof. Levy's normalization to minutes corresponding to what would occur around the spring / fall equinox, the variation by season is understated slightly.

Taking for granted the accuracy of Shmuel, renowned for his expertise in astronomy, Prof. Levi's table constrains interpretation of Shmuel's statement. I assume that at the very least the opinions of Shmuel and Rambam, both of whom lived in the Middle East and were careful observers of the heavens, must be reconciled with Prof. Levi's observations.

| Equinox | Fourth star | Third star | Second star | First star |
|----------------|--------------------|-------------------|--------------------|-------------------|
| Spring | 16-18-21 | 15-18-21 | 14-16-19 | 6-9-12 |
| Summer | 22-26-29 | 21-25-28 | 16-18-21 | 13-16-18 |
| Fall | 23-26-28 | 19-23-26 | 17-20-23 | 12-14-17 |
| Winter | 21-25-28 | 18-22-26 | 17-20-23 | 17-20-22 |

Prof. Levi's Table

With this table as background, we now return to the analysis of Shmuel's opinion. After discussing the appearance of the horizon and the length of the *bein ha-shemashot* period, the *gemara* states the

opinion of R. Yehudah in the name of Shmuel who asserts: “*one star – daytime, two stars – bein ha-shemashot, and three stars – night.*”

Prepositions have been omitted in order not to bias the semantics. This is followed by the opinion of R. Yosi (*bar Adin*) asserting that the stars in question are neither stars that appear in the day (i. e., large stars or planets) nor small stars that only appear significantly after sunset (sometime after the time of *tzait ha-kokhavim* and prior to the time of *tzait kol ha-kokhavim*) but medium stars. How might Shmuel’s statement be reconciled with the previous discussion in the *gemara*?

I do not consider further an implausible suggestion that R. Yosi *bar Adin*’s assertion that Shmuel’s statement is referring to medium stars applies only to the third part (or second and third parts) of the text. Under this interpretation, the first part of Shmuel’s statement concerning one star, includes not only medium stars but large stars or planets as well, visible before sunset. Were that the case, the statement would be informing us that the appearance of a planet before sunset does not indicate that the *bein ha-shemashot* period has begun. In addition to being forced to argue the first part and the third part of Shmuel’s statement refer to different types of stars, such an assertion would hardly be necessary; the *gemara* gives no hint of the *bein ha-shemashot* period beginning before sunset. (The isolated opinion of R. Eliezer mi-Mitz, who begins the *bein ha-shemashot* period before sunset, is difficult to reconcile with Shmuel’s statement, in any case).

Let us first consider how to interpret Shmuel's statement examining alternative positions in five areas. Were the alternatives in the five areas below logically independent, there would be **72** options:

- I. Are Sirius and Canopus large or medium stars? (2 options)
- II. Shmuel is
 - consistent only with the opinion of Rabi Yosi,
 - consistent with both Rabbah's and R. Yosef's interpretations of R. Yehudah or
 - consistent only with R. Yosef's interpretation. (3 options)
- III. Are stars or darkness defining? (2 options)
- IV. Does "*one star – daytime*" mean that up until the time of one star appearing it is definitely daytime, or when one star appears, it *may* still be daytime? (2 options)
- V. "*Two stars – bein ha-shemashot*," means
 - two stars appearing defines the beginning of the *bein ha-shemashot* period,
 - two stars appearing means that the period of *bein ha-shemashot* has always already begun or
 - we are to treat the appearance of two stars as the beginning of the period of *bein ha-shemashot*, perhaps as a *harhakah*. (3 options)

Multiplying the two or three options associated with each item, gives $2 * 3 * 2 * 2 * 3 = 72$ possible combinations.

Fortunately, some alternatives are inter-dependant, immediately reducing the number of options to **28**. Options III. – V. yield 7 (not 12) realistic alternatives:

1. Darkness is defining, and the period of *bein ha-shemashot* begins before the second star and after the first star.
2. Darkness is defining, and we treat the appearance of a second star as the beginning of the *bein ha-shemashot* period, perhaps as a *harhakah*.
3. Darkness is defining, and the period of *bein ha-shemashot* can on occasion begin even before the first star.
4. Stars are defining for both night and the period of *bein ha-shemashot*.
5. Stars are defining for night only, and the period of *bein ha-shemashot* *always* begins before the second star and after the first star.
6. Stars are defining for night only, and we treat the appearance of a second star as the beginning of the period of *bein ha-shemashot*, *perhaps* as a *harhakah*.
7. Stars are defining for night only, and the period of *bein ha-shemashot* can begin even before the first star.

Commentators maintaining that three stars define the end of the day, often also consider two stars as defining the beginning of the *bein ha-shemashot* period, (alternative 4 above). While they could also consider

/ adopt alternatives 5, 6 or 7, some, who assume stars are defining, simply extend that assumption to the second star defining the beginning of the period of *bein ha-shemashot*, as well.

Similarly, options I. and II. yield only 4 (not 6) realistic alternatives:

1. Sirius and Canopus are large stars, and Shmuel's statement is only consistent with the opinion of R. Yosi.
2. Sirius and Canopus are medium stars, and Shmuel's statement is only consistent with the opinion of R. Yosi.
3. Sirius and Canopus are medium stars, and Shmuel's statement can be interpreted consistently with R. Yosef's, but not Rabbah's, interpretation of R. Yehudah.
4. Sirius and Canopus are medium stars, and Shmuel's statement can be interpreted consistently with both Rabbah's and R. Yosef's interpretations of R. Yehudah.

Note that this formulation dismisses the possibility of reconciling the *sugyah* with the opinion of Rabi Yehudah while also asserting that Sirius and Canopus are large stars.¹⁶⁹ The only stars potentially visible before Rabi Yehudah's *bein ha-shemashot* interval of the time needed to walk either $\frac{2}{3}$ or $\frac{3}{4}$ of a *mil* prior to *hashekhah* are Sirius and Canopus. Thus, "one star day" forces the assumption that according to Rabi Yehudah, Sirius (and hence Canopus) is a medium star. Absent these two stars, a

¹⁶⁹ Again, I am assuming that the first star is not a planet, as that would be uninformative.

larger number of medium stars first appear in close (time) proximity to each other, but only after the start of the period of *bein ha-shemashot*.

Let us begin by examining how Shmuel's assertion might align with the three dominant opinions in the *sugyah*, the opinion of Rabi Yosi and Rabbah and R. Yosef's interpretations of the opinion of Rabi Yehudah. One alternative aligns the entire statement with the opinion of Rabi Yosi. Instead of R. Yosi being identified with R. Yosi bar Adin, an *amora* commenting on the statement of Shmuel, based on some texts, the entire statement is assumed to follow only the opinion of Rabi Yosi, cases 1 and 2 above. A second viewpoint aligns Shmuel with the view of R. Yosef, case 3 above. It is assumed that Shmuel's statement cannot be aligned with the view of Rabbah, as in case 4 above, given the assumption that according to Rabbah, the period of *bein ha-shemashot* starts precisely at sunset. Of course, to argue that Rabbah disagrees with R. Yosi bar Adin and the star is a "moving star" or a planet that is visible even before sunset, is, as I have argued, forced.¹⁷⁰

Reading the entire statement of Shmuel as reflecting the viewpoint of only Rabi Yosi makes the extension of the day past sunset non-problematic. If the appearance of the second star defines the beginning of the period of *bein ha-shemashot* as is often assumed, then the 2 to 3

¹⁷⁰ There is also no suggestion to align the statement with Rabi Nehemiah since his opinion is not normative. However, were the statements of Shmuel interpreted so that the interval of *bein ha-shemashot* begins between the first and second star, Prof. Levi's chart shows remarkable consistency with a time to walk $\frac{1}{2}$ of a *mil* except in the winter when the first star appears late and the other stars appear in relatively rapid succession. Nonetheless, this alternative is not considered.

minute interval between the second and third star is much more consistent with *ke-heref ayin* than the time needed to walk either $\frac{2}{3}$ or $\frac{3}{4}$ of a *mil*. Nonetheless, 2 to 3 minutes is hardly *ke-heref ayin* as normally interpreted; instead, we would likely interpret “*two stars – bein ha-shemashot*,” as simply a *harhakah*. There is a point of view expressed in the *Yerushalmi*¹⁷¹ that Rabi Yosi’s *ke-heref ayin* occurs within the interval of R. Nehemiah. Thus, it is also possible that according to Rabi Yosi, two stars begin the period of *bein ha-shemashot* and it ends very shortly thereafter, with the third star only providing confirmation of the transition to the next day. However, this is all rather forced; regardless of how we might interpret “*two stars – bein ha-shemashot*,” Shmuel appears to be working with a defined interval for *bein ha-shemashot*, while Rabi Yosi is not. More importantly, aligning only with Rabi Yosi after the *gemara* clearly decides in favor of Rabi Yehudah with respect to *Shabbat* is strained. Despite these arguments, which make this position rather dubious, aligning Shmuel’s assertion only with Rabi Yosi has a number of adherents. That position leaves most of the alternatives outlined unresolved, and they could follow a number of options:

1. Sirius and Canopus may be either medium or large stars.
2. Either darkness or stars may be defining.

¹⁷¹ The opinion does not appear in the *Bavli* and does not have any support from *rishonim*.

However, it is highly likely¹⁷² that

1. one star may or may not appear in the daytime, and
2. “two stars – *bein ha-shemashot*” is only a *harhakah*.

Before investigating alignment with Rabbah and R. Yosef’s interpretations of the opinion of Rabi Yehudah, we will first look at the 7 alternatives associated with options III. - V. Examining the spring and winter periods in Prof. Lev’s chart, the meaning of “one star – day” requires clarification. Note that the first star appears in the winter, slightly after three stars appear in the spring. Of course if stars are defining this is anomalous, but by definition not problematic. However, if stars are merely indicative and darkness is defining, then this phenomenon requires explanation. One possibility is that this statement of Shmuel follows only Rabi Yosi, there is no *bein ha-shemashot* period, and the discrepancy is to be explained away as being too slight, perhaps the result of some change in the visibility of stars between the time of the *gemara* and now. However, as I argued above, having the *gemara* conclude with a definitive ruling against Rabi Yosi with respect to *Shabbat*, and then quote an opinion that follows only Rabi Yosi especially in the name of the same *amora*, R. Yehudah, whose students’ debate occupies much of the *sugyah*, is forced. Assuming the statement of Shmuel must be aligned with some view of Rabi Yehudah and hence assumes a 12 to 18-minute period of *bein ha-shemashot* (during at least some season of the year) forces our next definitive

¹⁷² There are multiple opinions about when Rabi Yosi’s precise point of *hashekhah* occurs; each would give Shmuel slightly varied semantics.

conclusion: **“One star - day” means it may still be the previous day if only one star is visible.** While one star can on occasion be visible prior to the period of *bein ha-shemashot* as occurs in the spring, the first star that appears in the winter certainly occurs after the start of Rabi Yehudah’s period of *bein ha-shemashot*. **The inverse: “as long as a star is not visible, the period of *bein ha-shemashot* has not begun” is incorrect.**

To this point, we can conclude that according to Rabi Yehudah, one star may be visible during the daytime and Sirius and Canopus are medium stars. Prof. Levi’s table further constrains the options: **Rabi Yehudah’s interval of *bein ha-shemashot* (according to either Rabbah or R. Yosef) must extend back prior to the appearance of a second star.** Examine the longest interval between the second and third star; it is significantly less than even the shortest potential interval for *bein ha-shemashot*, the time needed to walk $\frac{2}{3}$ of a *mil*. As a result, two stars cannot define the beginning of the *bein ha-shemashot* period. The *bein ha-shemashot* period begins either during the interval between the first and second star or, particularly during the winter, even prior to the appearance of first star, options that do not appear to have been widely considered. Alternatives 1, 4, and 5 can be dismissed; alternatives 2 and 6 are still remotely possible, but only if one assumes that in the spring two stars are visible before the beginning of the *bein ha-shemashot* period, a very remote possibility that cannot be dismissed. Under those options, we treat two stars as the beginning of the period of *bein ha-shemashot*, but only as a *harḥakah*. Thus, while we cannot conclude

whether stars or darkness is defining, we can conclude (excluding the unlikely possibility of alternatives 2 and 6,) that according the opinion of Rabi Yehudah:

1. Sirius and Canopus are medium stars.
2. “*One star – day*” indicates that up until the time of one star appearing it may still be daytime and prior to the beginning of *bein ha-shemashot*.
3. “*Two stars – bein ha-shemashot*” indicates that once two stars appear, the period of *bein ha-shemashot* has already begun.

With a clear understanding of Shmuel’s statement and understandably unable to resolve whether stars or darkness is defining, only one issue remains open: Is Shmuel consistent with both Rabbah’s and R. Yosef’s interpretation of Rabi Yehudah’s opinion, or consistent only with R. Yosef’s interpretation.

If one assumes that Rabbah’s period of *bein ha-shemashot* begins precisely at sunset, Rabbah is inconsistent with Shmuel. Thus, a prevalent opinion aligns Shmuel’s statement only with R. Yosef, who begins the period of *bein ha-shemashot palga de-danka*, the time needed to walk $\frac{1}{2}$ of $\frac{1}{6}$ th or $\frac{1}{12}$ th of a *mil*, after Rabbah. However, looking at Prof. Levi’s table makes this approach untenable, as well. If R. Yosef’s interval begins $\frac{1}{12}$ th of the time needed to walk a *mil* after Rabbah’s, then it begins between 1.5 and 2 minutes after sunset. However, the very earliest an expert can see a star is 6 minutes after sunset, a full 4

minutes later. This would then rule out R. Yosef as well, and leave Shmuel's statement inconsistent with either interpretation of Rabi Yehudah.

This inability to align Shmuel with either of the two interpretations of Rabi Yehudah's normative opinion, or even to align Shmuel with the opinion of Rabi Yosi in a fully satisfactory way, motivates a careful re-examination of the text. Before proceeding, one further difficulty with Rabbah's interval of the time needed to walk $\frac{3}{4}$ of a *mil* needs to be addressed. Assuming that the start of the period of *bein ha-shemashot* for the *geonim* is sunset, there has been considerable effort¹⁷³ to align the appearance of three stars within the time it takes to walk $\frac{3}{4}$ of a *mil* after sunset. The only solution provided is to assume that the time needed to walk $\frac{3}{4}$ of a *mil* applies only around the spring equinox,¹⁷⁴ and even then to make yet further assumptions to arrive at so short an interval. Particularly if the time needed to walk $\frac{3}{4}$ of a *mil* is 13.5 minutes and even if it is a bit under 17 minutes, three stars can rarely be seen so soon after sunset and then

- only with great difficulty,
- by experts, perhaps aided by telescopes, and
- in a pristine environment absent urban sources of light, like the Judean desert.

¹⁷³ R. Benish, R. Willig, Prof. Levi among many others struggle with this issue.

¹⁷⁴ Having seen no earlier discussion of this issue, I believe this interpretation originated with the *Gaon* in *O. H.* 261. The alternative proposed below may have been the more conventional interpretation.

Under any circumstance, 13.5 minutes is implausible.¹⁷⁵

This helps to motivate a fundamentally different alternative. The *gemara* in *Pesaḥim* 94a that equates the time needed to walk 40 *milin* to the daytime period must assume an average day around either the spring or the fall equinox. In the Middle East, during a winter day of approximately 10 hours or a summer day of approximately 14 hours, the distance covered in one day would vary significantly. However, unlike the *gemara* in *Pesaḥim* 94a that can only apply to a 12-hour daytime period, the *gemara* in *Shabbat*, defines the end of *Shabbat* using terms like *ḥashekhah*, *hiḥsif ha-elyon ve-hishveh le-taḥton* and the appearance of three stars, all of which apply uniformly throughout the year.¹⁷⁶ What possible value could there be in introducing (especially in an era before clocks) a time-based approximation that is a lower bound, season dependent, rarely applicable, and then only under idealized conditions? In what context would such information be

¹⁷⁵ One can also assume some worsening of atmospheric conditions perhaps the result of pollution, slightly decreasing visibility. I spoke with a chemist in my synagogue, Dr. Irwin Goldblatt, who verified this as a possibility. I have no basis to determine how accurate this might be, but it is hard to imagine that it is consequential.

¹⁷⁶ Note as well, that the *gemara* in *Pesaḥim* 94a applies to

- the **average** length of a day,
- which occurs around **both the spring and fall equinox**.

However, the days that would be referred to in *Shabbat* 34b would be

- a **minimum**, as opposed to an **average**, and
- the appearance of three stars, within the time needed to walk $\frac{3}{4}$ of a *mil* after sunset, occurs **only in the spring and not the fall**.

While neither of these two differences is itself convincing, both lend further support to the thesis developed.

useful? Thus, I propose that the time needed to walk $\frac{3}{4}$ of a *mil* should be interpreted very differently than normally assumed.

- First, the time needed to walk $\frac{3}{4}$ of a *mil* it is not counting forward from sunset but counting back from the point at which the period of *bein ha-shemashot* ends. This is more consistent as well with the primary focus of the *gemara* to determine the beginning of the *bein ha-shemashot* period on Friday evening.
- Second, the time needed to walk $\frac{3}{4}$ of a *mil* is an upper bound on the length of the *bein ha-shemashot* period.

There are multiple alternatives for the beginning of the period of *bein ha-shemashot* that can address some or all of these issues, but all require that we relax the assumption that sunset defines the beginning of the period of *bein ha-shemashot* precisely, according to the *geonim*. In section 8, when discussing alternatives for the beginning of the period of *bein ha-shemashot*, we will consider these alternatives in greater depth. If the beginning of the period of *bein ha-shemashot* according to Rabbah is prior to six minutes after sunset, then his position remains inconsistent with Shmuel. However, as long as Rabbah's period of *bein ha-shemashot* begins after 4 minutes, Shmuel is at least (minimally) consistent with R. Yosef. Finally, to make the time needed to walk $\frac{3}{4}$ *mil* an upper bound it is likely that the period of *bein ha-shemashot* begins between 5 and 15 minutes after sunset.¹⁷⁷ Assuming that one knew the

¹⁷⁷ A critical supporting notion for this approach, discussed at length in subsequent sections, is that the orientation of the *gemara* when describing the period of *bein ha-shemashot* is not counting forward from sunset, as is common place in the current

point of *ḥashekhah*, then a maximum interval would let somebody know how early one would potentially have to observe a beginning to the period of *bein ha-shemashot*. Thus, in the summer when days end later, a requisite level of darkness that triggers the beginning of the period of *bein ha-shemashot* is reached approximately at the time needed to walk $\frac{3}{4}$ *mil* before the end of the day.¹⁷⁸ During other seasons of the year, the length of the period of *bein ha-shemashot* is (slightly) shorter; if one could not otherwise approximate the beginning of the period of *bein ha-shemashot*, then using the maximum provides a conservative estimate.¹⁷⁹

Following any of these approaches, the reading of Shmuel according to either R. Yosef or both Rabbah and R. Yosef is entirely informative and should be read assuming:¹⁸⁰

1. Sirius and Canopus are medium stars.

idiom, but counting back from its end. The end of the period of *bein ha-shemashot* is apparently known; the beginning of the period of *bein ha-shemashot* on Friday night is what is being debated.

¹⁷⁸ Restating Prof. Levi's chart to remove the normalization to the spring equinox, and assuming *Shabbat* ends in the summer approximately 26 to 29 minutes after sunset, then the period of *bein ha-shemashot* begins approximately between 10 and 15 minutes after sunset. The period between two depression angles that might define the period of *bein ha-shemashot* is longest in the summer. This critical fact is not widely recognized or understood.

¹⁷⁹ The large range on the beginning of the period of *bein ha-shemashot*, of approximately 5 to 15 minutes after sunset, accommodates multiple points at which *Shabbat* may end (19 to 29 minutes) coupled with multiple assumed times to walk $\frac{3}{4}$ of a *mil* (13.5 to 18 minutes).

¹⁸⁰ This still leaves open the question of whether stars or darkness are defining, despite a strong personal bias for darkness being defining,

2. Regardless of whether three stars define or are merely indicative of the end of *Shabbat*, neither one nor two stars can be defining.
3. One medium star can appear in the daytime prior to the period of *bein ha-shemashot*, but can also occur well after the beginning of the period of *bein ha-shemashot* as occurs in the winter season.
4. The appearance of two stars always signifies that the period of *bein ha-shemashot* has begun.¹⁸¹

Shmuel's statement is both informative and insightful, telling us that:

5. Sirius may appear prior to the start of the period of *bein ha-shemashot*.
6. However, if two stars are seen, immediately assume that the period of *bein ha-shemashot* has begun.

Interpreting Rabbah as beginning the period of *bein ha-shemashot* after sunset is certainly not consistent with current practice. However, for Rabbeinu Tam and his followers, including the opinion of the *Shulḥan Arukh*, Rabi Yehudah was assumed to be referring to a second sunset defined by a level of disappearing illumination from the sun, occurring approximately one hour after what is typically called sunset, i. e., the first sunset. While our approach to the statement of Shmuel is novel, our approach to sunset posits similar semantics for the term *mi-she-tishkeh ha-ḥamah* as asserted by Rabbeinu Tam and his followers **even**

¹⁸¹ Under a remote possibility, explained in footnote 183, this may only be a *harḥakah*.

for the view of the *geonim*. The only change is that the point of nightfall is brought forward appreciably from Rabbeinu Tam's 72 or 90 minutes after sunset to only 20 to 30 minutes after sunset, in accordance with the view of the *geonim*. In other respects, we read this text of the *gemara* similarly to the vast majority of commentators who followed Rabbeinu Tam and assumed that *mi-she-tishkeh ha-ḥamah* does not refer to sunset proper.

To summarize the logical and textual advantages of this approach:

- Shmuel's statement completely aligns with the position of Rabi Yehudah.
- Every part of Shmuel's assertion is entirely informative.
- The *sugyah's* primary focus is the beginning of the period of *bein ha-shemashot*, as opposed to its end. The *gemara* assumes that the end to the period of *bein ha-shemashot* is known; each of the disputants are addressing when the period of *bein ha-shemashot* begins on Friday night. If the time needed to walk $\frac{3}{4}$ of a *mil* were meant to be added to the time of sunset, it would be addressing the end of the period of *bein ha-shemashot*, as opposed to its beginning.
- The significant issue of ever seeing stars as early as at the time needed to walk $\frac{3}{4}$ of a *mil* after sunset is entirely moot.
- The three alternative fractions of the time needed to walk a *mil*, $\frac{1}{2}$, $\frac{2}{3}$ and $\frac{3}{4}$, all have identical semantics counting back from the assumed point of *ḥashekhah*. Both the time needed to walk $\frac{1}{2}$ and

$\frac{2}{3}$ of a *mil* are subtracted from the (assumedly known) point of *hashekhah* and not added to a (yet) unknown point that is the beginning of the period of *bein ha-shemashot*. In fact, all of the intervals, more than likely, provide an upper bound to the length of the *bein ha-shemashot* period. The third interval, the time needed to walk $\frac{3}{4}$ of a *mil*, now has the same meaning. It too is subtracted from the point of *hashekhah* and provides an upper bound on the length of the *bein ha-shemashot* period. The *amoraim* are quantifying the opinion of Rabi Yehudah in contrast to Rabi Nehemiah's interval, the time needed to walk $\frac{1}{2}$ *mil*.¹⁸²

- The period of *bein ha-shemashot* has some practical consequence providing a useful upper bound as opposed to a purely theoretical lower bound.
- If someone were countering the position of Rabi Yosi, who says the period of *bein ha-shemashot* is instantaneous, it is more likely that he would say that "it can be as **long** as..." as opposed to "as **short** as..."
- The period of *bein ha-shemashot* provides a conservative approximation.

Relative to the five alternatives for interpreting Shmuel, consistent with the above reading of the text and assuming the observations of Prof. Levi, we can conclude:

¹⁸² This point, disputed by some commentators, is discussed at length in the section 7.

1. Sirius and Canopus are medium stars; were they considered large stars, given the length of the period of *bein ha-shemashot*, no medium star is ever visible before *bein ha-shemashot*.
2. Shmuel is consistent with both Rabbah's and R. Yosef's interpretations of R. Yehudah; the option to align Shmuel with R. Yosef while assuming Rabbah's start to the period of *bein ha-shemashot* is sunset proper is untenable.
3. We cannot conclude definitively whether stars or darkness is defining.
4. "One star – daytime" means that up until the time of one star appearing it **may** still be daytime; in the winter, for example no stars will appear before the period of *bein ha-shemashot* begins.
5. "Two stars – *bein ha-shemashot*" means that the appearance of two stars indicate that the period of *bein ha-shemashot* has (almost always) already begun.¹⁸³

Analogous to this approach, we can also address Rambam in *Kiddush Ha-ḥodesh* 2: 8, 9, something that has long eluded explanation. Rambam in *Terumot* 7:2 simultaneously mentions all three characterizations of nightfall in a single Mishnah:

¹⁸³ The reason for hedging with the phrase "almost always," and the remote possibility mentioned previously of "two stars – *bein ha-shemashot*" being only a *harḥakah*, is R. Kapach's opinion of Rambam's view that places the beginning of the period of *bein ha-shemashot* at 15 minutes after sunset. Under that scenario, two stars may be seen by an expert knowing precisely where to look (slightly) before the beginning of the period of *bein ha-shemashot* in the spring. I prefer to discount visibility by an expert and read the statement without caveats. Furthermore, as will be discussed in section 8, R. Kapach's opinion is among the most extreme; a broad consensus brings the beginning of the period of *bein ha-shemashot* a few minutes closer to sunset.

1. Time, approximately twenty minutes, is the approximate duration of the interval of *bein ha-shemashot*.
2. *Ḥashekhah*, a required level of darkness, is what Rambam considered the defining the point of transition between days.
3. The appearance of three stars provides an indication that the point of transition, defined by *ḥashekhah*, has occurred.

While some editions of *Mishnah Torah* omit the phrase about approximately $\frac{1}{3}$ of an hour, all include the latter two characterizations. Whether, as I have indicated, Rambam is as clear that *ḥashekhah* is defining can be argued. It is remarkable that commentators, trying to deal with Rambam in *Kiddush Ha-ḥodesh* 2: 8, 9, do not raise the possibility of *ḥashekhah* being defining and stars merely being an indicator, in order to explain an otherwise difficult Mishnah. Rambam first states categorically that if after examining witnesses (on the 30th of the month) the court does not declare the 30th day *rosh ḥodesh* before *ḥashekhah*, they can no longer do so. Clearly, Rambam considers *ḥashekhah* as the point after which a court cannot declare *rosh ḥodesh*, since a court is restricted from functioning at night. In the following Mishnah, Rambam discusses an instance when the court can declare a new moon prior to the beginning of the month, versus the normal circumstance when a new month is declared on what becomes the first day of the month.

Months have either 29 or 30 days. Normally, on the night after the 29th day, if witnesses see a new moon, they come to the court the following day, and what would have been the 30th day of the previous month becomes the first day of the new month. Rambam discusses the case when the court itself sees the moon on the evening of the 29th day, close to the onset of the 30th day. Rambam distinguishes two cases:

- The court observes the moon prior to the appearance of the first star.¹⁸⁴
- The court observes the moon after the appearance of two stars.

In the former case, the court can immediately declare *rosh ḥodesh*; in the latter case, the court must wait until the following morning. Since a court is restricted from functioning at night, the court cannot declare *rosh ḥodesh* except during the daytime period, and after two stars have appeared, time for such a declaration is ostensibly lacking. Rambam's formulation raises numerous questions.

- First, why does Rambam have to create all of this new language? Given the previous Mishnah, Rambam could simply say if *beit din* can declare *rosh ḥodesh* on the eve prior to the 30th before *ḥashekhah*, they can proceed, and if not, they can wait for the following day.

¹⁸⁴ Attempts to adjust the text to address some of the complexities mentioned do not appear to be supportable and are not addressed.

- Second, why does Rambam mention only the period **before** the emergence of one star and the period **after** the emergence of two stars?
- Third, why not allow a declaration of *rosh ḥodesh* after the emergence of two stars until (close to the time that) a third star is visible?

Parallel to the issues raised above, we must also consider Rambam's opinion on other issues:

- Are Sirius and Canopus included in Rambam's formulation of the stars appearing after sunset prior to the beginning of the 30th day?
- Are stars or darkness defining?
- Are the restrictions on a *beit din* operating at night different for the declaration of *rosh ḥodesh* than they are for monetary or capital cases?¹⁸⁵
- Does the period of *bein ha-shemashot* limit the time of a *beit din*'s deliberation in general and, in particular, for the declaration of *rosh ḥodesh*? Perhaps vis-à-vis *beit din*, we follow the opinion of Rabi Yosi?

In the laws of *Shabbat*, Rambam states one Mishnah, 5:3, in terms of both *ḥashekhah* and *bein ha-shemashot* and a second, 5:4, in terms of only *bein ha-shemashot*. Here, in *Kiddush Ha-ḥodesh* 2: 8, 9, the term *bein ha-shemashot* does not appear, and it is clear from Mishnah 8 that

¹⁸⁵ A conjecture in this area is discussed in section 9.

until *ḥashekhah*, *beit din* can operate, even into the period of *bein ha-shemashot*.¹⁸⁶ Mishnah 9 is simply an application of Mishnah 8 expressed in terms that are more practical, parallel to the two *mishnayot* in *hilkhot Shabbat*.

Unlike *hilkhot Shabbat* that involves the notion of *bein ha-shemashot*, in *hilkhot Kiddush Ha-ḥodesh*, Rambam is operating around a single point of *ḥashekhah*. Relative to our fundamental question of whether stars or darkness is defining, were Rambam to consider stars as defining then all this complexity could have been avoided and simply stated in terms of whether *beit din* has time for a declaration of *rosh ḥodesh* prior to a third star appearing. Given the formulation chosen, it seems highly plausible that Rambam considers darkness defining.¹⁸⁷ Having established the principle in the prior Mishnah, Rambam reads as follows:

- Until one star appears, it is always the case that *beit din* has time to declare *rosh ḥodesh*.¹⁸⁸
- Once one star appears until the time of a second star, the amount of time varies based on the season of the year and *beit din* will have to judge whether sufficient time exists for a formal

¹⁸⁶ A period of *safek ḥashekhah* need not be raised as well, as *beit din* can be trusted to decide if the point of *ḥashekhah* has been reached and how to act in case of any uncertainty.

¹⁸⁷ As pointed out by R. Tukitzinsky, the defining moment of darkness, *hiḥsif ha-elyon ve-hishveh le-tahton*, always occurs between the appearance of the second and third star.

¹⁸⁸ Examining Prof. Levi's table one can perhaps argue about the winter season, but note that the Rambam carefully wrote **before** one star appears.

declaration. If Sirius and Canopus were not considered medium stars, it is extremely unlikely that there would ever be sufficient time for a formal declaration.

- Once a second star appears, Rambam suggests that *beit din* adjourn and declare *rosh ḥodesh* during the following day. This represents common sense since nothing is lost by delaying. The same day is declared as *rosh ḥodesh*, and thereby *beit din* avoids the possibility of a declaration made outside the requisite time, after the point of *ḥashekhah*.

Since both these *mishnayot* in *Kiddush Ha-ḥodesh* do not operate with a period of *bein ha-shemashot*, no implication can be drawn as to when Rambam begins the period of *bein ha-shemashot*. Independent of these *mishnayot*, R. Kapach argues based on multiple sources that the period of *bein ha-shemashot* starts a few minutes¹⁸⁹ after sunset. As I noted in the opening paragraph of this monograph, Rambam refers to the period after sunset as “*hu ha-nikre bein ha-shemashot*,” “that is **called** *bein ha-shemashot*.” R. Kapach would argue that although in practice we begin *Shabbat* at sunset; the precise onset of the period of *bein ha-shemashot* occurs a few minutes later. Thus, Rambam likely also read *mi-she-tishkeh* as distinct from sunset proper, with semantics similar to Rabbeinu Tam, although he undoubtedly disagreed with Rabbeinu Tam

¹⁸⁹ In multiple places in his extensive commentary on Rambam, R. Kapach suggests that Rambam begins the period of *bein ha-shemashot* 15 minutes after sunset as defined both by R. Avraham ben Ha-Rambam and by current usage. This and other aspects of Rambam’s approach are beyond the scope of this monograph.

about the end of *Shabbat*, following a view closely aligned with that of the *geonim*.¹⁹⁰

¹⁹⁰ It is assumed by some commentators that once you demonstrate that Rambam did not start the period of *bein ha-shemashot* at sunset, he must therefore agree with Rabbeinu Tam. Of course, this implication is hardly conclusive and given our approach (and more importantly that of R. Kapach), aligning Rambam with Rabbeinu Tam is entirely implausible.

Section 6. How is the duration of the *bein ha-shemashot* period to be adjusted at different locations and during different seasons (if at all)? Might this depend on whether the period of *bein ha-shemashot* is:

- 1) a period of uncertainty that is its own unique *halakhic* category, either**
 - a) a combination of both day and night, or perhaps**
 - b) a category of its own, or**
- 2) a period with a definitive transition point that we are uncertain how to pinpoint, either**
 - a) practically or perhaps**
 - b) because of some element of halakhic uncertainty, or**
- 3) an example of the Rabbis establishing a fence?**

While there is extensive debate on the theoretical nature of the period of *bein ha-shemashot*, our focus is more practical: how might the period of *bein ha-shemashot* be calculated? In section 3, we discussed alternatives on whether to adjust, based on latitude and season, the period between *alot ha-shaḥar* and sunrise where both endpoints relate to natural occurrences, and the case for adjustment is straightforward. In the case of the *bein ha-shemashot* period, the issues are much less clear-cut. While the end of the *bein ha-shemashot* period is defined by a natural event, whether its beginning is as well, is unclear, as will be explained.

In the following two sections, we will cover various opinions about when the period of *bein ha-shemashot* begins and ends.¹⁹¹ Leaving that aside for a moment, the issue addressed in this section is whether the length of the *bein ha-shemashot* interval itself should be adjusted based on latitude and season, or is its length fixed.

The conceptual approaches to the period of *bein ha-shemashot* do not determine conclusively how the period of *bein ha-shemashot* is to be calculated; at best, they are suggestive. Both of the first two approaches can be expressed as an interval defined or approximated by two distinct physical occurrences such as,

1. the interval from sunset to three stars, as is almost universally practiced today and as I began this monograph with the citation from Rambam,
2. the interval from the appearance of two stars to the appearance of three stars, an option that is difficult to support as discussed in the previous section, or more generally
3. the interval either from or after *mi-she-tishkeh ha-ḥamah* equivalently expressed as **depression angle A** until, at or after *ḥashekhah* expressed as **depression angle B**. This formulation

¹⁹¹ I previously discussed the rare practice of adjusting the end of *Shabbat* according to Rabbeinu Tam, as opposed to the more usual tradition that uses a fixed 72 / 90 minutes after sunset. On the other hand, the view of the *geonim* has almost always incorporated at least an adjustment based on latitude. While there does not appear to be any clear rationale for this disparity, that issue is not our focus here. Regardless of adjustments to the end of the period of *bein ha-shemashot* as certainly occurs according to the view of the *geonim*, the focus here is whether the length of the *bein ha-shemashot* interval itself should be adjusted.

encompasses multiple options for both angle A and B including the approach of most modern calendars where $A = 0$, in other words, sunset, and $B = 8.5$, the depression angle equating to three small stars, relatively adjacent to each other.¹⁹²

If one is defining an interval, it is reasonable to assume, similar to the discussion in section 3, that its duration adjusts based on location and season, as is often practiced today. Thus, for those living further from the equator than the Middle East, and during the summer and winter even at distances from the equator similar to the Middle East, the period of *bein ha-shemashot* is longer.

Especially since the widespread acceptance of the position of the *geonim*, a longer period of *bein ha-shemashot* has become common practice in locations further from the equator than the Middle East. However, regardless of its conceptual framework, one could argue that the *gemara's* description of the beginning of the *bein ha-shemashot* period was intended only for the Middle East. Assuming invariance to the length of the *bein ha-shemashot* period is at least as plausible, and as I will argue significantly more so, than leaving invariant the period from *alot ha-shaḥar* to sunrise. Thus, even those who conceptualize *bein ha-shemashot* as a period of uncertainty with well-defined endpoints, the duration of the interval may still be tightly bounded, with the physical

¹⁹² As one goes further from the equator, the time between a smaller and larger depression angle grows (non-linearly). Thus, any such formulation, unconstrained by other criteria, will lengthen the period of *bein ha-shemashot*.

description of when it begins intended only to be used in the Middle East.

Conceptually, moving in the direction of a *harhakah*, it becomes more likely that the duration of the *bein ha-shemashot* period should not lengthen based on season and latitude.¹⁹³ Particularly those who followed Rabbeinu Tam, and did not have an obvious anchor for the beginning of the *bein ha-shemashot* period, tended to limit the length of the *bein ha-shemashot* to the time needed to walk $\frac{3}{4}$ of a *mil*, without adjustment. For example, R. Yaakov Lorberbaum¹⁹⁴ in his commentary, *Derekh Ha-ḥaim*, sets the beginning of the *bein ha-shemashot* period 13 and $\frac{1}{2}$ minutes before the end of *Shabbat*.¹⁹⁵ R. Sofer also states

¹⁹³ This is similar to the opinion of R. Isserlein limiting the interval during which *ḥametz* is forbidden on *erev Pesah* to two hours before midday.

¹⁹⁴ He is better known as the *Nesivos*, so named, after the famous *sefer* he authored.

¹⁹⁵ The precise opinion of R. Lorberbaum is subject to debate. Some have suggested that the numbers printed in the *Derekh Ha-ḥaim*, in the section on the beginning of *Shabbat*, are in error. I have no reason to doubt that the numbers are correct as printed but require an appreciation of how R. Lorberbaum was calculating. I assume, following the opinion of R. Pimental, R. Lorberbaum ended *Shabbat* based on the appearance of stars versus waiting a full 72 minutes; that would typically occur around 45 to 60 minutes after sunset according to *Ha-zemanim Be-halakhah*, volume 2, chapter 46, depending on the exact location for which he was writing. Assuming he was writing for the area whose latitude was similar to Lissa or even Zolkiev, where *Derekh Ha-ḥaim* was first published, an end to *Shabbat* at 51 minutes after sunset, similar to Pressburg and Frankfurt although 2-3 degrees further north, is plausible. Under that assumption, his beginning of the period of *bein ha-shemashot* at 37.5 minutes after sunset is exactly 13.5 minutes prior to the end of *Shabbat*. This assumption is consistent with what he then suggests that 57 minutes earlier (approximately 20 minutes before sunset and approximately 72 minutes prior his assumed end of *Shabbat*) is when one can begin *Shabbat* without explicit declaration. From *pelag ha-minḥah*, a yet earlier point, one can start *Shabbat* only with an explicit declaration. His use of 57 and hence approximately 72 minutes appears original, perhaps a variant of R. Eliezer mi-Mitz. His calculation of *pelag ha-minḥah* apparently used a seasonal variation that correlates the time between *pelag ha-minḥah* and *tzait ha-kokhavim* to the length of the daytime period, similar to the

categorically that the period of *bein ha-shemashot* begins at most 17 minutes¹⁹⁶ prior to the end of *Shabbat*. His community ended *Shabbat* using an approach like that of R. Pimental. On the *Shabbat* addressed in his famous *teshuvah*, a baby was born at 8:30PM, 27 minutes after sunset and 25 minutes before the end of *Shabbat*.¹⁹⁷ Thus, two *posekim*, whose communities followed a *modified* version of the approach of Rabbeinu Tam,¹⁹⁸ limited the *bein ha-shemashot* period to approximately $\frac{1}{4}$ of an hour.

On the other hand, neither the argument in the previous section where the time needed to walk $\frac{3}{4}$ of a *mil* was considered a maximum nor thinking of the period of *bein ha-shemashot* as a *harḥakah* necessarily imply that at locations further from the equator the interval of *bein ha-shemashot* might not lengthen. The time needed to walk $\frac{3}{4}$ of a *mil*, even **when considered as a maximum, was perhaps intended only to**

approach of R. Pimental that was addressed previously. A discussion concerning those who amend the times given in the *Derekh Ha-ḥaim* was covered in a separate paper that appeared in the Torah U'Maddah journal, vol. xx, 2013

¹⁹⁶ In *teshuvah* 80, he supported 22.5 minutes as the time needed to walk a *mil* and $(.75 * 22.5 =)$ 16.875 (rounded to 17) minutes as the maximum length of the *bein ha-shemashot* period.

¹⁹⁷ Like R. Benish, I assume that the *teshuvah* contains an error where “an hour” must be replaced by “a *mil*,” otherwise, the *teshuvah* makes little sense. R. Benish’s attempt in the additions at the back of volume 2 of *Ha-zemanim Be-halakhah*, to defend the original text under the assumption that *Ḥatam Sofer* did not know of the *geonim*’s position but only that of R. Eliezer mi-Mitz is difficult to accept for two reasons. First, his *rebbe*, R. Adler appears to have followed the custom of Frankfurt and the *geonim* for the start of *Shabbat*. Second, in his commentary on *Shabbat* 34a, R. Sofer refers to R. Eliezer mi-Mitz’s position as the **third** alternative. (Parenthetically, while Frankfurt ended *Shabbat* at the appearance of three stars, apparently R. Adler personally followed the standard *zeman* of Rabbeinu Tam for determining the end of *Shabbat*.)

¹⁹⁸ In reality, these communities were more closely following the position of the *geonim*.

apply in the Middle East. However, when we encounter a significantly longer period of twilight and as a result, a lengthened period of uncertainly, the period of *bein ha-shemashot* might lengthen as well. Thus, even if *bein ha-shemashot* is rooted in *harḥakah*, one could still argue, that the **fence** was intended to begin at some defined level of darkness around the point that *penai mizrah maadimin* ceases, the opinion of R. Yosef, or the time needed to walk $1/12^{\text{th}}$ of a *mil* earlier, the opinion of Rabbah. Nonetheless, the *Shulḥan Arukh* implies a fixed period of *bein ha-shemashot*, and that comports with generations of practice, as well. Both current practice that equates the period of *bein ha-shemashot* to the variable period between two physically defined endpoints, sunset and the appearance of three small stars, and a practice that limited the period of *bein ha-shemashot* to a fixed-length interval are well supported.

Whether or how the period of *bein ha-shemashot* should be adjusted based on latitude and season appears independent of the basic dispute of the *geonim* and Rabbeinu Tam and is only weakly linked to how the period of *bein ha-shemashot* is conceptualized. One could argue that since both R. Lorberbaum and R. Sofer viewed themselves as following Rabbeinu Tam's opinion, their invariant interval of the period of *bein ha-shemashot* is somehow linked to that opinion. However, I fail to see any conceptual linkage between a variable versus invariant length to the *bein ha-shemashot* interval and the fundamental dispute between the *geonim* and Rabbeinu Tam. It should also be noted that both R. Lorberbaum and R. Sofer, following a *pesak* similar to R. Pimental, were

in reality implementing an end to *Shabbat* in line with the opinion of the *geonim*.¹⁹⁹

In two complex and controversial opinions, R. Nosson Adler and R. Feinstein, both vary the length of the *bein ha-shemashot* period.²⁰⁰ Prior to both of these two *posekim*, R. Pimental supported explicitly a variable length to the *bein ha-shemashot* period, using a technically complex and problematic method (based on the length of the daytime period between sunrise and sunset).²⁰¹ Details of his approach are not pursued further.

In the *teshuvah* in which he established 50 minute after sunset as the end of *Shabbat* in the New York area, R. Feinstein took R. Pimental's approach²⁰² one-step further. Similar to R. Pimental, R. Feinstein adjusts Rabbeinu Tam's end to *Shabbat* based on the appearance of stars, equating 50 minutes after sunset in New York to 72 minutes after sunset in Lithuania. However, R. Feinstein then goes further, adjusting the time needed to walk a *mil* so that the time needed to walk 4 *milin* equals the number of minutes after sunset that *Shabbat* ends. Thus, based on his 50-minute period for Rabbeinu Tam's end of *Shabbat* in the

¹⁹⁹ Unless one insists that the period of *bein ha-shemashot* begins at exactly sunset according to the *geonim*, these same positions should be valid according to the *geonim* as well.

²⁰⁰ If the beginning of the period of *bein ha-shemashot* were a point after sunset, then calculating its variation by season and latitude would likely require a precisely defined appearance of the sky, equated in modern terms to a specific depression angle. This area has received scant attention from *posekim*; I return to this point in the epilogue.

²⁰¹ For further discussion of R. Pimental's approach, see footnote 127.

²⁰² I covered R. Feinstein's approach in both the introduction and section 3.

New York area, R. Feinstein calculated the time needed to walk a *mil* for New York as 12.5 minutes ($\frac{1}{4}$ of 50 minutes), and the time needed to walk $\frac{3}{4}$ of a *mil*, at slightly more than 9 minutes. Using that shortened *mil*, he maintained that the *geonim's* end to *Shabbat* remained the time needed to walk $\frac{3}{4}$ of a *mil*, or 9.375 minutes after sunset. In addition to the issues previously raised that are common to the approaches of both R. Pimental and R. Feinstein, R. Feinstein's methodology raises two additional issues, one fundamental and one computational.

First, fundamentally, the time needed to walk (one or) 4 *milin* is not linked to location or latitude or to the length of time between sunset and the appearance of stars. Intervals defined by physical phenomena are currently measured by clock-time; in the times of the *gemara*, they were approximated by the interval required to walk a particular distance or perform a specific activity. We might adjust an interval based on latitude and season and then recalculate its clock or *mil*-based equivalent; the reverse, adjusting the length of time required to walk a *mil* and then expecting physical phenomena, like the appearance of three stars to correspond is inexplicable.²⁰³ As a result, the time R. Feinstein ascribed to the *geonim's* end to *Shabbat*, less than 10 minutes after sunset, is so early that at most one star may be visible, only during certain seasons of the year and only to an expert, who knows where to look. This cannot be the opinion of the *geonim* applied to New York.

²⁰³ As discussed at end of section 3, R. Feinstein may have seen language that talked in terms of adjustments to the time needed to walk a *mil* in order to demonstrate, as mentioned previously, that *pelag ha-minḥah* always occurs at the time needed to walk $\frac{1}{6}$ th of a *mil* before sunset, not only on 12 hour days. He may have extended that approach that is itself, as previously indicated, imprecisely worded.

Similarly, it would follow that *alot ha-shaḥar* is 50 minutes before sunrise as well, again contradicted by reality and drastically different from R. Feinstein's own *teshuvah*²⁰⁴ on the topic. To be clear, adjusting the time needed to walk a *mil* is akin to saying that 72 minutes on clocks in Lithuania equals 50 minutes on clocks in New York; this approach is significantly more disconcerting than allowing one to end *Shabbat* after 50 minutes in New York according to Rabbeinu Tam. I have not read nor can I conceive of a defense of this position.²⁰⁵ Surprisingly, I have not seen this issue even raised, despite widespread use of both the *teshuvah* and its methodology.

Second, computationally, using a linear approximation is problematic since extrapolation to larger depression angles is not linear.²⁰⁶ Assume two latitudes, for example, 30 and 40 degrees, and two depression angles, for example, 4 and 8 degrees. While the number of minutes after sunset required to reach a 4-degree depression angle may be only 3 minutes greater at 40 degrees than at 30 degrees, the number of minutes after sunset to reach a depression angle of 8 degrees might be 10 minutes greater at 40 degrees than at 30 degrees. Thus, the final “ $\frac{3}{4}$ *mil* equivalent” interval prior to *hashekhah* is of a longer duration than the first “ $\frac{3}{4}$ *mil* equivalent” interval after sunset. Disregarding this non-

²⁰⁴ *Igrot Moshe O. H.* 4:6.

²⁰⁵ What was intended was to reduce the time after sunset that *Shabbat* ends relative to Lithuania. That remains problematic, similar to the issue raised with R. Pimental's approach.

²⁰⁶ Latitude, seasonality and the magnitude of the depression angle all affect the length of time between sunset and reaching a particular depression angle in a non-linear manner. R. Feinstein uses the same invariant interval of about ten minutes at both the beginning and end of the *bein ha-shemashot* interval.

linearity becomes more consequential at locations further from the equator, particularly when applied as the basis for a leniency. Despite these difficulties, R. Feinstein clearly assumes adjustment based on latitude to the length of the *bein ha-shemashot* period.

R. Sofer transcribed R. Adler's rulings for the period of *bein ha-shemashot*,²⁰⁷ without providing explanation or justification. Prior attempts²⁰⁸ to explain R. Adler's conceptual basis have never been entirely satisfactory; some appear completely unsustainable. R. Adler ruled that the *bein ha-shemashot* period is either 24 or 35 minutes, choosing whichever is the greater stringency in a case of a *de-oraysah*, and the greater leniency in a case of a *de-rabbanan*. I will not speculate on the conceptual rationale used to derive these numbers.²⁰⁹

Independent of its rationale, there is significant uncertainty even on how to understand the calculation that R. Adler suggested. Four options present themselves:

- Options 1 and 2: R. Adler is **counting forward** from the beginning of the period of *bein ha-shemashot*, either **option 1) from sunset**

²⁰⁷ *Hiddushai Hatam Sofer*, at the end of *Seder Moed*. As argued below, it does not comport entirely with R. Sofer's position in the *teshuvah* referenced.

²⁰⁸ *Ha-zemanim Be-halakhah*, pages vol. 2, chapter 46, footnote 77.

²⁰⁹ A number of options are mentioned by *Ha-zemanim Be-halakhah*. Some, quoted by R. Benish, try to equate 24 to 35 minutes in Frankfurt with various intervals for the period of *bein ha-shemashot* in the Middle East. It is also unclear why or if the numbers are invariant year-round; it may be the result of R. Adler having calculated an upper bound. As noted below, however, the numbers appear to align with practice.

or **option 2)** from some other start to the *bein ha-shemashot* period after sunset.

- Options 3 and 4: R. Adler is **counting back** from the end of the *bein ha-shemashot* period, either **option 3) from the appearance of three small stars**, 50 to 55 minutes after sunset, as practiced in Frankfurt or **option 4) from an invariant 72 minutes after sunset**.

Three of the four options can be easily excluded. If R. Adler were counting forward from sunset, as some who censored this passage may have thought, he would be ending *Shabbat* exceedingly early.²¹⁰ As well, the passage seems to be trying to establish the start of the *bein ha-shemashot* period, implying it is not an obvious point like sunset.

Furthermore, were this even remotely possible, his student, R. Sofer would never have allowed a *brit* for a baby born *Shabbat* afternoon, 27 minutes after sunset, on the following *Shabbat*.²¹¹ R. Adler could also

²¹⁰ In a taped lecture on Jewish censorship in literature in modern times, recorded 05/12/99, Prof. Leiman uses this practice of R. Adler that *Ḥatam Sofer* transcribed, as an example of something that was deleted by a Satmar *Ḥasidic* group who considered it too troublesome for publication. It is unclear whether Prof. Leiman is criticizing the interpretation or acknowledging its correctness and only criticizing the audacity of its censorship. The latter appears more reflective of his tone.

²¹¹ I assume that R. Sofer would not allow a *brit* for a baby born *Shabbat* afternoon during the period of *bein ha-shemashot* even prior to the period of *safek ḥashekhah*. It is possible that in practice R. Sofer did not make any distinction between *bein ha-shemashot* and the period of *safek ḥashekhah*; perhaps he treated them as identical even conceptually. In general, I assume that *bein ha-shemashot* represents a longer interval extending beyond *safek ḥashekhah*. Often in practical contexts, the two terms, *bein ha-shemashot* and *safek ḥashekhah*, are treated as coincident. Given the lack of any easily defined events for the beginning of the period of *bein ha-shemashot*, the ability to distinguish it from the point of *safek ḥashekhah* may have

not be counting from some unknown beginning of the period of *bein ha-shemashot*; that would be entirely circular, since it is that beginning point to the *bein ha-shemashot* period that he is attempting to specify. Thus, the times are counting back not forward. From his language, it is clear that R. Adler was counting back from a well-known point to establish the beginning of the period of *bein ha-shemashot*. If that point was the practiced time that three small stars appear, approximately 50 to 55 minutes after sunset, or approximately 25 minutes after the time the baby in *teshuvah* 80 was born, again one has to assume that R. Sofer would at least raise the possibility of the opinion of R. Adler, his revered *rebbe*. This leaves only one alternative: R. Adler was counting back from the conventional time to walk 4 *milin*, 72 minutes after sunset. A point 35 minutes earlier, or 37 minutes after sunset, is one possible beginning to the period of *bein ha-shemashot*, and 24 minutes earlier, or 48 minutes after sunset, is the other. Computationally, 37 minutes after sunset almost coincides with what we assume is R. Lorberbaum's position²¹² for the beginning of the period of *bein ha-shemashot*, and 48 minutes after sunset is a few minutes earlier than the practiced end of *Shabbat* in the Frankfurt community, perhaps subtracting a few minutes that correspond to *tosefet Shabbat*. Note that both these *zemanim* might vary by season in a manner that is unexplained. Despite the fact that this suggestion has both textual and conceptual challenges, I find it the only explanation that at least corresponds to the 18th century practice of

led to the two *zemanim* being fused at least in practice, certainly by those following Rabbeinu Tam. Whether and how that might have then influenced modern practice that follows the *geonim* would require a detailed study.

²¹² I am unsure of the location that R. Lorberbaum was writing for, but if it was Zolkiev as I hypothesized, it is at approximately the same latitude as Frankfurt.

many cities in that region (other than Frankfurt²¹³) and the later *pesak* of R. Sofer. Independent of rationale, R. Adler apparently specified the beginning of the period of *bein ha-shemashot* with consideration for both the latest accepted time in many communities, 37 minutes after sunset, but where that represents a leniency, one must wait until close to the end of the day.

The application of either R. Sofer's or R. Lorberbaum's *pesak* to another location is straightforward; one would simply subtract an invariant period of time equal to time required to walk $\frac{3}{4}$ of a *mil*, either 16.875 or 13.5 minutes, from the varying point at which *Shabbat* ends at that location.²¹⁴ R. Adler's *pesak* is more problematic since using 72 minutes and a period of *bein ha-shemashot* beginning 24 to 35 minutes earlier,

²¹³ Frankfurt was one of the few cities that even prior to the time of the *Gaon* started *Shabbat* around sunset, and R. Adler probably followed that custom as well. If R. Adler was addressing the beginning of *Shabbat* as opposed to other biblical commandments, then he was likely not directing his comments to the Frankfurt community. Additionally, starting *Shabbat* around sunset may have been viewed as a stringency that would not apply to situations where an early start to the period of *bein ha-shemashot* would be a leniency.

²¹⁴ One issue common to many responsa in this area is establishing the beginning of the period of *bein ha-shemashot* by counting back from the time that three small, adjacent stars appear, the practiced end of *Shabbat*, as opposed to counting back from the earlier point when three medium stars appear, the actual end of *Shabbat*. It is both more logical and consistent with the *sugyah* in *Shabbat* to determine the beginning of the period of *bein ha-shemashot* by subtracting from the (approximate) appearance of three medium stars than to subtract from the practiced end of *Shabbat*, the appearance of three small stars, incorporating various stringencies. This delayed starting point to the period of *bein ha-shemashot* often creates an unintended leniency and has no apparent defense. The magnitude of the leniency increases as one moves to latitudes further from the equator where the time between the first appearance of three medium and three small stars grows further apart, particularly in the summer. This problematic method of calculation occurs in a number of scholarly and popular publications.

appears to be an unwarranted *leniency* in the spring and fall seasons unless again 24 to 35 minutes was a yearly maximum.

In summary,

- both R. Lorberbaum and R. Sofer clearly support an invariant interval of *bein ha-shemashot*, while
- current practice adjusts the length of the *bein ha-shemashot* period, as did R. Pimental, R. Adler and R. Feinstein..

Neither the basic argument of Rabbeinu Tam and the *geonim* nor the alternative conceptualizations of the period of *bein ha-shemashot* necessarily imply whether the *bein ha-shemashot* interval is to be adjusted or left invariant.

Section 7. When does the period of *bein ha-shemashot* end? How are the criteria specifying the end of the *bein ha-shemashot* period interpreted by various authorities?

As explained earlier, the *Gaon* distinguished *tzait (kol) ha-kokhavim*, the end of the evening twilight period mentioned in *Pesaḥim*, from *tzait ha-kokhavim* discussed in *Shabbat* (and the beginning of *Berakhot*).

According to Rabbeinu Tam, they coincide. According to the *Gaon* and the *geonim*, *Shabbat* ends when there is still residual illumination from the sun. From the conceptual viewpoint of Rabbeinu Tam, the end of *Shabbat* coincides with the point at which illumination from the sun is either exceedingly minimal or has disappeared completely.

The thesis to be developed is that for the *gemara*, the time when *Shabbat* ends is not in dispute. Perhaps most fundamental to any discussion of the end of the period of *bein ha-shemashot* is recognizing that despite the significant disagreement between Rabbeinu Tam and the *geonim* that has dominated discussion for centuries, the *Bavli* does not record any significant disagreement on this point. Reading the *Bavli* in *Shabbat* and the majority of the opinions of the *Yerushalmi* in *Berakhot*,²¹⁵ **the issue in dispute is not when the period of *bein ha-shemashot* ends but rather when it begins; the focus of the *Bavli* in *Shabbat* is Friday night.** The two physical descriptions, “*hiḥsif ha-*

²¹⁵ In the *Yerushalmi* there is a single opinion supporting two stars versus three stars as the end of the period of *bein ha-shemashot*. There is also one opinion that Rabi Yosi's end of *Shabbat* occurs during R. Nehemiah's *bein ha-shemashot* interval of the time needed to walk $\frac{1}{2}$ *mil*, a position not supported by any of the *rishonim* commenting on Rabi Yosi in the *Bavli*.

elyon ve-hishveh le-tahton” and three medium stars, while subject to debate between Rabbeinu Tam and the *geonim* as to their interpretation, are accepted by the *gemara* as the definitive point at which the period of *bein ha-shemashot* and *Shabbat* concludes. With only a few alternative viewpoints, most commentators do not see any significant disagreement about the end of *Shabbat* among the three major opinions²¹⁶ cited:

1. Rabi Yehudah defines the end of *Shabbat* as the point when “*hihsif ha-elyon ve-hishveh le-tahton.*”
2. Rabi Yosi maintains that the period of *bein ha-shemashot* is exceedingly short, or perhaps just a point along a continuum.
3. R. Yehudah in the name of Shmuel specifies that the appearance of three stars signify the beginning of *lailah*.

The majority of *rishonim* maintain that Rabi Yosi and Rabi Yehudah argue about an interval at most equal to the time needed to walk 50 *amot*²¹⁷ or 1/40th of a *mil*, about ½ of a minute. The *gemara* cites the opinions of Rabi Yosi and Rabi Yehudah together with a discussion of

²¹⁶ The *gemara* decides in favor of Rabi Yehudah as far as the start of *Shabbat*. Whether that is just stringency and we definitively follow R. Yosi as a matter of law, or there is actually an extended period of uncertainty, relates to the options defining *bein ha-shemashot* addressed in the previous section.

²¹⁷ One of the *baalei tosefot* disagrees and places the point at which nightfall occurs according to Rabi Yosi significantly later. However, see *Ha-zemanim Be-halakhah*, chapter 40 where he lists a significant group of commentators that place Rabi Yosi’s end of *Shabbat* either very slightly later (less than the time needed to walk 50 *amot*, the opinion of various *baalei tosafot*, Ramban, R. M. Yaffe, etc.) or coincident with that of Rabi Yehudah (the opinion of Rashi, *Tur*, *Gaon* of Vilna, etc.).

the appearance of three stars without any suggestion that they are in conflict.

The one possible exception is the opinion of Rabi Nehemiah²¹⁸ who states that the duration of the period of *bein ha-shemashot* is the time needed to walk $\frac{1}{2}$ of a *mil*. Associated with Rabi Nehemiah's position in the *Bavli* in *Shabbat* and in a more detailed story in the *Yerushalmi* in *Berakhot* is a scenario involving the sun, *har Carmel* and one immersing himself in the Mediterranean Sea.²¹⁹ The scenario in the *Bavli* is uncertain along multiple dimensions:

- Where was the person when the scenario begins - on top of the hill or on the shore below?
- Is he looking up at the sun's illumination to the east on the top of *har Carmel*, or is he watching the sun as it sets to the west from the top of *har Carmel*?
- If he is on the hill, does that imply that because of his elevation, he is able to see the sun for about 2 to 3 minutes longer than one at sea level, or does the story begin before or at sunset?

²¹⁸ Unlike the *rishonim* mentioned above, an opinion in the *Yerushalmi* suggests that the point Rabi Yosi considers nightfall falls either somewhere during the *bein ha-shemashot* interval of Rabi Nehemiah or at its conclusion. The former point of view clearly places the end of *Shabbat* in dispute. The latter makes Rabi Yosi's and Rabi Nehemiah's end of the period of *bein ha-shemashot* identical. Clearly, the *Bavli* seems to side with the latter opinion but only if Rabi Nehemiah is just arguing about the start of the period of *bein ha-shemashot*, a point I clearly support. That point is implicitly disputed by those who assume that Rabi Nehemiah starts *Shabbat* precisely at sunset.

²¹⁹ I assume as described in the additional notes to *Ha-zemanim Be-halakhah*, volume 2, page 660, that *har Carmel* refers to a hill about 900 feet from the summit to the base with its base is situated another 300 feet from the Mediterranean Sea.

- If he is on the beach looking towards the hill, has the eastern horizon already darkened, but only *har Carmel* is still light because of its elevation, or has no part of the eastern sky darkened?
- Does he return (*oleh*) from immersion in the sea, or does he return to the top of *har Carmel*?
- When he returns, is that the beginning or the end of Rabi Nehemiah's period of *bein ha-shemashot*?

I could go on, but it is clear that it would be difficult to determine Rabi Nehemiah's viewpoint since he does not position his interval relative to that of Rabi Yehudah, and almost every relevant detail of the story is ambiguous. Furthermore, Rabi Nehemiah uses the term *mi-she-tishkeh ha-ḥamah*, a term that Ramban reads as different from *shekiyah* (see the next section) and hence we do not know for certain when his period of *bein ha-shemashot* begins.

While the scenario summarized succinctly by the *Bavli* in *Shabbat* is unclear, a similar story in the *Yerushalmi Berakhot* is somewhat clearer. Ramban equates the two stories and thus we know that

- the person is on top of *har Carmel*,
- the sun (or its bright illumination) is still visible in the west, at least from that elevation,
- he goes down to the Mediterranean Sea,
- immerses himself in the sea

and he can be certain that he has immersed himself prior to the beginning of the *bein ha-shemashot* period.

Ramban's interpretation is consistent with (though not definitively supportive of) the viewpoint that the end of *Shabbat* is not in dispute. Despite Ramban's interpretation, some assume²²⁰ that the period of Rabi Nehemiah, like that of Rabbah, begins at sunset. This viewpoint would create a dispute as to the end of *Shabbat* as well. That would mean that according to Rabi Nehemiah (as interpreted by the *geonim*) *Shabbat* ends at most 12 minutes after sunset before two stars are visible (even to an expert knowing precisely where to look). This would have placed his view²²¹ in dispute with many of the views in the *Yerushalmi* in *Berakhot* where his opinion is assumed consistent with those who end *Shabbat* when three stars are visible. Thus, starting Rabi Nehemiah's period exactly at sunset appears to be impossible to maintain. However, one cannot eliminate the possibility that Rabi Nehemiah's interval of *bein ha-shemashot* starts after sunset but still ends earlier than the other viewpoints. For example, the beginning of Rabi Nehemiah's period, could match that of R. Yosef and end slightly earlier (the time needed to walk $\frac{1}{6}$ th of a *mil*), after or coincident with

²²⁰ See for example Rashba, *Shabbat* 34b.

²²¹ According to an opinion in the *Yerushalmi* that places Rabi Yosi's point of *ke-heref ayin during* Rabi Nehemiah's interval of the time needed to walk $\frac{1}{2}$ *mil*, the end of *Shabbat* is also in dispute.

the appearance of the second star.²²² However, were that Rabi Nehemiah's position, it is again somewhat surprising that the *Yerushalmi* does not correlate two stars with his viewpoint. Alternatively, perhaps Rabi Nehemiah ends *Shabbat* when only an expert can see three stars, but that approach also places the beginning of Rabi Nehemiah's period of *bein ha-shemashot* after sunset. Both of these rather remote possibilities are difficult to exclude unequivocally.

However, even including the position of Rabi Nehemiah, whose opinion is rejected in any case, it is very plausible, and perhaps compelling, to read the texts of the *gemara* assuming that the end of the period of *bein ha-shemashot* and *Shabbat* is largely undisputed. Certainly excluding the position of Rabi Nehemiah, the dispute centers only on when the period of *bein ha-shemashot* begins.

To a contemporary reader this is often surprising (and often overlooked), especially because of the general popularity in our culture, both western and Moslem, of reporting (in the newspaper) or announcing (by the call of the *mugrab* approximately 10 minutes after²²³) the time of sunset. In an almost mirror image of the *gemara*, we have the universally accepted practice in modern times to begin *Shabbat* at sunset. **Sunset is the time known to all of us as the**

²²² See Prof. Levi's table in section 5, to verify that the time needed to walk $\frac{1}{4}$ of a *mil* (the difference the time needed to walk $\frac{1}{2}$ *mil* and $\frac{3}{8}$ *mil*) is very close to the time between the second and third star again varying based on how visible stars need be.

²²³ There are references to *posekim* who used of the call of the *mugrab* for determining if the period of *bein ha-shemashot* has begun. In an earlier era, the "Arab clock" began its 24-hour period 10 minutes after sunset, at which time the call of the *mugrab* was heard.

beginning of the period of *bein ha-shemashot and Shabbat*; when *Shabbat* ends is in dispute. To the *gemara* it appears that the opposite was the case.

In modern times, we tend to discuss the period of *bein ha-shemashot* with sunset as the anchor point. For the *gemara*, the anchor point was likely nightfall. The period of *bein ha-shemashot* extends back from the point of nightfall, when *Shabbat* ends, to the beginning of the period of *bein ha-shemashot*. Of course, if we assume that we know the starting point for the period of *bein ha-shemashot*, then we could derive the time when *Shabbat* ends. However, it is that start to the period of *bein ha-shemashot* that the *gemara* in *Shabbat* appears to be debating, an issue that I address in the next section.²²⁴

The *gemara* provides two descriptions of the end of *Shabbat*,

- the point at which the sky has reached a particular configuration, (the opinion of Rabi Yehudah and according to almost all *rishonim* within 36 seconds or less²²⁵ of Rabi Yosi's opinion as well), and
- the appearance of three (medium) stars.

²²⁴ See the last three chapters of *Ha-zemanim Be-halakhah* for a discussion of the terms used by the early *mishnayot*. The term sunset occurs only four times in early *halakhic* literature; the prevalent term is *hashekhah*, denoting a level of darkness as the sun's illumination dissipates after sunset.

²²⁵ As stated previously, most *rishonim* assume the two points are coincident or that Rabi Yosi's interval is less than the time needed to walk 50 *amot* later. Since, a *mil* is 2000 *amot*, the time needed to walk 50 *amot* is at most $50 / 2000$ or $1/40$ of the maximum time to walk a *mil* which is $1/40$ of 24 minutes or $(24 * 60 / 40 =)$ 36 seconds. Assuming the more standard time to walk a *mil* of 18 minutes, the interval reduces to $(18 * 60 / 40 =)$ 27 seconds.

Despite debate concerning the appearance of one and two stars and the beginning of the period of *bein ha-shemashot*, the two points indicating or defining the end of *Shabbat* are assumed consistent.²²⁶

Those who follow the practice of the *geonim* assume that three medium stars appearing anywhere in the sky represent the end of *Shabbat*. As Prof. Levi's chart indicates,²²⁷ even a casual observer in Jerusalem will likely see three stars, depending on the season of the year, 20 to 30 minutes after sunset. The phrase defining the end of *Shabbat* in the *gemara*, "*hihsif ha-elyon ve-hishveh le-tahton*," is interpreted as meaning that looking towards the eastern half of the sky, the sky's apex has gotten as (**pale** or) **dark** (or perhaps **deep blue** as Prof. Levi suggests) as the eastern horizon. The term *hihsif* means darkened²²⁸ and *elyon* refers to the high point of the sky while *tahton* refers to the eastern horizon.²²⁹ R. Tukitzinsky²³⁰ made numerous observations of this period in Jerusalem and states that *hihsif ha-elyon ve-hishveh le-tahton*, as described above, always occurs between the appearance of the second and third star. Both these definitions remain applicable as one travels from the latitude of the Middle East, going either closer or

²²⁶ As was dealt with extensively in section 5, perhaps one is providing a definition of the end of the period of *bein ha-shemashot* and the other is a close approximation, indicating that the period of *bein ha-shemashot* has ended. For current purposes, we need only assume that the two descriptions of the end of *Shabbat* are approximately equivalent.

²²⁷ See section 5.

²²⁸ Precise semantics of the term *hihsif* is an argument between Rashi and the *Arukh*.

²²⁹ We are not measuring illumination as we would in the morning with *alot ha-shaḥar* or *mi-she-yakir*; we are observing the appearance of the sky.

²³⁰ *Bein Ha-shemashot*, pages 17 - 21.

further from the equator. Of course, as one travels much further from the equator, well before one reaches either the North or the South Pole, during some months of the year, *hihsif ha-elyon ve-hishveh le-tahton* will not occur and three stars will not appear, the sky never becoming sufficiently dark.²³¹

For Rabbeinu Tam, both the appearance of stars and the definition of *hihsif ha-elyon ve-hishveh le-tahton*, as defined above, present an almost insurmountable challenge, as they occur in the Middle East well before 72 minutes after sunset. Those who defend Rabbeinu Tam focus on a discussion in the *gemara* between Abaye and Ravah, looking to the west as opposed to the east. They are then forced to read the entire set of descriptions given by the *gemara* as referring not to the eastern horizon but to the western horizon, where the sun's illumination is visible well beyond 30 minutes.

For the *geonim*, the discussion between Ravah and Abaye is explained by looking to the side of the horizon that is the cause, the west, as opposed to (just) the east, where one normally observes the effect. Both the appearance of stars and the darkening eastern sky result from the sun continuing to descend further below the horizon in the west, thereby providing less illumination. Though not stated explicitly by the *gemara*, Ravah could probably tell by looking to the west that the sun was so far below the horizon that its illumination was sufficiently

²³¹ As noted previously, some propose *halakhic* midnight, *hatzot ha-lailah*, as an alternative for the end of *Shabbat* under those circumstances.

reduced. The reason that necessitated looking to the west was, perhaps, the presence of clouds that obscured the view of the eastern side of the sky. Note that even on a cloudy day, the extent of the sun's illumination on the western horizon is often more easily visible.²³²

To accommodate Rabbeinu Tam's viewpoint, each of the statements has to be interpreted differently. Raavyah²³³ suggests that the stars referred to must appear in the western sky. In the Middle East the sun continues to provide some illumination in the west until approximately an hour past sunset; therefore, stars would not be visible close to the western horizon until then. I have not read accounts of the observation of stars in the western horizon around this point in time, but for those who maintain that the time needed to walk 4 *milin* is 72 minutes, this solution is plausible. However, for those who maintain that the time needed to walk 4 *milin* is 90 minutes, this is problematic; three stars are likely to be visible well beforehand, given that even the western sky is completely dark at least ten minutes earlier.

A more difficult issue is *hihsif ha-elyon ve-hishveh le-tahton*. The *gemara*, as interpreted, states that the apex (*ha-elyon*) darkens to the level of the horizon (*ha-tahton*). **That occurs only in the eastern sky.** In the west, exactly the opposite occurs; the last point to **darken** is not

²³² It is not critical to decide whether Ravah and Abaye were observing the sky at the beginning of the period of *bein ha-shemashot* on Friday or at its conclusion on Saturday night; the former is clearly much more consistent with the focus of the *gemara*.

²³³ Raavyah, *Shabbat* 199, in vol. 1 page 268, is discussed in *Ha-zemanim Be-halakhah*, chapter 43, footnotes 64 and 66.

the apex but the western horizon. It has been suggested²³⁴ that Rabbeinu Tam would have to read the entire statement as referring not the sky, but to the sun. The top (*elyon*) refers to the top of the sun as it sets while the bottom (*tahton*) refers to the bottom of the sun. What the *gemara* is saying is that night begins when the top of the sun ceases to provide illumination just as the bottom had ceased providing illumination earlier. A brilliantly original but difficult reading of the *gemara*.

Most difficult is the disagreement between Abaye and Ravah looking to the eastern and western horizon. For the *geonim*, while the impact of the setting sun that defines both the beginning of the *bein ha-shemashot* period as well as *hashekhah* occurs in the east, the cause of the increasingly reduced illumination is the sun setting in the west. Hence, we can understand the reason for looking in either direction. However, according to the opinion of Rabbeinu Tam, Abaye and Ravah are looking towards the eastern and western horizon at least 50 minutes after sunset. Looking to the east provides no information; at that time, the eastern sky has been dark and completely unchanged to the naked eye for a significant period. Only observation of the western sky is still relevant; observing both directions at that time is inexplicable.

Evidently, the disagreement between Abaye and Ravah validates the opinion of the *geonim* with respect to the end of *Shabbat*.²³⁵

²³⁴ See *Ha-zemanim Be-halakhah*, chapter 43, pages 210 and 211.

²³⁵ Similar to aligning the argument of Rabbah and R. Yosef with the argument of the *geonim* and the Rabbeinu Tam referenced in *Am Mordechai*, chapter 2, some have tried to address this issue, as well. Aligning the dispute between Abaye and Ravah

Rabbeinu Tam himself, living in France, might have read the *gemara* similarly to the *geonim*. He likely assumed what he saw around the period of *bein ha-shemashot* was not different from observations in the Middle East; Rabbeinu Tam simply assumed that even Rabbah's beginning of the period of *bein ha-shemashot* does not begin at sunset. What is problematic is reconciling Rabbeinu Tam's opinion with observations in the Middle East.²³⁶

Independent of the significant issue of reading the *gemara* to comport with the opinion of Rabbeinu Tam, (an issue that neither of the options below addresses,) it is still necessary to apply Rabbeinu Tam's position to other latitudes and seasons. As discussed above, this drives one of two options (collapsing the multiple options detailed in the beginning of section 3):

on which direction to turn with the dispute between Rabbeinu Tam (looking west) and the *geonim* (looking east) is difficult to reconcile with the text. More fundamentally, it does not address the issue of the lack of any utility in looking east close to the time of Rabbeinu Tam's interval of *bein ha-shemashot*. I have not seen any discussion of this issue, and believe it is the greatest observational challenge to the opinion of Rabbeinu Tam.

²³⁶ The historical record around two major figures that immigrated to Eretz Israel and followed Rabbeinu Tam, Ramban and R. Yosef Caro, is unclear. I have seen nothing written of Ramban. Some have suggested that R. Yosef Caro's *hilkhot milah*, *Y. D.* 262:4-6, written in Israel, reflects a changed point of view; it does speak more of the appearance of stars than intervals of time. However, that is not conclusive, and is perhaps an early version of R. Pimental's approach. It is also entirely plausible that prior to the widespread use of clocks, observable events were more critical than an expression of time. His use of the term *mi-she-tishkeh ha-ḥamah* in *Y. D.* 266:9 does not imply support for any particular approach.

- **Option 1:** Apply latitude (and season) adjustments to the opinion of Rabbeinu Tam as practiced as a personal stringency by R. Soloveitchik and a few others. This option has no record of ever having been practiced except by a small number of individuals and only in recent generations. I am aware of no one who has even hinted that this could have been the practice of either Rabbeinu Tam or the *Shulḥan Arukh*. While this approach harmonizes the position of Rabbeinu Tam with our current understanding of the impact of latitude and season, it does not reconcile Rabbeinu Tam's reading of the *gemara* in *Shabbat* with observations in the Middle East.

- **Option 2:** Make no latitudinal adjustment to Rabbeinu Tam's opinion. This latter approach clearly corresponds to established practice in many communities, supported by a number of authorities.²³⁷ In conversation, I have heard two related arguments advanced in defense of not adjusting for latitude. First, we are simply keeping times equivalent to those practiced in Israel. Second, since the laws of *zemanim* will break down anyway as we approach the north or south poles, using times as practiced in Israel is a credible alternative. Linking to practice in Israel would be more compelling if a majority of Jewish

²³⁷ *Ha-zemanim Be-halakhah*, chapter 46 summarizes practice in different locations. Making no latitudinal adjustment is the prevalent practice by those following the opinion of Rabbeinu Tam in the United States and was apparently common in northern Europe, while in other parts of Europe, *Shabbat* ended (earlier not later) with the appearance of three small, adjacent stars, typically about 20 minutes earlier. *Ha-zemanim Be-halakhah* does not mention option 1.

communities in and around Israel²³⁸ followed Rabbeinu Tam as opposed to the *geonim*. It is only in recent times that some, who immigrated to Israel from Europe, instituted the practice of Rabbeinu Tam. Even R. Karelitz followed the opinion of the *geonim*, albeit with a number of stringencies.²³⁹ As well, even in Israel, adjustments for seasonality must be made. Thus, given actual practice in and around Israel, this rationale is at best theoretical and more likely a *post facto* rationalization. As well, not making adjustments because *zemanim* break down near the poles where the very notion of day and night is not well defined is hardly compelling.

The approach of making no latitudinal adjustment, but making seasonal adjustments applied consistently to both the morning and the evening *zemanim*, can be maintained at least in theory as long as one is willing to disregard the textual challenges from the *gemara's* description of the end of *Shabbat*. As noted earlier, this might imply disallowing prayers prior to the time needed to walk 4 *milin* before sunrise even if the point of *mi-she-yakir* has passed. More critically, it would allow ending *Shabbat* the time needed to walk 4 *milin* after sunset even if three medium stars have not yet appeared. I have never read or heard of such a practice. Additionally, an unadjusted 72 / 90 minutes must address issues discussed previously, resulting from the required symmetry between the interval from sunset until the conclusion of *Shabbat* (and

²³⁸ See *Ha-zemanim Be-halakhah*, chapter 45.

²³⁹ See *Ha-zemanim Be-halakhah*, chapter 51, page 534.

other days of the week) with the interval from *alot ha-shaḥar* until sunrise.

On the other hand, R. Soloveitchik's approach to Rabbeinu Tam, while an elegant conceptual solution, is inconsistent with generations of practice, and again, does not address the challenges from the text of the *gemara* in *Shabbat*.

Claiming to uphold the approach of Rabbeinu Tam, R. Pimental suggests that the appearance of three stars always take precedence over the time needed to walk 4 *milin* (72 minutes). While this and similar suggestions, including R. Feinstein's, have gained acceptance in many communities, they cannot be reconciled with the conceptual approach of Rabbeinu Tam who equates the endpoints of the *sugyot* in *Pesaḥim* and *Shabbat*. As indicated, this approach often introduces other anomalies regardless of how one chooses to deal with the morning *zemanim*. Even if one were to unilaterally define *alot ha-shaḥar* as the morning period counterpart to three stars, it will often occur after *mi-she-yakir*. As far as I know, the most stringent time for ending *Shabbat* in Israel according to the *geonim*, taking the equivalent of R. Karelitz's practice of waiting 40 to 45 minutes after sunset, corresponds to only a depression angle of 9.7 (or according to some perhaps 10.1) degrees. On the other hand, the latest time for *mi-she-yakir* is normally a depression angle of 10.2 degrees.²⁴⁰ The more typical *pesakim* for *mi-*

²⁴⁰ The opinion of R. Feinstein for *mi-she-yakir*, 35 to 40 minutes before sunrise in New York, is well outside the limits of customary *pesak*.

she-yakir and *ḥashekhah*, expressed using depression angles, differ by 2 or more degrees, from greater than or equal to 10.5 degrees to less than or equal to 8.5 degrees, respectively.

While R. Pimental and R. Feinstein both assume they are working within the context of Rabbeinu Tam's opinion, they are in practice adopting the position of the *geonim*. Only positions like R. Soloveitchik's or an invariant interval of 72 or 90 minutes reflect the conceptual viewpoint of Rabbeinu Tam.

In a practice going back at least to the students of Rabbeinu Yonah, the end of *Shabbat*, either because of uncertainty over the definition of a medium star or because of *tosefet Shabbat*, was extended from the appearance of three medium stars to the appearance of three small stars, relatively adjacent to each other.²⁴¹ Translated into a depression angle, three medium stars appear when the sun is approximately 6 degrees below the horizon. The appearance of three small adjacent stars is currently associated with a depression angle of 8.5 degrees, the stringent opinion of R. Tukitzinsky and the basis of most modern calendars. Because of non-linear growth in the twilight period, by increasing the depression angle that defines the end of *Shabbat*, these

²⁴¹ This opinion requiring closely aligned stars is discussed in the *Yerushalmi*, but it appears to have been followed only because of Rabbeinu Yonah. See R. Tukitzinsky's lengthy discussion in the second chapter of *Bein Ha-shemashot*. Another potential motivation, to better align Rabbeinu Tam's position with observation, cannot be dismissed. If one assumed a requirement to wait 72 minutes, stars seen before that time must be discounted as either large or not meeting some additional condition by appearing in the wrong part of the sky or being too dispersed.

stringencies have a more pronounced effect both in the summer and winter and especially at latitudes further from the equator.²⁴²

In summary, unless one is willing to accept inventive but exceptionally forced suggestions that,

1. assume the term *hihsif ha-elyon ve-hishveh le-tahton* is referring to the sun and not the sky,
2. require that the three stars referred to by the *gemara* appear close to the western horizon, perhaps a more reasonable, albeit unstated assumption, and
3. in any case, still need to explain what would possibly be gained by looking to the east at so late a point,

the *gemara* strongly supports the opinion of the *geonim*. Furthermore, the focus of the *gemara* is on *erev Shabbat*, attempting to determine the time when the period of *bein ha-shemashot* begins. Despite it being central to the dispute between Rabbeinu Tam and the *geonim*, for the *gemara*, when the period of *bein ha-shemashot* ends is not in substantial dispute.

It is ironic that while we have two clear, well established times for the end of *Shabbat* for Rabbeinu Tam, 72 and 90 minutes corresponding to depression angles of 16 and 20 degrees respectively, for the *geonim's*

²⁴² At the equator for example, the difference between the time of appearance of three medium stars and three small stars is approximately 10 minutes. In London that difference is almost 16 minutes in the spring and yet longer in the summer.

end to *Shabbat*, there is little consensus on how to determine the precise moment of *hashekhah*. Unlike Rabbeinu Tam, whose position on the end of *Shabbat* corresponds to a point when light has (almost) disappeared completely, the continuous change around the point of *hashekhah* makes a precise definition more difficult. Using depression angles, the most widely followed current *pesak* for the end of *Shabbat* is when the sun is 8.5 degrees below the horizon. However, relative to practice in many European communities,²⁴³ 8.5 degrees is a stringency for calculating an acknowledged stringency, the appearance of three small, adjacent stars. Indicating the end of *Shabbat* by the appearance of three small, adjacent stars was used both by those who followed the opinion of the *geonim* as well as those using R. Pimental's (re) definition of the opinion of Rabbeinu Tam. The depression angle that equates to the end of *Shabbat*, absent any stringencies, is approximately 6 degrees and the period of *bein ha-shemashot* precedes that point. Alternatives for the calculating the beginning of *bein ha-shemashot* follow in the next section.

²⁴³ See *Ha-zemanim Be-halakhah*, chapter 46. In the modern day, because of extensive illumination resulting in a limited ability to observe stars, many not perceiving an 8.5-degree depression angle as a stringency. My sense is that *hihsif ha-elyon ve-hishveh le-tahton* is not impacted as significantly by illumination and occurs before the time that a depression angle of 8.5 degrees is reached. R. Benish gives many depression degree equivalents for various *zemanim* observed in Europe; a vast majority are less than 8.5 degrees, particularly during some times of the year.

Section 8. How does the beginning of the *bein ha-shemashot* period relate to what we call sunset? What alternatives might be considered for the beginning of the *bein ha-shemashot* period?

This topic is normally considered uncomplicated. The *geonim* and most of their followers, at least as a stringency with respect to the start of *Shabbat*, follow the opinion of Rabbah. That opinion is assumed to equate the beginning of the period of *bein ha-shemashot* with sunset as defined by R. Avraham Ben Ha-Rambam, and identical to the way sunset is defined in a modern newspaper or almanac. R. Feinstein, R. Willig and many other *posekim* over the previous century tacitly assume that the beginning of the period of *bein ha-shemashot* and sunset coincide, at least under normal circumstances.²⁴⁴ As mentioned earlier, in the Middle East seeing three stars as early as 13.5 minutes after sunset is impossible under normal circumstances, and then only with the use of a telescope. Assuming that the end of *Shabbat* in the spring is between 18 and 21 minutes after sunset, then one can assume a longer period for the time needed to walk a *mil* of 24 or perhaps 22.5 minutes, separate slightly the beginning of the *bein ha-shemashot* period from sunset or both. Additionally, if we assume the appearance of three stars includes being seen with difficulty and only by one who knows where to look, something that I find less than plausible, then we can maintain the

²⁴⁴ This assumption is so deeply rooted, that a recent booklet distributed in Jerusalem attacks those in a valley who use the calendar's defined time for sunset at sea level despite the fact that their view of the sun is obscured. Alternate opinions that were followed in Jerusalem for setting the day of a *brit milah*, for a baby born slightly after sunset are not cited.

equivalence between sunset and the start of the period of *bein ha-shemashot*.²⁴⁵

While the position of Rabbah according to the *geonim* normally equates the beginning of the period of *bein ha-shemashot* with sunset, according to the conceptual approach of the Rabbeinu Tam and his school, the beginning of the period of *bein ha-shemashot* occurs 54 minutes after sunset, at the earliest.²⁴⁶ The *Shulḥan Arukh* that assumes the time needed to walk a *mil* is only 18 minutes sets the start of the period of *bein ha-shemashot* 4.5 minutes later, at 58.5 minutes after sunset. Regardless of how the various phenomena described in the *gemara* are to be interpreted, under no circumstances can *mi-she-tishkeh ha-ḥamah* refer to sunset. Those who follow the opinion of the Rabbeinu Tam read the *gemara* that states that the period of *bein ha-shemashot* begins *mi-she-tishkeh ha-ḥamah* to mean not *shekiyah* but some point thereafter. Ramban²⁴⁷ stresses the difference between *shekiyat ha-ḥamah*, sunset, and *mi-she-tishkeh ha-ḥamah* referring to some later point after the sun has set. Of course, all that this language motivates is distancing sunset from the beginning of the period of *bein ha-shemashot*; it can be applied as well and presumably even better²⁴⁸ to argue for a (slightly) delayed

²⁴⁵ It should be noted that if the appearance of three stars is viewed as defining the end of *Shabbat*, observation by an expert is more plausible. In any case, the inability to align Shmuel's assertion with the main conclusions of the *gemara* and the other issues raised in section 5 would remain.

²⁴⁶ 54 minutes equates to a 72-minute interval after sunset until the end of the day, with the period of *bein ha-shemashot* beginning ($\frac{3}{4} * 24 =$) 18 minutes earlier.

²⁴⁷ *Torat Ha-adam* Mossad Ha-rav Kook edition, pages 251 - 255.

²⁴⁸ Referring to a point as being after sunset is more likely to refer to a point directly after sunset, approximately 10 minutes later, than to a point approximately one hour later.

start to the period of *bein ha-shemashot* even according to a view like that of the *geonim*, without adopting the full view of Rabbeinu Tam. Thus, even if we insist that we are to follow the opinion of Rabbah according to the *geonim* and reject Rabbeinu Tam's opinion as to the end of *Shabbat*, we can still accept that the language of *mi-she-tishkeh ha-ḥamah* is referring to a point after sunset. Reading Rabbah under that circumstance is no more difficult than for those who follow the opinion of the Rabbeinu Tam; Rabbah begins the period of *bein ha-shemashot* some number of minutes after sunset.

While some communities still use the opinion of Rabbeinu Tam when setting times for afternoon and evening prayers, no communities remain that start *Shabbat* after sunset. How late communities started *Shabbat* in the past, and when that might have involved a violation at a biblical or a rabbinic level is rarely addressed. I will return to this issue in the epilogue; in any case, it is difficult to specify precisely, and this area still awaits an extensive historical study. It is entirely likely that four related factors would have moved the beginning of the period of *bein ha-shemashot* to well before 50 minutes after sunset even according to Rabbeinu Tam:

1. Positions like that of R. Pimental and R. Feinstein were adopted in various communities. While conceptually aligned with Rabbeinu Tam, in practice these communities equated the end of *Shabbat* with the appearance of three stars that occurred well before 72 minutes after sunset; on *erev Shabbat* the period of *bein*

- ha-shemashot* would normally begin at the very least 13.5 minutes earlier.
2. Given uncertainly about the definition of a medium star, it is likely that the appearance of **any** two or three stars would have indicated that the period of *bein ha-shemashot* or *Shabbat* had begun and work must cease.
 3. Given uncertainly, the lack of clear observational definitions and for many generations without the *benefit* of clocks, observant Jews would err on the side of safety in starting the period of *bein ha-shemashot*.
 4. *Tosefet Shabbat* would extend the beginning of *Shabbat*.

Our focus is to discover what flexibility exists for the start time of *Shabbat*, even assuming the opinion of the *geonim*. The three phrases in Rabi Yehudah's formulation of the period of *bein ha-shemashot*, Rabbah and R. Yosef's approximations to the length of the period of *bein ha-shemashot*, as well as Shmuel's statement about 1, 2 and 3 stars must all be considered in determining the start of the period of *bein ha-shemashot*.

Of the three phrases of Rabi Yehudah:

1. *Mi-she-tishkeh ha-ḥamah* according to Ramban means some point **after** sunset; anything more precise cannot be derived.
2. *Kol zeman she-penai mizraḥ maadimin*, all the time (including a short interval after *mi-she-tishkeh ha-ḥamah*) when illumination

from the sun below the western horizon is still causing a glowing (reddish) impact (in the east or in clouds) is suggestive of a point approximately ten minutes after sunset, but not conclusively.

3. *Hihsif ha-tahton ve-lo hihsif ha-elyon*, the paleness / darkness of the sky, moving from east to west, has not progressed to the apex of the sky, is the period of *bein ha-shemashot* according to everyone. However, it describes the sky during the *bein ha-shemashot* period as opposed to describing the point when the period of *bein ha-shemashot* begins.

Fortunately, the statement of Shmuel and both Rabbah and R. Yosef's approximations to the length of the period of *bein ha-shemashot* help identify more concretely the point at which the period of *bein ha-shemashot* begins.

However, before addressing this issue further, it is useful to summarize the discussion in Section 6 on how the beginning of the period of *bein ha-shemashot* could be set (both according to Rabbeinu Tam and the *geonim*). Two conceptual approaches exist, each with a variety of implementation alternatives:

- Some physical event triggers the beginning of the *bein ha-shemashot* period. In modern terms, we would normally associate that event with a depression angle, but that is not critical. In the opinion of many current *posekim*, that event is sunset; some have asserted that it is the emergence of two stars. That same physical

event applies at various latitudes and across all seasons. As a result, the length of time between the beginning of the period of *bein ha-shemashot* and *hashekhah* would vary, lengthening both at locations further from the equator and during the summer and winter seasons.

- The intent of the period of *bein ha-shemashot* is to establish a *fence* around *hashekhah* and the Rabbis chose a physical event that specifies or conservatively approximates the beginning of the period of *bein ha-shemashot* in the Middle East. The duration of that interval might then be considered invariant, providing an upper bound for the length of the *bein ha-shemashot* period everywhere. As a result, the start of the period of *bein ha-shemashot* could always be established by subtracting that upper bound on the length of the *bein ha-shemashot* period from the varying point of *hashekhah*.²⁴⁹

I leave this question open; not surprisingly, I found no discussion in the *halakhah* of any physical triggers for *bein ha-shemashot* with respect to the opinion of Rabbeinu Tam. As noted earlier, in the modern era of clocks and *zemanim* specified using time intervals, both R. Adler and R. Feinstein clearly adopted varying intervals, while R. Lorberbaum and R. Sofer considered the duration of the *bein ha-shemashot* period invariant. Despite the fact that these *posekim* assumed they were working within the framework of Rabbeinu Tam, I see no fundamental reason not to

²⁴⁹ Clearly, given a longer period of twilight, the length of the *fence* could be increased; that option is accounted for in the first approach above.

apply their positions to the opinions of the *geonim* as well. As I have noted, since three of the four *posekim* referenced above utilized approaches like that of R. Pimental, they were in reality largely consistent with the *geonim's* end time for *Shabbat*, adjusted for their communities.

We now turn to alternatives for the beginning of the period of *bein ha-shemashot* according to the approach of the *geonim*. While the same two options apply for the opinion of the *geonim*, there is a somewhat richer literature both from a small number of European authorities, and from *posekim* in the Middle East where the opinion of the *geonim* was, with isolated exceptions, always followed.

Disregarding for a moment the precise times involved, our assumption is that at the biblical level one cannot work past *ḥashekhah*. The period Friday night prior to *ḥashekhah*, referred to as *safek ḥashekhah*, represents a classical *safek de-oraysah*. The start of the period of *bein ha-shemashot* is yet earlier and work during any part of the period of *bein ha-shemashot* prior to the point of *safek ḥashekhah* is a rabbinic level violation.²⁵⁰

Since the *gemara* decides that we follow the opinion of Rabi Yehudah with respect to the beginning of the period of *bein ha-shemashot*, it is

²⁵⁰ As discussed in footnote 211, a number of opinions assume *bein ha-shemashot* and *safek ḥashekhah* are identical, a position many adopt at least practically. That would imply that the entire *bein ha-shemashot* period is a *safek de-oraysah*. Some even assume *safek ḥashekhah* begins prior to the period of *bein ha-shemashot*, a difficult opinion that is not considered.

critical to assess the alternative views to which he might subscribe. First, let us consider the opinion of R. Yosef who defines the beginning of the period of *bein ha-shemashot* just after *penai mizrah maadimin* ends, approximately the point at which the lower part of the eastern horizon pales or darkens. Again, there is some ambiguity as to the exact point in time to which that description applies. In particular, what defines precisely the subsiding level of illumination or exactly how high above the eastern horizon has the sky darkened at the start of R. Yosef's period of *bein ha-shemashot*? For those who assert that Rabbah's start to the period of *bein ha-shemashot* is exactly at sunset, we can determine the meaning of R. Yosef's statement by observing the sky above the horizon the time needed to walk 1/12th of a *mil* after sunset. Other than that, it is difficult to determine the exact time implied based only on the language of R. Yosef's formulation.²⁵¹

While we normally decide like Rabbah in disagreements with R. Yosef, commentators who equate *mi-she-tishkeh ha-ḥamah* with sunset, often assume that the opinion of R. Yosef is primary.²⁵² A key reason for preferring R. Yosef is his assumed alignment with the statement of Shmuel that considered it (possibly) day when only one star is visible after sunset. Under the assumption that Rabbah begins *Shabbat* at sunset proper, we must reject Rabbah in favor of R. Yosef. However, as

²⁵¹ Nonetheless, to assume that R. Yosef's interval of *bein ha-shemashot* starts approximately 10 to 15 minutes after sunset appears reasonable given observation of *penai mizrah maadimin*.

²⁵² For a more complete analysis of this topic, see Rashba *Shabbat* 34b - 35a. Note that since Rashba is working within the context of Rabbeinu Tam it is not directly relevant to this discussion.

was discussed in section 5, the appearance of the first star over Jerusalem occurs, at the earliest, and only to an expert, 6 minutes after sunset, and is more generally visible a number of minutes later. This implies that choosing R. Yosef gains nothing, since his position is inconsistent with Shmuel as well. However, if we move Rabbah's start of the period of *bein ha-shemashot* to 4 or 5 minutes after sunset, then R. Yosef's start to the period of *bein ha-shemashot*, approximately two minutes later, is 6 or 7 minutes after sunset. This would align R. Yosef, but not Rabbah, with Shmuel, but only if you assume that potential visibility by an expert is what Shmuel meant by one star being visible. While visibility by an expert who is aware of exactly where to look is strained as an indicator for the end of *Shabbat*, since we interpret "*one star - daytime*" as "if you see one star, do not assume that the day is over," then visibility by an expert is somewhat more plausible. This slight 4 or 5-minute advance also allows Rabbah's time to walk $\frac{3}{4}$ *mil* to better align with the ability to observe three stars in the spring without assuming telescopes or other less than intuitive circumstances.²⁵³

Both reported practice in Jerusalem for the *brit milah* of a baby born slightly after sunset²⁵⁴ and R. Shneur Zalman of Liadi²⁵⁵ support this minimal change in the opinion of Rabbah, and both give the same

²⁵³ Prof. Levi, *Halakhic Times*, Hebrew section, page 39.

²⁵⁴ See *Minhagei Eretz Yisroel* by R. Yaakov Gliss, pages 102 and 282.

²⁵⁵ The *siddur* of R. Shneur Zalman of Liadi specifies four minutes, not five.

reason: at 4 to 5 minutes after sunset, the sun is no longer visible from the highest elevations around Jerusalem.²⁵⁶

R. Haim Volozhin set the beginning of the *bein ha-shemashot* period at (more than) 6 minutes²⁵⁷ after sunset based on the statement of Shmuel that the appearance of one star can occur during the daytime.

Particularly in the Middle East, there were multiple views among *posekim* during the 19th century that extended the start of the period of *bein ha-shemashot*, some going as far as approximately 10 minutes after sunset, when the Arab clock is set to 6:00PM and the *mugrab* announces the time for prayer.²⁵⁸

Assuming either of the latter two views, Shmuel's statement is consistent with Rabbah as well. However, they also begin to

²⁵⁶ Lacking visibility of the sun even from even the highest elevations around Jerusalem as an indicator for the beginning for the period of *bein ha-shemashot* is no less intuitive than the use of conditions at sea level, particularly given the fact that sea level is a more modern concept. Regardless of rationale, this approach provides an identifiable beginning for the period of *bein ha-shemashot*.

²⁵⁷ See the addition to *Maaseh Rav* section 19. Six minutes is expressed as 1/10th of an hour to be applied in both the morning and evening, although "*one star - daytime*" is given as the reason for the slight delay after sunset. I assume that this applies to Israel, but no reference to any adjustment is given. At 6 minutes after sunset, any adjustments by use of an equivalent depression angle would be minimal in any case. One can speculate that his divergence from the standard opinion of the *Gaon* may result from his asserting the normative *halakhah*, while the *Gaon* may have insisted on sunset proper only *le-migdar miltah*.

²⁵⁸ See the second chapter, third section of *Zemanim Ke-hilkhatam* by R. Dovid Burstyn and chapter 10 of *Ha-zemanim Be-halakhah* for a variety of sources that went beyond the 4 to 5 minutes quoted in *Minhagei Yisroel*. Those sources often discuss obstructed views and sea level in justifying their positions. While these views are supportive of the position developed in this chapter, their justifications are different and are not within the scope of this monograph.

constrain²⁵⁹ the length of time to walk a *mil* to 18 minutes and / or force an assumption of very clear visibility of three stars if one insists that the time needed to walk $\frac{3}{4}$ of a *mil* is a minimum length to the period of *bein ha-shemashot* applicable around the spring equinox.

According to R. Kapach's interpretation of Rambam, the period of *bein ha-shemashot* begins 15 minutes after sunset. I see no clear justification for so late a start to the period of *bein ha-shemashot*, and I disagree as well with the detailed rationale that R. Kapach provides.²⁶⁰ Nonetheless, R. Kapach makes a convincing case that according to Rambam, the start of the period of *bein ha-shemashot* begins a number of minutes after sunset.

Any viewpoint that starts the period of *bein ha-shemashot* between 10 and 15 minutes after sunset provides yet another reason for rejecting the time needed to walk $\frac{3}{4}$ *mil* as a lower bound on the length of the *bein ha-shemashot* interval applicable only in the spring, beyond what was argued in section 5.²⁶¹ By 22 or 23 minutes after sunset in the

²⁵⁹ Adding 6 + 13.5 is still reasonable but 6 + 16.875 is slightly later than the third entry in Prof. Levi's table. As discussed below, this also provides support to considering the time needed to walk $\frac{3}{4}$ *mil* period as a maximum.

²⁶⁰ His use of two stars as defining the start of the period of *bein ha-shemashot* is particularly difficult. However, I can easily recast (and hopefully improve) his viewpoint using degrees of darkness as I have argued in section 5. R. Kapach expresses this view throughout his commentary on *Mishnah Torah*; see in particular his commentary on relevant *mishnayot* in *hilkhot Shabbat* and *Kiddush Ha-ḥodesh*.

²⁶¹ As I argued earlier: What possible value could there be in introducing a time-based approximation that is a lower bound, that is season dependent, rarely applicable, and then only under idealized conditions? In what context would such information be useful?

Middle East in the spring, *Shabbat* has ended, implying a beginning to the period of *bein ha-shemashot* before 10 minutes after sunset.

Consider carefully a day in Israel close to the summer solstice where we assume *Shabbat* ends between 24 and 29 minutes after sunset, without the normalization to spring that Prof. Levi introduces in his table, reproduced in section 5. Now assume the time needed to walk $\frac{3}{4}$ of a *mil*, either 13.5 or 16.875 minutes, is taken as the maximum length of the period of *bein ha-shemashot*. This gives a range for the start of the period of *bein ha-shemashot* from 7 to 15 minutes after sunset, a figure that I believe represents a preferred reading of the *gemara*. This provides another mechanism for defining when the period of *bein ha-shemashot* begins. For example, if the time of *ḥashekhah* in the summer is precisely 25 minutes after sunset, then using the *Shulḥan Arukh's* position of 13.5 minutes as the time needed to walk $\frac{3}{4}$ *mil*, the period of *bein ha-shemashot* would begin 13.5 minutes earlier, approximately 11 to 12 minutes after sunset. Note however, that in the spring and fall, this same level of darkness (and appearance of the sky) will occur approximately 1 minute earlier at about 10 to 11 minutes after sunset. However, in the spring and fall, *ḥashekhah* occurs approximately 2 to 3 minutes earlier. Thus, the period of *bein ha-shemashot* is longest in the summer, by approximately 1 to 2 minutes. For all of us accustomed to clocks and sunset, this example, both consistent with the *gemara* and astronomically accurate, requires time to grasp.²⁶²

²⁶² Once it is understood that larger depression angles (the point of *ḥashekhah*) exhibit more seasonal variation than smaller depression angles (the beginning of the period of *bein ha-shemashot*), this approach will feel more intuitive.

This alternative meets both objectives stated in the prologue. First, the explanation of the *gemara* is straightforward,

- given the more natural interpretation of the time needed to walk $\frac{3}{4}$ *mil* as a maximum,²⁶³
- reconciling Rabbah completely with the statement of Shmuel and
- without assuming visibility only by an expert.

Second, this approach helps with our second objective of creating as late a start to the period of *bein ha-shemashot* as possible, partially justifying practice in Europe over the centuries. The few extra minutes are not what is most valuable; more importantly (only) this approach completely justifies the opinions of R. Lorberbaum and R. Sofer among others, who implicitly treat the time needed to walk $\frac{3}{4}$ of a *mil* as an invariant maximum, subtracted from the end of *Shabbat* to determine the beginning of the period of *bein ha-shemashot*.

Consider the application of these approaches to other seasons and latitudes. The first conceptual approach would calculate the depression angle associated with a level of darkness at say 4 to 7 minutes after sunset, around the spring equinox in the Middle East, and then use that angle at other latitudes and during other seasons. This would not justify

²⁶³ Furthermore, if Rabbah meant sunset precisely, it is puzzling why the language of the *gemara* would have been so complex? This would motivate any of the three approaches.

practice that began *Shabbat* more than 6 to 10 minutes after sunset except at latitudes in the very northern regions of Europe.

Assume that we were to choose a beginning to the period of *bein ha-shemashot* in the Middle East 12 minutes after sunset around the spring equinox. Again, we would equate that to a depression angle and apply that angle at other latitudes and seasons. On its own, this would imply a start to the period of *bein ha-shemashot* less than 20 minutes after sunset at all major European locations. This would still be inadequate to defend the practice that we saw in the case of R. Sofer, with a *brit* for a baby born on *Shabbat* at 8:30PM, 27 minutes after sunset, being held on the following *Shabbat*. However, if we,

- conclude that **the time needed to walk $\frac{3}{4}$ mil is a seasonal maximum** to the period of *bein ha-shemashot*, and
- treat **the time needed to walk $\frac{3}{4}$ mil as latitude invariant as well,**

a position that has strong support from *posekim*, **then we can calculate the beginning of the period of *bein ha-shemashot* by subtracting its invariant length from the end of the period of *bein ha-shemashot*, the time of *hashekhah*.**

Computationally, we can now specify the beginning of the period of *bein ha-shemashot* in four different ways:

- 1) Equate a depression angle to the beginning of the period of *bein ha-shemashot*.
- 2) Calculate the beginning of the period of *bein ha-shemashot* by subtracting the time needed to walk $\frac{3}{4}$ of a *mil* from the point of *hashekhah*.²⁶⁴
- 3) Take the earlier of 1 and 2 as the beginning of the period of *bein ha-shemashot*.
- 4) Take the later of 1 and 2 as the beginning of the period of *bein ha-shemashot*.

Note that for much of northern Europe, options 2 and 4²⁶⁵ provide a start to the period of *bein ha-shemashot* approximately 25 to 35 minutes after sunset, depending on location and season.

When the time needed to walk $\frac{3}{4}$ of a *mil* is defined as a minimum, applicable only in the spring, then the standard method for calculating *the start of* the period of *bein ha-shemashot* by subtracting from *hashekhah* would more than likely have to be adjusted by latitude as well as season. Regardless of precisely how that process might be defined, it would imply an earlier start to the period of *bein ha-*

²⁶⁴ We could even subtract a slightly lesser amount if a method for such approximation is developed. One could theoretically calculate the length of the period of *bein ha-shemashot* during any season in the Middle East and then apply that shorter interval everywhere. Again, I suggest this detail only as method to defend prior practice yet more aggressively. Similarly, other detailed variants of the four alternatives can be specified; the four listed establish the fundamental alternatives that would apply both closer to and further from the equator than the Middle East.

²⁶⁵ Options 2 and 4 are identical at European latitudes.

shemashot in Europe, given that the length of the period of *bein ha-shemashot* would have to be lengthened before being subtracted. However, defining the time needed to walk $\frac{3}{4}$ of a *mil* as a maximum is a plausible, if not a preferable reading of the text and provides the basis for maximal leniency, supporting a later start to the period of *bein ha-shemashot*.

Note however, that defining the time needed to walk $\frac{3}{4}$ of a *mil* as a maximum in the Middle East does not in any way imply that the maximum is invariant and does not adjust with changing latitude. In order to justify subtracting an invariant interval from *hashekhah* it is necessary, but not sufficient, to interpret the time needed to walk $\frac{3}{4}$ of a *mil* as a maximum. Both R. Lorberbaum and R. Sofer supported going a step further, assuming that the time needed to walk $\frac{3}{4}$ of a *mil* is a maximum that can be subtracted from *hashekhah* **at any latitude** to determine the beginning of the period of *bein ha-shemashot*. Such a ruling does not and could not derive from the text of the *gemara*, which did not include attention to latitude. What is perhaps less obvious, is that defining the time needed to walk $\frac{3}{4}$ of a *mil* as a minimum more than likely precludes any approach that treats the length of the period of *bein ha-shemashot* period as invariant.

In practical terms, *posekim* can decide to opt for any of the four options and perhaps decide differently in various situations based on a

multiplicity of factors.²⁶⁶ Current custom for the start of *Shabbat*, as Rambam quoted at the outset stated, regardless of the particular *halakhic* position adopted, should not necessarily be viewed as the precise beginning to the period of *bein ha-shemashot* but as the beginning of an accepted period of *tosefet Shabbat*, with the full force of accepted practice.

In summary, we have multiple approaches to the beginning of the period of *bein ha-shemashot* coupled with multiple options on how to conceptualize the *bein ha-shemashot* period. With each of the approaches, from sunset to some number of minutes thereafter, we might equate a depression angle that would naturally vary the beginning of the period of *bein ha-shemashot* by latitude and season. As well, conceived of as a minimum, the time needed to walk $\frac{3}{4}$ *mil* has limited practical significance. **However, when conceived of as the maximum length of the *bein ha-shemashot* period throughout the year in the Middle East, and even more so when considered as latitude invariant as well, the time needed to walk $\frac{3}{4}$ *mil* has major import.**²⁶⁷ In latitudes farther than the Middle East from the equator, it

²⁶⁶ Questions in the area of a *brit* both on a weekday and especially on *Shabbat*, *niddah*, special situations at the start of *Shabbat*: a delayed traveler, delayed *hadlakat ha-neirot*, *amirah le-akum*, etc. all occur with some frequency and might be treated differently by a *posek*, dictating varying degrees of leniency.

²⁶⁷ Note that if the time needed to walk $\frac{3}{4}$ *mil* is considered only a seasonal maximum but assumed to vary by latitude, the interval between the beginning of the period of *bein ha-shemashot* and *hashekhah*, the end of the period of *bein ha-shemashot*, would grow proportionately faster than the earlier interval between sunset and the beginning of the period of *bein ha-shemashot*. As such, its practical impact in justifying practice is significantly reduced; practically it would be essentially identical to option 1, given previously.

limits the length of the period of *bein ha-shemashot* to the time needed to walk $\frac{3}{4}$ mil. Thus, as I have indicated throughout, we can fuse some of the notions of Rabbeinu Tam and his followers to the position of the *geonim* and create a hybrid that is

- a preferred reading of the text of the *gemara*,
- consistent with astronomic observation and
- (partially) justifies generations of practice.

Section 9. How do the two meanings of day – day as in “day of the week” and day as in “during the daytime” relate? Must the end of the daytime period coincide with the end of a day of the week?

In the previous sections, we have dealt with both,

- the undisputed start of the daytime period, *alot ha-shaḥar*, and
- the focus of much of our attention, the beginning and end of the *bein ha-shemashot* period, the latter being the transition point between days.

When the *Gaon* defined *tzait kol ha-kokhavim*, despite the fact that *alot ha-shaḥar* is a fundamental point in *halakhah*, he gave no examples of *halakhot* dependent on its evening counterpart. Given the *Gaon*'s approach, calculating the hours of the day from sunrise to sunset, *tzait kol ha-kokhavim* also has no computational relevance. However, as we have seen, those who calculate the hours of the day following the approach of Magen Avraham (and make latitudinal and seasonal adjustments) are apparently forced to use *tzait kol ha-kokhavim* as the evening endpoint corresponding to *alot ha-shaḥar*.

In spite of the fact that the *Gaon* gives no examples of such *halakhot*, this section focuses on finding potential *halakhic* import to the time of *tzait kol ha-kokhavim*.

Many²⁶⁸ have noted the two different meanings of the term *day* both:

- the day of the week, and
- the daytime period.²⁶⁹

In secular terms, days of the week currently begin and end at midnight, while the daytime period is an interval defined by sunrise and sunset or a somewhat longer interval based on some degree of light. To this point, most, if not all, of the end-of-day *halakhot* we encountered depended on the day of the week. According to the *geonim*, the day of the week transitions around *ḥashekhah / tzait ha-kokhavim*, and at least theoretically according to Rabbeinu Tam, the day of the week transitions at *tzait kol ha-kokhavim*. Conceptually,²⁷⁰ according to Rabbeinu Tam, **both** uses of the term **day** terminate at the identical point of *tzait kol ha-kokhavim*. For the *geonim* however, the end of a day of the week is *ḥashekhah / tzait ha-kokhavim*, while the end of the daytime period is perhaps *tzait kol ha-kokhavim*.

According to *the geonim*, the daytime period could have at least three endpoints,

²⁶⁸ These include R. Meltzer, R. Zimmerman and others identified with the *Brisker* methodology.

²⁶⁹ It is probable that this distinction can explain why one can advance a holy day like the *Shabbat* on Friday afternoon, while one cannot eat the Passover meal until nighttime. While there is flexibility in defining the start of a day of week, such flexibility cannot turn daytime into nighttime. See, for example, *tosefot* s.v. *ad she-teḥshakh* on *Pesahim* 99b where the term *lailah* is emphasized.

²⁷⁰ R. Soloveitchik's fundamental reformulation of the approach of Rabbeinu Tam explicitly equated his end to *Shabbat* to the point of *tzait kol ha-kokhavim*, both in theory and in practice.

1. a point around sunset,²⁷¹
2. *ḥashekhah / tzait ha-kokhavim*, the transition point between days of the week, as well as
3. *tzait kol ha-kokhavim*.

The latter two points are conceptually the same according to Rabbeinu Tam. According to both viewpoints, *halakhot* related to *korbanot*,²⁷² for example, may utilize an endpoint related to sunset as opposed to either *ḥashekhah* or *tzait kol ha-kokhavim*. Note that the possibility of additional transition points²⁷³ makes it difficult to attack either the position of Rabbeinu Tam or the *geonim* using the existence of *halakhot* that depend neither on sunset, *ḥashekhah* nor *tzait kol ha-kokhavim*.

For Rabbeinu Tam, given the conceptual alignment between both meanings of the end of the day, clearly *tzait kol ha-kokhavim* is important.²⁷⁴ However, for the *geonim*, it remains an area of conjecture.

²⁷¹ Theoretically, from the point the sun begins to disappear until it goes completely below the horizon at normal elevations, a span of about three minutes, or perhaps a few minutes later as in the position of R. Shneur Zalman of Liadi.

²⁷² In particular, other *halakhot*, whose *zemanim* are tightly tied to *korbanot* like specific *zemanai tefillah*, may be dependent on a time around sunset as opposed to some later point.

²⁷³ Both the beginning of the period of *bein ha-shemashot* and the earliest point of *safek ḥashekhah* represent two additional points. Yet other defined points, the point that a poor man begins his evening meal, the point that people go to sleep, etc. may be yet additional points or may correlate precisely to the other transition points, a topic I do not cover.

²⁷⁴ One area not covered in this monograph is alternatives for the latest time of *minḥah*. It is normally assumed that the argument of Rabi Yehudah and the Rabbis in *Berakhot* coupled with the argument on how to calculate the hours of the day, imply two pairs of possible points for *ad ha-erev* and *pelag ha-minḥah*. According to

According to the *geonim*, it is at least plausible that specific *halakhot* that depend only on the daytime period, and not the day of the week, might use *tzait kol ha-kokhavim* as a delimiter. I am not aware of any uncontested examples.²⁷⁵ However, I will mention some areas where the possible applicability of *tzait kol ha-kokhavim* as a delimiter cannot be dismissed summarily. Our approach is to isolate *halakhot* that have two characteristics:

- They have an affinity only to the daytime period versus a particular day of the week.
- There is some textual basis that might suggest support for allowing an extension beyond *ḥashekhah*.

Each of the four examples is highly speculative; the fourth does not even satisfy both characteristics.

the *Gaon* we have sunset and 75 minutes before sunset and according to Magen Avraham we have *tzait kol ha-kokhavim* and shortly before sunset. Sandwiched between those alternatives, there is another possibility. A simple reading of the *gemara* in *Berakhot*, might lead to the possibility that even according to Magen Avraham, *minḥah* cannot extend past some point around the end of each day of week, the end of the *geonim's bein ha-shemashot* period. This position is a modification of Magen Avraham as currently implemented and reminiscent of an older method of calculation that was abandoned. Whether and how that might create another potential calculation of *pelag ha-minḥah* as well as a third way to calculate the hours of the day, has been rarely considered. A related notion is rejected by R. Weiss in his *teshuvah* on *pelag ha-minḥah* in *Minḥat Yitzḥak* 4:53; a major variation of such an approach is outlined in the summary of section 4, in the epilogue.

²⁷⁵ Finding such uncontested examples, given that the *Gaon* did not provide any, would be more than surprising.

Perhaps the clearest example is the construction of the temple. Undoubtedly, work is not in any sense tied to a particular day. Additionally, Rambam *Hilkhot Beit Ha-behirah* 1:12 uses unique language, defining the time when work is permitted as both in the day and not in the night as well as from *alot ha-shaḥar* until *tzait ha-kokhavim*. Perhaps this implies that work can continue past *ḥashekhah*, the appearance of only three stars, until the night or complete darkness. On the other hand, it is more likely that Rambam was only referring to the verse in Nehemiah, and one should not draw any implication from the fact that “three” did not modify the word “stars.” However, since there is not even a stated preference for work to begin only after sunrise, one might conjecture that work can continue until the point corresponding to *alot ha-shaḥar* in the evening, until *tzait kol ha-kokhavim*.

A second possibility is Rif and Rambam who both rule that a mourner who is within the time needed to walk 40 *milin* on the day of burial observes *shivah* with the leader of the family assuming he arrives prior to the end of the *shivah* period. R. Willig²⁷⁶ supports the assertion in *Orot Haim* who uses this as proof that Rif held that the entire 40-milin distance is walked in the interval from sunrise to sunset, and hence the time needed to walk a *mil* is 18 minutes. Rambam, however, must allow the interval to walk 40 *milin* to extend from *alot ha-shaḥar* until *tzait kol ha-kokhavim* as Rambam clearly maintains that one walks only 30 *milin*

²⁷⁶ R. Willig, *Am Mordechai, Berakhot*, chapter 2.

during the sunrise to sunset period.²⁷⁷ It is certainly possible that Rif is to be interpreted similarly.²⁷⁸ For Rambam and perhaps for Rif as well, that arrival time might extend until *tzait kol ha-kokhavim*. While hardly compelling and in opposition to practiced *halakhot* that count the first day of mourning only if it includes a brief period before either *bein ha-shemashot* or *hashekhah*, it represents a remote possibility, perhaps implied by Rambam's interval of the time needed to walk 40 *milin*. Alternatively, as mentioned earlier in section 2, a mourner and / or a messenger might be assumed to walk faster or perhaps during an extended period given the pressing circumstances of a death.

A third possibility relates to courts that must rule in the daytime. Rambam clearly states that the court must complete a declaration of *rosh ḥodesh* prior to *hashekhah*. It is also likely that certain deliberations, like capital cases, similar to *negaim* that must be examined during sunlight must only occur between sunrise and sunset. However, we can also argue that parallel to allowing specific judgments as early as *alot ha-shaḥar*, *tzait kol ha-kokhavim* might be the delimiter at the other end of the day. Thus, there may be a stricter definition requiring daytime for specific deliberations, nightfall with respect to the declaration of a new month and perhaps *tzait kol ha-kokhavim* for other matters. The Rambam's language allowing civil judgments to continue

²⁷⁷ The view of Rambam, *Hilkhot Aveilut* 7:4, was discussed earlier in Section 2.

²⁷⁸ See as well Rosh MK 22b. He allows for a three-day interval. The logic of his ruling appears to be based on the assumption that even if burial occurs at the end of the day, a messenger travels on the next day, and the mourner can still arrive by the end of the third day. His use of the term *hashekhah* is not supportive of this conjecture.

into the night, ostensibly past *hashekhah*, might still be delimited by *tzait kol ha-kokhavim*. This case is also tentative at best.

Finally, Ramban²⁷⁹ indicates that prior to sunset there cannot be *tosefet Shabbat*. For those following either Rabbeinu Tam or the *geonim*, *pelag ha-minḥah* is normally assumed the earliest point on Friday afternoon for the start of *tosefet Shabbat*. On Saturday night perhaps, *tzait kol ha-kokhavim* might act as the post *Shabbat* endpoint for *tosefet Shabbat* for those following the opinion of the *geonim*. While this use of *tzait kol ha-kokhavim* relates to the day of week as opposed to the daytime period, it has the advantage of having been suggested previously.^{280 281}

²⁷⁹ This is a part of Ramban's argument in support of Rabbeinu Tam. The same argument would also support starting *Shabbat* (much) earlier than the opinion of Rabbeinu Tam as long as there would still be time for a meaningful period of *tosefet Shabbat* after sunset.

²⁸⁰ See R. Shechter's notes on the *shiurim* of R. Soloveitchik on *masekhet Shabbat* page 87. I have also heard this quoted in the name of R. Moshe Soloveitchik.

²⁸¹ This assumes that it becomes sufficiently dark for *tzait kol ha-kokhavim* to occur; otherwise, in locations further from the equator when only *tzait ha-kokhavim*, but not *tzait kol ha-kokhavim* occurs, *ḥatzot ha-lailah* has been suggested for both the end of *tosefet Shabbat* according to the position of the *geonim* and *Shabbat* itself according to the position of Rabbeinu Tam.