

Preface

This monograph focuses on two related topics:

- ❖ How is the time of *alot ha-shaḥar* determined?
- ❖ How are *shaot ha-yom* calculated when following what is currently known as the approach of the Magen Avraham?

In modern times, the method most often used for determining *shaot ha-yom* is that of the *Gaon* of Vilna. Unlike the *Gaon's* method, which calculates from sunrise, the time when the ball of the sun is first visible, the Magen Avraham calculates from the earlier point of *alot ha-shaḥar*, around the time when the sun's illumination is first apparent. Regardless of whether *alot ha-shaḥar* is used to calculate *shaot ha-yom*, it is universally accepted that *alot ha-shaḥar* marks the *halakhic* beginning of the daytime period. The first sections of the monograph discuss alternate opinions for specifying the time of *alot ha-shaḥar*. Those sections are largely tutorial; hopefully, their organization (as well as their mathematical / logical precision) will be helpful in clarifying the area. After covering major *halakhic* alternatives for the time of *alot ha-shaḥar*, the unquestioned morning endpoint used in calculating the Magen Avraham's approach, the remainder of the monograph concentrates on two areas:

- ❖ What *halakhic* alternatives are there for the evening endpoint to be used in the Magen Avraham's calculation?
- ❖ What other method may have been used in implementing the calculation?

Both the method of calculation and the evening endpoints suggested are not ordinarily considered.

Many prominent rabbis and their communities calculated the opinion of the Magen Avraham using an evening endpoint that was deemed to be incorrect over a century ago. While the method of calculation was clearly faulty, discarding it in its entirety was not required. A hypothesis on how that method of calculation might have originated (and then incorrectly modified,) as well as support for that hypothesis, is a major focus of this monograph. An implementation of this former practice, using both the precision of clocks and depression angles,¹ provides an alternative method to calculate the opinion of the Magen Avraham.

Three observations about the opinion of the Magen Avraham, each of which would benefit from a more extensive historical study, would benefit from further study. First, in the Middle East, with a largely unbroken tradition, there is powerful support for the position of the Magen Avraham in both Jerusalem and other areas. That tradition was frequently abandoned elsewhere. Second, support for the position of the Magen Avraham by all *ḥakhmai seforad* and other *rishonim* is incontrovertible. These *rishonim*'s support for the position of the Magen Avraham as well as their support for a longer (90-minute) period between *alot ha-shaḥar* and sunrise is rarely acknowledged. Third, calculating the position of the Magen Avraham becomes problematic, at latitudes encountered in northern Europe. I

¹ While a brief introduction to depression angles is included in this monograph, a more thorough description is provided in XXXX

strongly suspect that the northern migration of Jewry was a major contributor. Given our focus on *halakhic* issues, these three areas are beyond our scope.²

Introduction

Two conceptual methods for calculating *shaot zemaniot*,³ the hours of the day to be used in most *halakhic* contexts, are those ascribed to R. Abraham Abele Gombiner, more widely known as the Magen Avraham, and R. Elijah ben Shlomo Zalman, the *Gaon* of Vilna.⁴ *Shaot zemaniot* divide the daytime period into 12 *halakhic* hours. Unlike clock hours that are a fixed sixty minutes in length, *halakhic* hours vary based on the length

² My articles in the TuMJ (2013) on “Zemannim: On the Introduction of New Constructs in Halakha” and on the Seforim blog, January 3, 2019, on “Lighting Shabbat Candles 40 Minutes before Sunset in Jerusalem” are preliminary attempts to deal with issues that are addressed more completely in this monograph.

³ O. H. 58. *Shaot zemaniot* define important times in the morning like the end of the third and fourth hour. In the afternoon, *shaot zemaniot* are used to define ***minḥah ketanah***, 3 and ½ hours after *ḥatzot* or 2 and ½ hours before nightfall, ***plag ha-minḥah***, the midpoint between *minḥah ketanah* and nightfall, 1 and ¼ hours before nightfall, and perhaps as well ***minḥah gedolah***, ½ hour after *ḥatzot*.

⁴ Both methods had earlier proponents. In the 17th century *sefer Minḥat Kohen*, the first comprehensive treatment of *zemanim* authored by R. Avraham Pimental, he refers to the dispute between R. Yisroel Isserlein, author of *Trumat Ha-deshen* (14th / 15th century), and R. Mordekhai Yaffe, author of the *Levushim* (16th century), in place of the Magen Avraham and the *Gaon*, respectively. The position of the Magen Avraham is provably that of Ramban (13th century) and other *ḥakhmai seforad*. Earlier attribution of the position of the *Levushim* is subject to debate.

of the daylight period on a specific day, a function of both location and the season of the year. Near the equator, throughout the year days are always approximately 12 hours long. As one travels further from the equator, variation in the length of the daylight period increases particularly during the summer and winter; *shaot zemaniot* vary accordingly.⁵

The *Gaon* calculated *shaot zemaniot* from *hanetz ha-ḥamah*,⁶ the point the sun begins to rise, until *shkiyat ha-ḥamah*, the point that it has fully set.⁷ However, with respect to the opinion ascribed to the Magen Avraham there is little unanimity on the precise method of calculation. This monograph discusses multiple conceptual and practical approaches for implementing

⁵ We avoid any discussion of regions in the Arctic and Antarctic circles, where the daytime or nighttime periods can last up to 6 months. Those regions introduce other fundamental questions. See for example, *Contemporary Halakhic Problems*, by R. J. David Bleich, volume 5.

⁶ This method makes the day slightly longer than measuring

- from midpoint to midpoint,
- from when the sun is beginning to rise to beginning to set, or
- from when it has completely risen to completely set.

Using the latter two methods would slightly miscalculate *ḥatzot*. Both the method used as well as a midpoint to midpoint calculation correlates precisely with the observed time for *ḥatzot*.

⁷ The impact of the sun's rays bending is ignored by the *halakha*; only what is seen versus the actual position of the sun is the basis for measurement.

the opinion of the Magen Avraham.⁸ These approaches derive from positions in **three** fundamental and independent areas that together establish the precise endpoints to be used in implementing the Magen Avraham's calculation.⁹

Although this monograph is organized around the opinion of the Magen Avraham, the first two areas covered present the primary options for specifying the time of *alot ha-shaḥar*; those areas are relevant as well to those who follow the opinion of the *Gaon*.

The approach of the *Gaon* begins the calculation of *shaot zemaniot* with sunrise, while the Magen Avraham's calculation begins earlier at *alot ha-shaḥar*. For the moment, assume that *alot ha-shaḥar* is defined around (or by) the first light of day. Well before the sun appears above the horizon in

⁸ The method of organization chosen is itself non-obvious. We ask the reader's indulgence for the reasoning behind this chosen method of organization. As should become clear as well, the method chosen does not introduce or exclude any alternative.

⁹ Which calculation R. Gombiner himself used is not of primary interest. Recently, Rabbi D. Y. Boorstyn, in *Zemanim Ke-hilkhatam*, page 74, footnote 10, takes the startling position that R. Gombiner did not calculate using the approach attributed to him. A digression into the details of the Magen Avraham's personal practice is outside the scope of this monograph. However, a part of R. Boorstyn's argument rules out specific approaches based on his assumption that they are inconsistent with *ḥatzot* being calculated accurately. Some of the alternatives developed in this monograph that maintain the accuracy of the calculation of *ḥatzot*, might weaken R. Boorstyn's argument and support the universally held assumption that the Magen Avraham calculated using one of the alternatives given for the approach ascribed to him.

the east, its light is already apparent; *alot ha-shaḥar* corresponds to that earlier point.

The *gemara* in *Pesaḥim* 94a is the primary text for specifying the time of *alot ha-shaḥar*. The *gemara* discusses the average distance covered by a typical person walking for the entire daytime period. Since the daytime period on a long summer day is longer than during a short winter day, the average distance a person can walk during the daytime period varies during different seasons of the year. All commentators therefore assume that the *gemara* is referring only to travel on an average day, which occurs near both the spring and fall equinox. At those two times of the year, daytime and nighttime periods are both 12 hours long. For those days (only), the *gemara* implicitly provides the time of *alot ha-shaḥar*; however, how that time is to be determined remains uncertain.¹⁰ Depending on how the *sugyah* is to be understood, the two primary opinions for **the time of *alot ha-shaḥar*** on days in the Middle East that are 12 hours long are 72 and 90 minutes before sunrise. These two opinions and others less frequently cited constitute the **first disagreement** on how to determine the time when *alot ha-shaḥar* occurs.

Even assuming for the moment, we can resolve the first disagreement, we only know the time of *alot ha-shaḥar* for a day near either the spring or fall equinox and presumably only for the location(s) addressed in the Middle

¹⁰ Note that the *gemara* does not define *alot ha-shaḥar*; it merely specifies when it occurs.

East.¹¹ The **second disagreement** focuses on determining the time of *alot ha-shaḥar* more generally. This disagreement can be formulated both **conceptually and practically**. **Conceptually**, how do we define the time of *alot ha-shaḥar*? For example, does the time of *alot ha-shaḥar* always correlate with the first light of the day¹² as we normally assume, or perhaps it begins at some invariant period prior to sunrise? **Practically**, how do we establish the time of *alot ha-shaḥar* on other days of the year in the Middle East and every day of the year elsewhere in the world? As is apparent scientifically, we need to determine **if and how *alot ha-shaḥar* varies based on latitude, the season of the year, or both**.

The impact of both latitude and season on the exact number of minutes before sunrise that a defined level of illumination is apparent is indisputable. Nonetheless, the relevancy of latitude and season to determining the time of *alot ha-shaḥar* remains widely disputed. Assume it was decided, for example, that *alot ha-shaḥar* occurs 90 minutes before sunrise in Jerusalem on a day near the fall equinox that happened to occur one (13 month) year on the 21st of *Elul*. How many minutes before sunrise is *alot ha-shaḥar* on that same day in Vilna, located at latitude much further from the equator than Jerusalem? In Vilna on that day the impact of the rising sun can be observed well before 90 minutes before sunrise.

¹¹ Most often Jerusalem is assumed, as will be done throughout this monograph. Choosing another location in the Middle East would not make a significant difference.

¹² We will not analyze in detail important opinions (of *rishonim*) that might advance or delay that point slightly from a few minutes before the point that any light is visible to the naked eye to a later point when light is more apparent.

Similarly, when is *alot ha-shaḥar* in Jerusalem on a different date of the year, say on the 4th of *Shevat*? In Jerusalem, 90 minutes before sunrise on that day, the amount of visible light is slightly greater than the amount of light visible on the 21st of *Elul*.¹³ In general, how does the duration of the interval between *alot ha-shaḥar* and sunrise vary, if at all, with changes in either latitude or season or both?

Note that using the *Gaon's* approach for calculating *shaot zemaniot* sidesteps this issue completely; sunrise and sunset and the duration of time between vary naturally at distinct locations / latitudes and during different seasons of the year. Correspondingly, the *Gaon's* calculation of *shaot zemaniot* adjusts as that interval changes, varying based on both latitude and season. Since the entire sunrise to sunset interval is a part of the Magen Avraham's calculation, variation based on that interval is always included. However, if and how the period between *alot ha-shaḥar* and sunrise and its evening equivalent incrementally affects the length of *shaot zemaniot* is disputed by *posekim*.¹⁴

These two disagreements jointly determine the time of *alot ha-shaḥar*, the starting point for the Magen Avraham's calculation of *shaot zemaniot*. The

¹³ This indisputable fact is on occasion misunderstood as will be discussed further. Note that as the days get shorter in the winter, the length of time between *alot ha-shaḥar* and sunrise increases. Accurate scientific measurements verify this fact.

¹⁴ Given the clear definition of sunrise this question is more often stated not in terms of defining *alot ha-shaḥar* but equivalently in establishing the duration of the interval between *alot ha-shaḥar* and sunrise. This equivalent formulation is much more common; it is used in this monograph as well.

third disagreement is the least intuitive and here the reader's indulgence is requested. The question is **what point in the evening should be used in conjunction with *alot ha-shaḥar*** in implementing the Magen Avraham's calculation? Most readers will assume that if *alot ha-shaḥar* is X minutes before sunrise, then the evening point must be X minutes after sunset. Any other alternative appears problematic.¹⁵ As we will explain, this assumption of required symmetry is perhaps not necessary. Other slightly variant alternatives exist and may have been the basis for the practice of many *posekim* particularly prior to the 20th century. Partially because of a major dispute in Jerusalem over century ago on how to implement the Magen Avraham's calculation of *shaot ha-yom*, in recent years this approach has had minimal, if any, use.

To summarize, according to the opinion of the Magen Avraham *shaot zemaniot* are calculated from *alot ha-shaḥar* until some point in the evening. The three areas to be addressed are:¹⁶

1. When is *alot ha-shaḥar* on the average 12-hour day that occurs around the equinox in the Middle East?

¹⁵ Such an alternative should create problems calculating *ḥatzot* both in the day and at night, like the significant debate over R. Shlomo Zalman Auerbach's ruling about the latest time to eat *matzah* at the *seder*. R. Auerbach used asymmetric endpoints, sunset and *alot ha-shaḥar*, to determine *ḥatzot ha-lailah*.

¹⁶ As noted, one immediate benefit of this organization is that the first two areas are completely applicable even to those who follow the opinion of the *Gaon*. The third area as well is potentially relevant more broadly than just the opinion of the Magen Avraham, a topic that is not pursued further.

2. Does the duration of the interval between *alot ha-shaḥar* and sunrise vary, by season and latitude, and if so, how?
3. What point in the evening is used to calculate the position of the Magen Avraham?¹⁷

The former two areas contain nothing novel beyond rigorous proofs and arguments for the use of depression angles. The latter area provides a basis, albeit with a (slight) modification, for a method previously used but now largely abandoned. Also included later in this monograph is a proposed implementation of the Magen Avraham's position that circumvents a major reason that led to its widespread rejection.¹⁸

ALOT HA-SHAḤAR

The *gemara* in *Pesaḥim* discusses whether the period between *alot ha-shaḥar* and sunrise (and the equivalent period between sunset and *tzait ha-*

¹⁷ Potential variation by latitude and season of the point in the evening need not be covered explicitly. When the evening and morning endpoints are set symmetrically, as they normally are, it is always the case that either both points are adjusted or both points are not. The alternatives to be developed that use (earlier) asymmetric points in the evening are always assumed to be adjusted by latitude and season.

¹⁸ R. Joseph Soloveitchik did not concern himself with the position of the Magen Avraham and R. Moshe Feinstein, adopting the position of the *Gaon*, behaved similarly, although in a less pronounced way.

kokhavim) equals the time to walk 4 or 5 *milin*.¹⁹ Assuming²⁰ the oft-quoted statement, “A person walks 40 *milin* in one day,” the majority of *rishonim* divide the 40 *milin* distance covered as follows: 4 or 5 *milin* in each of the (equal) periods from *alot ha-shaḥar* to sunrise and sunset to *tzait ha-kokhavim* and 30 or 32 *milin* in the period from sunrise to sunset. Those who maintain a 5-*milin* interval, are left with a daytime period of the time it takes to walk 30 *milin*, while those that maintain a 4-*milin* interval, are left with a 32-*milin* daytime period. Given that the day in question is universally assumed²¹ to occur around the spring (or fall) equinox, the interval from sunrise to sunset is 12 hours or (12 * 60 =) 720 minutes. Dividing 720 minutes by 30 yields a time to walk a *mil* of 24 minutes, which is the opinion of Rambam; dividing 720 minutes by 32 yields a time to walk a *mil* of 22.5 minutes, which is the opinion of many, if not most, other *rishonim*.²² This

¹⁹ We often will use the phrase “the time to walk X *milin*”. Undoubtedly, a person walking for an entire day will cover fewer *milin* per hour than one who walks for a shorter period. In every context, we calculate taking the number of *milin* walked in a daytime period and then divide by the required number. Thus, if one walked 32 *milin* in a daytime period of 12 hours, the time to walk a single *mil* is 22.5 minutes even though one can assume that the time required to walk only one *mil* is significantly less than 22.5 minutes.

²⁰ A novel alternative to this assumption is discussed below.

²¹ As noted previously, since the length of the day varies by season, it is universally accepted that the *gemara* in *Pesaḥim* refers to the two days of the year that are average, when the length of the day and night are equal.

²² Prof. Levi in *Halakhic Times* argues that all *rishonim* maintained either 22.5 or 24 minutes; R. Ovadiah Yosef proposed several counterexamples. See footnote 27.

latter calculation is the basis of the 90-minute interval²³ from *alot ha-shaḥar* to sunrise. For Rambam and a few others, it would therefore appear that the *alot ha-shaḥar* to sunrise period would be the time to walk 5 *milin*,²⁴ or 120 minutes.

While our text of the *gemara* concludes definitively that the interval between *alot ha-shaḥar* and sunrise is 4 *milin*, the Mishnah in *Pesaḥim*²⁵ and the final *pesak* of Rambam, state that the period from sunrise to midday is approximated by the time that one walks 15 *milin*. The sunrise to sunset interval is thus 30 *milin*, ostensibly implying that the time of *alot ha-shaḥar* is the time to walk 5 *milin* or 120 minutes before sunrise. However, that view has limited support.²⁶ It is not (even) supported by Rambam in *Peirush Ha-mishnayot* and, as we will see, it is challenging given astronomical observation, something of which Rambam was certainly aware.

²³ The time to walk 4 *milin*, each 22.5 minutes, is 90 minutes.

²⁴ The time to walk 5 *milin*, each *mil* requiring 24 minutes, is 120 minutes.

²⁵ The Mishnah does not mention a numeric distance, just the distance from Jerusalem to Modiim. However, assuming the uncontested view of Ullah, the distance from Jerusalem to Modiim is the time to walk 15 *milin*. How this corresponds to actual distances between various locations is beyond the scope of this monograph.

²⁶ R. Ovadiah of Bartenura and perhaps R. Shneur Zalman of Liadi. See *Ha-zemanim Be-halakhah*, chapter 21, footnotes 39-43, where the position of R. Shneur Zalman of Liadi is examined in detail and aligned with the normally assumed interval of 72 minutes.

While both 22.5 and 24 minutes and their normally assumed correspondence to *alot ha-shaḥar* 90 and 120 minutes before sunrise are currently mentioned on occasion, 18 minutes has become the *halakhic* standard as the time to walk a *mil*, as in, for example, the “18-minute *matzah*.” An 18-minute interval for the time to walk one *mil*, multiplied by 4 derives the prevalent and normally assumed interval of 72 minutes between *alot ha-shaḥar* and sunrise. Although the 18-minute time to walk a *mil* is adopted by the *Shulḥan Arukh* and most *aḥronim*, there are only a few isolated references²⁷ to this opinion prior to R. Yisroel Isserlein in

²⁷ Rav Ovadiah Yosef, *Yeḥaveh Daat* volume 2, page 38, attempting to find support for the *Shulḥan Arukh*'s position of 18 minutes as the time to walk a *mil*, finds only a few earlier references, importantly including R. Saadyah Gaon. Those references are often indirect; instead of a direct reference to 18 minutes, the reference is to 72 minutes often expressed as one and 1/5th hours, which is assumed to be derived from 4 intervals of 18 minutes. Note two cautions with respect to R. Yosef's arguments. First, as is shown when discussing Rambam's position, supporting 72 minutes as the interval of time between *alot ha-shaḥar* and sunrise need not necessarily imply support for the time to walk a *mil* of 18 minutes; while it is almost always the case, it is not axiomatic, as R. Yosef appears to assume. In the context of this monograph, with focus on the determining the interval between *alot ha-shaḥar* and *hanetz ha-ḥamah*, the length of time to walk 4 *milin*, as opposed to focus on the length of time to walk 1 *mil*, this point is not relevant. The second caution is relevant and dependant on whether an **hour** refers to a standard clock hour or a *shaah zemanit* that is calculated based on the opinion of the Magen Avraham from *alot ha-shaḥar*, 72 or 90 minutes before sunrise until *tzait ha-kokhavim*, 72 or 90 minutes after sunset. In addition to 72 minutes, one and 1/5th hour can also equal either 86.4 or 90 minutes. If one is counting the day from *alot ha-shaḥar* 90 minutes before sunrise until *tzait ha-kokhavim*, 90 minutes after sunset, then the entire daytime period is (720 + 90 + 90 =) 900 minutes. Each *shaah zemanit* is (900 /

Trumat Ha-deshen written in the fourteenth / fifteenth century. R. Isserlein, in discussing the *halakhot* of *matzah*,²⁸ describes the time to walk a *mil* as 1/4 of an hour (15 minute) plus 1/20th of an hour (3 minutes) or 18 minutes.

To reconcile the time to walk a *mil* of 18 minutes with the *gemara* in *Pesaḥim*, a second element must be added to the debate. To this point, both sides in the 4 *milin* versus 5 *milin* dispute agree that the 40 *milin* that an average person walks in a day occurs between *alot ha-shaḥar* and *tzait ha-kokhavim*. However, to derive 18 minutes, both the period from *alot ha-shaḥar* to sunrise and the period from sunset to *tzait ha-kokhavim*, each of 4 *milin*,²⁹ **must occur outside of the 40-*milin* period**. Thus, a person walks 4 *milin* between *alot ha-shaḥar* and sunrise, 40 *milin* from sunrise to sunset, and 4 *milin* from sunset to *tzait ha-kokhavim*. Dividing the 720-minute period between sunrise and sunset by 40 results in 18 minutes as the time to walk a *mil*. Under this interpretation, the dispute in the *gemara* is two-fold:

1. What period does the period to walk 40 *milin* refer to – sunrise to sunset or *alot ha-shaḥar* to *tzait ha-kokhavim*?
2. Is the *alot ha-shaḥar* to sunrise period and its equivalent in the evening the time to walk 4 or 5 *milin*?

12 =) 75 minutes and 1 and 1/5th hours is 90 versus 72 minutes. If one is using 72 minutes, then each *shaah zemanit* is $((720+72+72) = 864 / 12 =) 72$ minutes and 1.2 hours is 86.4 minutes.

²⁸ *Trumat Ha-deshen* 123.

²⁹ Or 5 *milin* for that matter, although that opinion is very rare.

While our text of the *gemara* definitively concludes that the *alot ha-shaḥar* to sunrise period and its equivalent in the evening is the time to walk 4 and not 5 *milin*, some *rishonim* still maintained the 5-*milin* period. If that option is not precluded, then there remain **four** potential points of view.

Assuming a 4-*milin* period from *alot ha-shaḥar* to sunrise, the conclusion of our text of the *gemara*:

1. One walks 40 *milin* in one day beginning at sunrise. This is the standard opinion of the *Shulḥan Arukh* deriving a time to walk a *mil* of 18 minutes and a 72-minute interval from *alot ha-shaḥar* to sunrise.
2. One walks 40 *milin* in one day beginning at *alot ha-shaḥar*. This is the opinion of many *rishonim* and specific *aḥronim* deriving a time to walk a *mil* of 22.5 minutes and a 90-minute interval from *alot ha-shaḥar* to sunrise; it is also the so-called “*Brisker aḥtel*,” ($720 / 8 = 90$), an *aḥtel* meaning $1/8^{\text{th}}$ in Yiddish.

However, assuming a 5-*milin* period from *alot ha-shaḥar* to sunrise:

3. One walks 40 *milin* in one day beginning at sunrise. This is a rare opinion that derives a time to walk a *mil* of 18 minutes but a 90-minute interval from *alot ha-shaḥar* to sunrise.³⁰
4. One walks 40 *milin* in one day beginning at *alot ha-shaḥar*. This is the opinion of R. Ovadiah of Bartenura (and as normally assumed,

³⁰ This opinion is mentioned in *Pesaḥim* 11b *tosefot* sv. *Ve-eḥad omer*; see the *Gaon* in *O. H.* 459 as well.

Rambam) deriving a time to walk a *mil* of 24 minutes that ostensibly³¹ implies a 120-minute interval from *alot ha-shaḥar* to sunrise - the so-called “*Brisker szektel*,” ($720 / 6 = 120$.)

A 96-minute interval, 4 times a 24-minute *mil*, is cited on occasion.³² However, it is unsupportable and does not correspond to any accurate reading of the *gemara*. To derive a time to walk a *mil* of 24 minutes requires that there are 5 and not 4 *milin* between *alot ha-shaḥar* and sunrise. Thus, those who, despite the apparent conclusion of our text in the *gemara*,³³ maintain that the interval between *alot ha-shaḥar* and sunrise consists of 5 *milin*, must conclude that there are ($5 * 24 =$) 120, not 96, minutes between *alot ha-shaḥar* and sunrise.

Support for 22.5 minutes as the time to walk a *mil* and thus a 90-minute interval between *alot ha-shaḥar* and sunrise can be derived in two distinct ways:

³¹ Rambam does not make this ostensibly logical conclusion explicitly; as noted, Rambam commenting on the first Mishnah in *Berakhot* in *Peirush Ha-mishnayot* maintains a 72-minute interval. There have been multiple efforts to explain Rambam, including a complex attempt by R. Mosheh Sofer in his *ḥiddushim* on Shabbat 34a and another by R. Yeshayahu Karelitz in chapter 13, paragraph 2 of his commentary on *O. H.*, who assumes that Rambam changed his mind, at a later point in life. Both explanations are challenging, a topic not addressed in this monograph; an alternative explanation by R. Schlesinger is covered below.

³² *Melamaid Le-hoil* 30, *Igrot Mosheh O. H.* 4:62.

³³ R. Ovadiah of Bartenura and Rambam *Peirush Ha-mishnayot*, *Berakhot* 1:1, and *Mishneh Torah*.

- ❖ By the second interpretation of the *gemara* in *Pesaḥim* outlined above, as, for example, is specified in our text of Rashi.
- ❖ By derivation from a comment by Ramban³⁴ and many *ḥakhmai seforad* who specify that *plag ha-minḥah* occurs at the time to walk 1/6th of a *mil* before sunset.

While, the former method derives a 90-minute interval directly, the latter is more complex. It is mathematically provable that anyone who asserts that *plag ha-minḥah* occurs the time to walk 1/6th of a *mil* before sunset, must also maintain a time to walk a *mil* of 22.5 minutes, and cannot simultaneously maintain 18 minutes as the time to walk a *mil*.³⁵ The reader is strongly cautioned that it is not always understood that

³⁴ *Torat Ha-adam*, Chavel edition, page 251 - 255.

³⁵ **Linear equations have only one solution.** Ramban's assertion can be expressed as a linear equation:

$$(720 + 8X) / 12 * 1.25 = 4X + 1/6 X$$

to which X = 22.5 is the unique solution. Note the left side of the equation specifies the time between *plag ha-minḥah* and the end of the day as it is defined in the *gemara* – a *shaah zemanit* ((720 + 8X) divided by 12), multiplied by 1.25. The right side of the equation specifies the time between *plag ha-minḥah* and the end of the day given by Ramban and other *ḥakhmai seforad* as the time to walk 4 *milin* between sunset and *tzait ha-kokhavim* and an additional time to walk 1/6th of a *mil* by which *plag ha-minḥah* precedes sunset. However, if the time to walk a *mil* is 18 minutes, then *plag ha-minḥah* is ((720 + 144) / 12 * 1.25 =) 90 minutes before *tzait ha-kokhavim* or 18 minutes before sunset. Only a time to walk a *mil* of 22.5 minutes results in *plag ha-minḥah* occurring the time to walk 1/6th of a *mil* before sunset.

- ❖ maintaining 18 minutes as the time to walk a *mil*,
- ❖ while also maintaining that *plag ha-minḥah* occurs the time to walk 1/6th of a *mil* before sunset

is impossible; those views are provably incompatible.³⁶ In addition to conceivably not recognizing the necessity of maintaining a time to walk a *mil* of 22.5 minutes from the specification of *plag ha-minḥah* given by Ramban and many *ḥakḥmai seforad*, conceivably because many of the

³⁶ There is a flawed “proof” that asserts otherwise. See *Am Mordechai – Berakhot* chapter 2, where R. Mordechai Willig disputes this proof. The proof mixes use of the *Gaon* and the Magen Avraham, calculating *plag ha-minḥah* from *tzait ha-kokhavim* like the Magen Avraham while using *shaot zemaniot* defined using the *Gaon*’s approach. Thus, an hour is set to 60 minutes on a canonical 12-hour day in accordance with the approach of the *Gaon* and the time of *plag ha-minḥah* is derived by subtracting $(1.25 * 60 =) 75$ minutes from *tzait ha-kokhavim* at 7:12PM, as in the approach of the Magen Avraham. This ostensibly “proves” that 72 minutes and a time to walk a *mil* of 18 minutes is consistent with the opinion of Ramban - *plag ha-minḥah* is 75 minutes before 7:12PM, at 5:57PM, the time to walk 1/6th of a *mil* (3 minutes) before sunset. Of course, the hour should be $((720 + 144) / 12 =) 72$ minutes long and *plag ha-minḥah* is derived by subtracting $(1.25 * 72 =) 90$ minutes from *tzait ha-kokhavim* at 7:12PM and occurs at 5:42PM, the time to walk a *mil* before sunset. The reader should verify that this approach does not even allow the calculation of *ḥatzot*, subtracting 6 hours from *tzait ha-kokhavim* sets the time of *ḥatzot* at 1:12PM while adding 6 hours to *alot ha-shaḥar* sets the time of *ḥatzot* at 10:48AM. In addition to the mathematical error shown in the previous footnote and the logical error described in this footnote, R. Willig also notes that this argument contradicts a *gemara* asserting the interval from *minḥah gedolah* to *minḥah ketanah* as 3 hours. The reader should be aware that the existence of such errors necessitates checking all calculations carefully.

ḥiddushim of *ḥakhmai seforad* were not available, the derivation of a 90-minute interval based on the second criteria above is not widely found as well. In fact, as noted, 90 minutes, 1/8th of a 12-hour day, is often called a “*Brisker aḥtel*.” While the *Shulḥan Arukh* and most *aḥronim* adopted both 18 and 72 minutes as opposed to 22.5 and 90 minutes, several major *aḥronim* argued strongly for 22.5 and 90 minutes.³⁷

In addition to the methods listed above, there are two other derivations for 72 minutes. R. Pimental, despite his comprehensive work covering almost all aspects of *zemanim*, never engages in a discussion of the *sugyah* in *Pesaḥim*. Instead, he quotes only one line of the *sugyah*: R. Yehudah asserts that “*oveyo shel rakiya*,” referring to both the periods of *alot ha-shaḥar* to sunrise and sunset to *tzait ha-kokhavim*, are each 1/10th of the daytime period. Applying 1/10th to a daytime period of 720 minutes, R.

³⁷ R. Y. Reicher, the author of the *Ḥok Yaakov* and R. Mosheh Sofer supported this position. The argument about the position of the *Gaon* can make for its own essay with a formidable list of more recent *aḥronim* on both sides of the issue. Two indications that the *Gaon*'s position is 22.5 minutes are often cited. First, the *Gaon*'s gloss to *O. Ḥ.* 261 gives the length of the interval from *alot ha-shaḥar* to sunrise, at latitude around the Middle East, as one and one-half hours (90 minutes.) Second, in *O. Ḥ.* 459 he specifies that the period from *alot ha-shaḥar* to *tzait kol ha-kokhavim* is 15 hours, two 90-minute periods added to the 12-hour daytime period. Support for those ascribing to the *Gaon* an 18-minute time to walk a *mil* derives from his commentary on *Berakhot*.

Interestingly, when R. Feinstein (*Igrot Mosheh O. Ḥ.* 4:6) wants to support *alot ha-shaḥar* 90 minutes before sunrise as a leniency under pressing circumstance, he quotes no *rishonim*. In addition to quoting the *Ḥok Yaakov* and the *Gaon*, he reads the sequence of events in *masekhet Tamid* innovatively in support of this position.

Pimential derives a 72-minute interval and thus the time to walk a *mil* of 18 minutes. Clearly those who maintain a 22.5-minute *mil* and 90-minute interval must interpret R. Yehudah's assertion that “*oveyo shel rakiya*” are $1/10^{\text{th}}$ of the day as a variant of *milebar*³⁸ – $1/10^{\text{th}}$ of 900, adding not one but two 90-minute intervals to the 720 minutes between sunrise and sunset.

The other position clearly supporting 72 minutes is Rambam. Despite maintaining the time to walk a *mil* of 24 minutes, Rambam also maintained

³⁸ *Mi-le-gav* (from the inside) is the standard way that fractions are currently expressed; e.g., $\frac{1}{4}$ of 12 = 3. The alternative is to say that $\frac{1}{5}^{\text{th}}$ of 12 is 3, meaning that if you add 3 to 12 and take $\frac{1}{5}^{\text{th}}$ of the sum, 15, the result is 3. Thus, the biblical term “*ve-ḥamishiso*” meaning a fifth is what our standard use would call a fourth; in biblical usage, $\frac{1}{5}^{\text{th}}$ of 10 is 2.5. This alternative is referred to as *mi-le-bar* or from the outside, meaning from the sum. Normally we say that 90 is $1/10^{\text{th}}$ of 810, not of 720, *mi-le-bar*, adding 90 minutes to 810 and then taking $1/10^{\text{th}}$ of 900. When reading Talmudic texts, one would normally assume *mi-le-bar* as opposed to *mi-le-gav*. However, in this case an interval of 90 minutes must be added twice, 90 is $1/10^{\text{th}}$ of $(720 + 2 * 90 =) 900$. Another argument in favor of *mi-le-gav* is advanced by the *Gaon* in *O. H.* 459. When contrasting 5 *milin* to 4 *milin*, the *gemara* uses the fractions $\frac{1}{6}^{\text{th}}$ and $1/10^{\text{th}}$ of the daytime period, respectively. The *gemara* rejects a 5-*milin* interval for *alot ha-shaḥar* to sunrise, which it refers to as $\frac{1}{6}^{\text{th}}$ of a 30-*milin* day in favor of an interval corresponding to $1/10^{\text{th}}$ of a day. Clearly, $\frac{1}{6}^{\text{th}}$ means that the 5 *milin* walked between *alot ha-shaḥar* and sunrise is $\frac{1}{6}^{\text{th}}$ of the 30 *milin* walked from sunrise to sunset. Assuming an identical meaning to the numerator and denominator, then $1/10^{\text{th}}$ must mean that the 4 *milin* walked between *alot ha-shaḥar* and sunrise is $1/10^{\text{th}}$ of the 40 *milin* walked between sunrise and sunset. Since $\frac{1}{6}^{\text{th}}$ is clearly *mi-le-gav*, by implication, $1/10^{\text{th}}$ is as well.

that *alot ha-shaḥar* is 1 and 1/5th hours or 72 minutes before sunrise.³⁹ Similar to R. Pimental's use of 1/10th, an innovative solution was proposed by R. Schlesinger⁴⁰ to resolve this apparent contradiction. R. Schlesinger assumes, like the *Gaon*,⁴¹ that Rambam may not have had (or partially discounted) our text of the *gemara*. Instead, R. Schlesinger argues Rambam attempts to decide in accordance with the (only) two statements of *tannaim* that are quoted in the *sugyah*. First, Rambam took 24 minutes as the time to walk a *mil* as the undisputed opinion of R. Akiva.⁴² The only other undisputed statement of a *tanna* is R. Yehudah's assertion that both twilight periods, *alot ha-shaḥar* to sunrise and sunset to *tzait ha-kokhavim*, are each 1/10th of the daytime period. Since Rambam assumes the daytime period from sunrise to sunset is the time it takes to walk 30 *milin*, then each of the twilight periods is the time to walk $((1/10) * 30 \text{ milin} =) 3 \text{ milin}$. Given that the time to walk a *mil* is 24 minutes, the time to walk 3 *milin* is 72 minutes.⁴³ Usually one thinks of 72 minutes as 4 intervals of 18

³⁹ Other attempts to reconcile these opinions were mentioned in footnote 31.

⁴⁰ *Yeshuran* 5695.

⁴¹ *O. H.* 459.

⁴² In addition to dependence on R. Akiva's opinion in the Mishnah, Rambam must also assume the undisputed opinion of Ullah on the distance from Jerusalem to Modiim as the time to walk 15 *milin*.

⁴³ Assumedly, this would mean that Rambam does not maintain that one walks 40 *milin* in a day but rather $(30 + (2 * 3) =) 36 \text{ milin}$. However, Rambam himself in *hilkhot aveilut* when specifying *derekh reḥokah* assumes that one walks 40 *milin* in a day. Several solutions beyond the scope of this monograph have been proposed; some assume one

minutes or even as 1/10th of the daytime period; 72 minutes as 3 intervals of 24 minutes was to my knowledge never mentioned prior to R. Schlesinger.

While there are yet other opinions, 72 and 90 minutes are by far the most prevalent and as will be clear in the next section, both can be reconciled with accurate observation in Jerusalem in the period around the spring and fall equinox. For the remainder of this monograph we will only consider these two opinions.

VARIATION BY SEASON AND LOCATION

Whether and how variation by season and / or latitude impacts the time of *alot ha-shaḥar* (and other *zemanim*) is actively debated. However, if one assumes that *alot ha-shaḥar* and the first light of day are correlated, the conclusion is clear-cut; the use of what are called depression angles is mandated. While many *posekim* with unique expertise in this area⁴⁴ have a

can walk faster under pressing circumstance while others see this as an example of being lenient with respect to the laws of *aveilut* and relying on an opinion that is not normally accepted.

⁴⁴ Depression angles were discussed by R. Dovid Tzvi Hoffman, used in preparation of *zemanim* for R. Naftali Tzvi Berlin and were importantly used and advocated by R. Yechial Michel Tukitzinsky. Among contemporaries, many *posekim* including R. Belsky and R. Willig, most authors of *seforim* on *zemanim*, many calendars and almost all websites use depression angles extensively. In fact, as noted in footnote 47, some go so far as to discount opinions that conflict with observation, currently expressed in terms of depression angles.

strong preference for the use of depression angles, other *posekim* set the time of *alot ha-shaḥar* at a fixed number of minutes before sunrise.

A depression angle measures how far below the horizon the sun appears at a given moment, providing an accurate measurement of the level of (remaining) illumination; a larger angle indicates that the sun is further below the horizon with less illumination coming from the sun. If a depression angle of X degrees occurs at 4:30AM in London and 4:50AM in New York on the same or different days, then one can be certain that the level of illumination from the (rising) sun is identical at those two times.

To determine *alot ha-shaḥar* (as well as many other *zemanim*) in other latitudes and seasons of the year,⁴⁵ assuming *alot ha-shaḥar* is defined by the degree of illumination from the sun, one can utilize depression angles. The first step is to establish the number of degrees below the horizon the sun is located 72 and 90 minutes before sunrise in the Middle East around the spring / fall equinox. The second step is to use that same number of degrees as the angle to determine *alot ha-shaḥar* elsewhere and during other times of the year. In practice, the shorter 72-minute interval for *alot ha-shaḥar* corresponds to the sun being approximately 16 degrees below the horizon, while for the longer 90-minute interval corresponds to a depression angle of approximately 20 degrees.

In Israel around the spring / fall equinox, scientists consider the sun to provide no measurable light until approximately 80 minutes before sunrise

⁴⁵ Even before one reaches the Arctic and Antarctic circles, particularly as one moves more than 60 degrees from the equator, many *halakhot* must be carefully examined.

corresponding to a depression angle of approximately 18 degrees.⁴⁶ Since the *halakhah* often disregards miniscule, non-visible quantities, this provides observational support for the standard *pesak* of 72 minutes (and by implication for the time to walk a *mil* of 18 minutes.) In any case, 72 and 90 minutes are both reasonable approximations to this scientific delineation of the first time when measurable light from the sun is observable.⁴⁷

From everything I can determine, depression angles capture the *halakhic* notion of the degree of darkness and light accurately; no alternative for “measuring” *hashekhah* or *alot ha-shahar* has ever been formulated, nor has anyone ever proposed any problem that the use of depression angles might create. Depression angles naturally adjust *zemanim* based on latitude and season. Clearly, we do not need such precision; observation was adequate for generations. Nonetheless, **a depression angle is to darkness / illumination what a watch is to time.** Despite an admitted

⁴⁶ There is an interesting comment by R. Hoffman, *Melamaid Le-hoil* 30, like that of R. Pimental, relating the comment of R. Yehudah that *oveyo shel rakiya* are 1/10th of the day to 18 degrees being 1/10th of the 180-degree daytime movement of the sun.

⁴⁷ In a recent *sefer*, *Zemanim Ke-hilkhatam* by R. Boorstyn, it is suggested that the 90-minute period should not be used where it would create a leniency, as for example R. Feinstein did in *Igrot Mosheh O. H.* 4:6, because 90 minutes is counter-indicated by current scientific observation that cannot discern any light at so early a point. While I might be accused of apologetics, it may well be the case that *alot ha-shahar*, 90 minutes before sunrise, is not meant to identify the first point of light, but rather when people begin their day in anticipation of visible light about 20 minutes later. Given that 90 minutes was the opinion of many *rishonim* and *ahronim* and the science is in my mind confirming, this point is not addressed further.

bias on my part in favor of depression angles, there are several other approaches currently used by *posekim*. A thorough introduction to depression angles is provided in Appendix II, on www.thelehrhaus.com, dated, xxx.

In theory, one could postulate five possible approaches to calculating *alot ha-shaḥar*.

1. Determine the depression angle that correlates to *alot ha-shaḥar* at the spring and fall equinox in the Middle East and use that depression angle to determine *alot ha-shaḥar* at all other latitudes and seasons.
2. Use a 72 / 90-minute interval uniformly across all latitudes and seasons.
3. Adjust a 72 / 90-minute interval based on season in the Middle East and then use that time across all other latitudes.
4. Adjust the time of *alot ha-shaḥar* based on latitude but assume the interval is (treated as) constant independent of seasons.
5. Adjust using the assumption that the interval from *alot ha-shaḥar* to sunrise varies linearly with the length of the daytime period from sunrise to sunset.

Even though option 3 appears preferable to option 2, I know of no instance where it was ever used or even suggested and thus it is not considered further. Option 4 may result either from the desire to simplify option 1 and use a conservative, but fixed interval that is uniform throughout the year.⁴⁸

⁴⁸ Several rabbis, well versed in this area, have expressed support for this alternative because they prefer to keep practical *pesak* simple. R. Feinstein's famous 50-minute ruling for the end of *Shabbat* in the New York area, *Igrot Mosheh – O. H.* 4:62 is often

Alternatively, option 4 could be the result of a *posek* not realizing the impact of seasonal variation; at latitudes as far north as New York the variation in the length of the period between *alot ha-shaḥar* and sunrise is only about 15 minutes. In any case, we will disregard option 4 as well and consider it a practical alternative for implementing option 1. We deal with option 5 later in this section; it varies significantly from even casual observation.

With minor exception, only options 1 and 2 are normally encountered. However, in every case it is critical to avoid two pitfalls:

1. Making some adjustments based on physical observation (or equivalently by depression angles) and leaving other times fixed. Invariably this will lead to some anomaly. For example, some might leave *alot ha-shaḥar* fixed while adjusting *mi-she-yakir* based on observation. In more northern locations like Prague or London, *mi-she-yakir* will then often occur well before *alot ha-shaḥar* particularly during the summer.⁴⁹

used uniformly, most likely as it was intended. However, others adjust R. Feinstein's ruling seasonally, using 50 minutes as a maximum. See the www.myzmanim.com website, for example, where adjustments are made based on the guidance of R. Yisroel Belsky.

⁴⁹ This type of error was illustrated for example on an old OU website (2007) and other calendars that use depression angles for *mi-she-yakir* and a fixed 72 (or 90) minutes for *alot ha-shaḥar*.

2. While attempting to make adjustments that correlate with physical events, inadvertently doing so incorrectly; option 5 above, still encountered on occasion, is an example.

Despite a preference for *zemanim* that derive from observable physical events, conceptually one could argue that the twilight period is not to be adjusted, we simply use intervals around sunrise and sunset that equate to the duration of those intervals in the Israel (and only in the spring and fall.) This has the effect of treating what would normally be assumed as an approximation to an event, as defining the event himself.⁵⁰ Some defend this practice either based on the eventual breakdown of any methodology as the north and south poles are approached or by an appeal to tradition that in many cases made no adjustments. Thus, one could, at least in theory, not begin the day prior to 72 or 90 minutes before sunrise, regardless of the amount of light that might exist; considerably less likely,

⁵⁰ Arguing for *zemanim* given in terms of the time to walk a given interval is hardly intuitive. It is entirely more likely that these time equivalent measures are meant simply as approximations to *zemanim* defined by astronomic events. The fact that *masekhet Tamid* discusses events around *alot ha-shahar* and never raises a time-based approximation lends further (and powerful) support for the use of event-based *zemanim*. Widespread use of clocks, and the sense of precision they provide, may have contributed to this phenomenon.

Shabbat might end once a fixed 72 or 90-minute interval has passed, regardless of the number of stars visible.⁵¹

Option 5 and its occurrence in *halakhah* is a multifaceted subject, which is entirely incorrect. The interval between *alot ha-shaḥar* and sunrise is **longer not shorter in the winter** than it is in the spring or fall. In Jerusalem, for example, the same amount of light **72** minutes before sunrise on March 20 is already present **87** minutes before sunrise on June 21 and **78** minutes before sunrise in December 21. In the summer while directionally correct since the interval between *alot ha-shaḥar* and sunrise is longer in the summer than in the spring or fall, the correlation to the length of the daytime period (between sunrise and sunset) is directionally correct but imprecise.⁵² Fortunately *posekim* who employed this method would not rely on it for leniencies in the winter, where it is entirely incorrect. This method, still mentioned on occasion in various *halakhic* contexts,⁵³ is not considered further.

⁵¹ In locations close to the Arctic Circle, when it never gets dark enough for stars to appear waiting until *ḥatzot* (midnight) is normative. Waiting only 72 or 90 minutes after sunset to end *Shabbat* is likely a biblical violation in places like Saint Petersburg at times close to the summer solstice.

⁵² In Jerusalem, the ratio of the number of minutes between *alot ha-shaḥar* and sunrise in the summer and spring (or fall) is 87 / 72, or approximately 1.21. The ratio of the lengths of the respective days is 854 / 720 minutes, or approximately 1.19. That slight difference increases at latitudes further from the equator than Jerusalem.

⁵³ The reader is cautioned to carefully examine attempts by various *aḥronim* to show how this approach preserves some *halakhic* formulae for *plag ha-minḥah*. The fact that

Most of current practice uses either depression angles or a constant time interval, 72 or 90 minutes before sunrise. While not an absolute rule, many who follow the *geonim* with respect to the end of *Shabbat* and always adjust that time based on latitude (and on occasion season) tend to adjust *alot ha-shaḥar* as well. From a scientific perspective, this is entirely expected; the need for adjustment (based on something like a depression angles) is more apparent with a larger angle (*alot ha-shaḥar*) than a smaller one (the end of *Shabbat*.)⁵⁴ On the other hand, many who following Rabbeinu Tam wait a fixed 72 minutes after sunset for the end of *Shabbat*, also used a fixed 72 minutes before sunrise to establish the time of *alot ha-shaḥar*.⁵⁵

various *halakhic* formulae are preserved may represent elementary facts about ratios between numbers / formulae, mistakenly invested with *halakhic* significance.

⁵⁴ For example, on May 1st, using depression angles of 8.5 and 16 degrees, *Shabbat* ends 39 minutes after sunset in Jerusalem and 58 minutes after sunset in London, about 49% longer, while *alot ha-shaḥar* is 122 minutes before sunrise in London versus only 78 minutes in Jerusalem, about 56% longer. The difference resulting from the larger depression angle makes the need to adjust yet more evident. Choosing a city further north than London, Vilna for example, or a date closer to the summer solstice illustrates these phenomena yet more dramatically. A reader can repeat this exercise using a website like www.myzmanim.com with different cities at different times during the year.

⁵⁵ Conceptually Rabbeinu Tam's reconciliation of the *sugyot* in *Pesaḥim 94a* and *Shabbat 34b* forces use of the same interval (normally 72 minute) before and after sunrise and sunset for *alot ha-shaḥar* and the end of *Shabbat*, respectively. R. Joseph Soloveitchik in a *yarzeit shiur*, (*Shiurim Le-zekher Avi Mori*, volume 1, *Yom Ve-Lailah*)

The *sugyah* in *Pesaḥim* that uses the term “*oveyo shel rakia*,” *masekhet Tamid* that refers only to the appearance of varying levels of illumination like “*heyir kol penai mizrah*” as well as the various events defining of *mi-she-yakir*, argue strongly in favor of correlating *alot ha-shaḥar* to physical events and against viewing a “time to walk 4 or 5 *milin*” as definitional. The *Gaon*’s clear support for adjustment, given his assertion that *alot shaḥar* and its evening equivalent do not occur during the summer at very northern latitudes, is rarely quoted. In any case, despite strongly favoring the use of depression angles, for completeness, both views are maintained in the next sections.

outlined an original approach that reworked Rabbeinu Tam to a position, consistent with astronomical observation, creating a framework (based on complete darkness) implicitly using depression angles even per Rabbeinu Tam. While R. Soloveitchik preserved the equivalence that Rabbeinu Tam created between the length of time between *alot ha-shaḥar* and sunrise and the length of time between sunset and the end of *Shabbat*, he defined both periods by the same astronomical occurrence, the complete disappearance of the sun’s light, instead of a fixed number of minutes. *Ke-darkho be-kodesh*, R. Soloveitchik avoided focus on the science, while explicitly using a foundational mathematical notion. Most noteworthy, R. Soloveitchik did not address the significant observational challenges to the opinion of Rabbeinu Tam from an attempt to correlate the different events described in *Shabbat* 34a - 35b that occur at day’s end to an evening point that follows sunset by an interval equal to the length of the interval between *alot ha-shaḥar* and sunrise; that equivalence is fundamental to Rabbeinu Tam’s position.

THE EVENING POINT

Summarizing the first two major areas addressed to this point:

1. two options for *alot ha-shaḥar* around the spring and fall equinox in the Middle East – 72 and 90 minutes before sunrise, and
2. two options for *alot ha-shaḥar* during other seasons or at other latitudes – either a fixed time (72 or 90 minutes) before sunrise or the translation of 72 or 90 minutes before sunrise to their depression angle equivalents of approximately 16 or 20 degrees and the use of those depression angles as the basis for determining the time of *alot ha-shaḥar*.

Those four alternate times for *alot ha-shaḥar*, each some length of time before sunrise, corresponds to a symmetric endpoint the same length of time after sunset, which is to be used in calculating per the Magen Avraham. For shorthand, those four times will be referred to as 72 minutes fixed, 72 minutes adjusted, 90 minutes fixed, and 90 minutes adjusted.

In general,⁵⁶ morning *zemanim* using an adjusted period are earlier than their fixed counterpart. In the afternoon, the situation is considerably more complex. To illustrate, look at the relevant entries in the table at the end of the monograph. During the winter in Jerusalem a 90-minute interval, either adjusted or fixed, places *plag ha-minḥah* after sunset. Consider the shortest day in Jerusalem, which is slightly longer than 10 hours or 600

⁵⁶ We are referring to latitudes at least as far from the equator as the Middle East where almost all Jewish communities are located.

minutes, from sunrise to sunset. Adding (90 + 90 =) 180 minutes to 600 minutes and then dividing 780 minutes by 12 give a *halakhic* hour of 65 minutes. *Plag ha-minḥah* is approximately (1.25 * 65 =) 82 minutes before *tzait kol ha-kokhavim* and about 8 minutes after sunset.⁵⁷ This is particularly troubling because the *minhag* of Jerusalem follows both the Magen Avraham and a 90-minute interval for *alot ha-shaḥar*. If one travels as far north as Prague or London, 72 minutes will encounter similar difficulties, regardless whether the interval is adjusted or fixed.⁵⁸

Were the monograph to end at this point, most would assume the set of alternatives is complete despite some complications with the 90-minute alternative even in Jerusalem. However, looking more carefully at the opinion of *posekim* over hundreds of years, suggests that another option exists, one that would address the issue of *plag ha-minḥah* occurring after sunset around the time of the winter solstice in Jerusalem.⁵⁹

The existence of another option is surprising. Each of the four times in the evening corresponding to one of the four methods for calculating *alot ha-shaḥar* would appear to be almost axiomatic if *ḥatzot* must always be

⁵⁷ Using an adjusted 90 minutes, *plag ha-minḥah* is 13 minutes after sunset. Efforts to utilize option 5 in the previous section to avoid this problem are manifestly incorrect.

⁵⁸ In practice, outside of Israel, calendars most often only calculate the Magen Avraham *kenahug*, i.e. using 72 minutes fixed.

⁵⁹ As will become clear, this suggested alternative leaves the morning *zemanim* completely unaffected.

calculated correctly.⁶⁰ *Hatzot* is observable and occurs at the midpoint between sunrise and sunset. Thus, if sunrise is at 6:00AM and sunset is at 7:00PM, *hatzot* is exactly at 12:30PM. If one uses any two symmetric points, X minutes before sunrise and X minutes after sunset, the midpoint will always be at 12:30PM. However, if the calculation were to use asymmetric endpoints, for example, 4:30AM, 90 minutes before sunrise and 8:00PM, 60 minutes after sunset, then *hatzot would* be calculated to be at 12:15PM, 15 minutes too early.

Nonetheless, many *posekim*, perhaps less concerned with the accuracy of *hatzot*, but not necessarily, calculated differently. Their approach, as unusual as it may now appear, provides another method for calculating the opinion of the Magen Avraham. To appreciate their rationale, a summary of evening *zemanim* is required.

The *halakhot* concerning *zemanim* in the evening are more complex than those in the morning. In the morning, *alot ha-shaḥar* is the unquestioned beginning of the day. *Mi-she-yakir* is more than likely just the time where there is enough light to be certain that *alot ha-shaḥar* has occurred, while *hanetz ha-ḥamah* is the preferred time for specific *mitzvot*. In the evening, the level of disagreement between *posekim* is greater; theory and practice are both more diverse. In the morning, we are dealing with one occurrence, the beginning of the daytime period, while at night we are dealing both with the end of the daytime period as well as the transition

⁶⁰ See, for example, *Minḥat Yitzḥok* (4:53, 54) who asserts that any method that does not calculate *hatzot* correctly is flawed.

point between days of the week.⁶¹ How those two concepts relate depends on the well-known argument between Rabbeinu Tam and the *geonim*⁶² concerning the start and end of days of the week in general and of *Shabbat* in particular. This vast area will only be summarized,⁶³ with focus on its impact on the calculation of the opinion of the Magen Avraham.⁶⁴

⁶¹ In the last century, several *Brisker* conceptual Talmudists, with few highly debatable proof-texts, have hypothesized *halakhic* significance for both points. Until then, this was largely not the case. Those following the approach of Rabbeinu Tam assumed that the two points are identical; those following the *geonim* did not ascribe *halakhic* significance to the nighttime equivalent of *alot ha-shaḥar*. I am unaware of any comment by the *Gaon*, who introduced the expression *tzait kol ha-kokhavim*, giving it any *halakhic* function.

⁶² The position of the Rabbeinu Tam was dominant, adopted by later *rishonim* and undisputed in both the *Shulḥan Arukh* and by many of its earlier commentators. Both R. Elijah of Vilna and R. Shneur Zalman of Liadi, among others, fought to restore the opinion of the *geonim*. The *Gaon's* viewpoint appears consistently throughout his commentary on *Shulḥan Arukh*. R. Shneur Zalman abandoned his position in *Shulḥan Arukh Ha-rav* that supports Rabbeinu Tam; his *siddur* contains a letter that presents a view based on the *geonim* and comparable to that of the *Gaon*.

⁶³ In addition to defining the beginning and end of *Shabbat*, the dispute between the Rabbeinu Tam and the *geonim* as well as divergent views within each position directly impacts other *halakhot* including the latest time for *minḥah*, the earliest times for *maariv* and *kriat shema shel arvit*, the ideal time to light the *Ḥanukah menorah*, the day of a *brit* for a baby born shortly after sunset, the latest time for examination by a *niddah* for *hefsek taharah*, the end of a rabbinic fast day, etc.

⁶⁴ The relationship between the argument of Rabbeinu Tam and the *geonim* and the argument of the *Gaon* and the Magen Avraham is of *halakhic*, historical and conceptual

Following the language of the *Gaon*, we will refer to the evening equivalent of *alot ha-shaḥar* as *tzait kol ha-kokhavim*. Effectively, all stars are visible both at *alot ha-shaḥar* and *tzait kol ha-kokhavim* when the amount of detectable light from the sun is either minuscule or entirely absent. However, when omitting the word - *kol*, *tzait ha-kokhavim* refers to an earlier point when there is still some remaining illumination from the sun and only three (medium) stars can be seen.⁶⁵

interest. R. Willig in *Am Mordechai, Berakhot*, 2nd chapter, for example, considered the two disputes between the *Gaon* and both Rabbeinu Tam and the Magen Avraham as conceptually linked in both directions; he has since modified his position. Others, including R. Aaron Kotler in *Shut R. Aaron* vol. 2: 2, for example, assume the implication is valid in only one direction: those who follow the opinion of the Magen Avraham must also follow the opinion of Rabbeinu Tam. However, those who follow the *Gaon* with respect to *shaot zemaniot* can follow either the opinion of Rabbeinu Tam or the *Gaon* with respect to the transition point between days of the week. Despite conceptual arguments, the implication in either direction is contrary to established practice by either specific individuals and / or communities. The community in Jerusalem traditionally calculated *shaot zemaniot* like the Magen Avraham and ended *Shabbat* according to the opinion of the *geonim*; in fact, R. Kotler raised this problem with his own position. R. Mordechai Yaffe, R. Avraham Pimental and R. Joseph Soloveitchik (as a *ḥumrah*) ended *Shabbat* in accordance with the opinion of Rabbeinu Tam but calculated the hours of the day from sunrise to sunset. One can logically dispute either implication. The options to be developed in this section further undercut arguments for implications in either direction.

⁶⁵ Using the *Gaon's* formulation, introduces his bias towards variable, event based *zemanim* as opposed to fixed intervals. However, that bias is not material to the ongoing discussion.

For the *Gaon*:

- ❖ The earlier time of *tzait ha-kokhavim* defines the end of *Shabbat*.
- ❖ While *tzait kol ha-kokhavim* and *alot ha-shaḥar* are equidistant from *ḥatzot*, unlike *alot ha-shaḥar*, *tzait kol ha-kokhavim* has limited / no *halakhic* significance. The use of *tzait kol ha-kokhavim* for the calculation of *shaot zemanot* according to the Magen Avraham would be of no significance to the *Gaon*.

According to Rabbeinu Tam, the length of time between *alot ha-shaḥar* and sunrise and the length of time between sunset and the end of *Shabbat*, the endpoints of the *sugyot* in *Pesaḥim* 94a and *Shabbat* 34b are identical. Additionally, Rabbeinu Tam makes no reference to the notion of *tzait kol ha-kokhavim*. Rabbeinu Tam's position is both logically and observationally challenging. In a certain sense, the *Gaon's* fundamental challenge to Rabbeinu Tam is that he treats two distinct points (*tzait ha-kokhavim* and *tzait kol ha-kokhavim*) as identical.

In further support of the *Gaon's* position, in his sefer *Bein Ha-shemashot*, R. Yechial Michel Tukitzinsky argues strongly that in the Middle East the stars, horizon, and darkening sky described by the *gemara* around the end of *bein ha-shemashot*, the time of *tzait ha-kokhavim*, is completely inconsistent with their appearance much later in the evening, 72 minutes after sunset. In practice, *posekim* who followed Rabbeinu Tam had to choose between two different points, regardless if or how they dealt with their declared correspondence.

This created two primary options on how the opinion of Rabbeinu Tam was implemented in practice, which likely impacted the implementation of the opinion of the Magen Avraham as well. Those who stressed the equivalent length of the morning and evening intervals picked an equivalent point to *alot ha-shaḥar* in the evening, like Ramban in *Torat Ha-adam*, for both the end of *Shabbat* and to calculate *shaot zemaniot*. Almost all who today follow Rabbeinu Tam and wait 72 (or 90) minutes, either adjusted or (in practice, mostly) fixed, follow this approach.

However, many *posekim*⁶⁶ despite agreeing with Rabbeinu Tam conceptually⁶⁷ implemented an earlier end to *Shabbat* at approximately the

⁶⁶ These include R. Pimental in *Minḥat Kohen, ma'amar sheni* chapter 6, R. Yaakov Lorberbaum in *Derekh Ha-chaim* in the laws of *erev Shabbat*, R. M. Sofer in *teshuvah* 80 on a baby born after sunset on *Shabbat* and many other communities whose observance undoubtedly reflected the position of their rabbinic leaders that are listed in *Ha-zemanim Be-halakhah*, (Bnai Brak, 5756 / 1995), vol. 2, chapter 46 by R. Haim Benish. These *posekim* all ended *Shabbat* about 50 minutes after sunset, (in practice adopting a slightly shorter end to *Shabbat* than current practice given their more northern locations.) R. Feinstein in *Igrot Mosheh O. H.* 4:62 is part of this group as well; only for one who wants to be more stringent, does R. Feinstein suggest a fixed 72 minutes as the position of Rabbeinu Tam. Absent any stringencies, R. Feinstein considers 50 minutes after sunset as the end of *Shabbat* (even) according to the position of Rabbeinu Tam. How these *posekim* dealt with the conceptual challenge to their position from required equivalence in length to the interval from *alot ha-shaḥar* to sunrise is a complex topic in the study of *bein ha-shemashot*.

⁶⁷ Perhaps influenced more by the practical description of the end of *Shabbat* than by Rabbeinu Tam's conceptualization, these *posekim* implemented Rabbeinu Tam's end of

time they observed *tzait ha-kokhavim*. That approach also enables an alternative for calculating the opinion of the Magen Avraham, using an earlier point in the evening. While those followers of Rabbeinu Tam who would wait 72 / 90 minutes after sunset, either adjusted or not, would likely also use one of the symmetric alternatives, the group that only waited for the earlier appearance of stars, may have used that same point for calculating *shaot zemaniot*.

A story, whose minute details are in dispute, introduces this alternative in detail. The calendar of Jerusalem was heatedly debated over a century ago. The calendar implemented the opinion of the Magen Avraham using *alot ha-shaḥar* and the normal end time of a day of the week to calculate all the *halakhic* hours of the day. This calendar had strong precedents in several major European cities reflecting the *pesak* of prominent rabbinic authorities. R. Tukitzinsky approached his wife's grandfather, R. Shmuel Salant with the following problem: the calendar is miscalculating an

Shabbat in a manner that was similar in practice to that of the *Gaon*. Comparing the end of *Shabbat* for the *posekim* in the previous footnote, in every case:

- ❖ The appearance of the sky, regardless of the number of minutes after sunset, determines the end of *Shabbat*.
- ❖ Their rulings are in remarkable alignment with how the *Gaon* is implemented currently using depression angles, as can be verified comparing each *pesak* for the community in question with the times given on www.myzmanim.com. In fact, the time for the end of *Shabbat* according to the *geonim* provided on the website, is later than what R. Pimental, R. Lorberbaum and R. Sofer gave as the time for the end of *Shabbat* according to their application of Rabbeinu Tam.

observable point of *ḥatzot*⁶⁸ That fact was both observable and provable since the time between *alot ha-shaḥar* and sunrise (a depression angle of about 20 degrees) is significantly longer than the interval between sunset and the end of the day (a depression angle of only 8.5 degrees.) If, for example, sunrise and sunset were at 6:00 AM and 6:00PM respectively, and *alot ha-shaḥar* was at 4:30AM and three stars were visible at 6:30PM, then *ḥatzot* (clearly at 12 noon) would be calculated to occur at 11:30AM, 30 minutes too early. R. Yosef Chaim Sonnenfeld was neither able to resolve the problem or accept the change. Therefore, the calendar remained unchanged for another few years, until R. Yitzḥak Goldberg of Minsk visited Jerusalem and strongly supported the position of R. Tukitzinsky, finally ending the dispute.

R. Tukitzinsky then created an astronomically accurate calendar, supported in part by the *gemara* in *Pesaḥim* 94a and explicitly by Ramban's specification of the time of *plag ha-minḥah*. However, that methodology for calculating *shaot zemaniot* differed from what was done for generations as reflected in both the Jerusalem calendar and in other European calendars, as well. His methodology was straightforward; he simply used the nighttime equivalent of *alot ha-shaḥar*, *tzait kol ha-kokhavim* that occurs when the sun is again 20 degrees below the horizon. For example, if *alot ha-shaḥar* occurred on a specific day 95 minutes before sunrise, then *tzait kol ha-kokhavim* would occur 95 minutes after sunset. Using those endpoints, *ḥatzot* as calculated would coincide with the astronomically

⁶⁸ Those who did not seem concerned about a precise point of *ḥatzot* included R. Nosson Adler, quoted in the *minhagim* of the Zurich community, among many others.

observed point of 12 noon as opposed to being calculated to occur 30 minutes earlier.

This debate might seem odd to most of us at least conceptually; the position of R. Tukitzinsky is almost too obvious. As mentioned, all four current methods of calculation use symmetric endpoints; but this was not so over a century ago.

Assume for a moment, that we can address the need to calculate *ḥatzot* accurately. Consider the oft quoted verse in Nehemiah (4:15): *Ve-anaḥnu osim be-melakhah...mei-alot ha-shaḥar ad tzait ha-kokhavim*, specifying the daytime period, as interpreted explicitly by the *yerushalmi* at the beginning of *Berakhot*, accepted (even) by Rabbeinu Tam and his followers, and the *bavli* and other *rishonim* as well.^{69 70} It also may be the

⁶⁹ R. Kapach's position in multiple places in his commentary on Mishneh Torah is that Rambam, while ending *Shabbat* like the opinion of the *Gaon*, calculated *shaot zemanot* like the Magen Avraham. On the other hand, some claim Rambam held the latter position of the *Gaon* and calculated from sunrise not *alot ha-shaḥar*; see for example, *Mi-pininai Ha-rav* p. 31, where R. Schechter notes that R. Joseph Soloveitchik attributed this position to his grandfather, R. Chaim Soloveitchik. Following the approach of R. Kapach, one can argue that Rambam followed a practice akin to that being developed. While the opinion of Rambam is not addressed further in this monograph, it may represent the oldest source for the practice of using asymmetric endpoints, a topic to which I hope to cover in a future monograph.

⁷⁰ According to Rabbeinu Tam, calculating *shaot zemanot* from *alot ha-shaḥar* until the appearance of three stars is completely reasonable and should be expected. The issue is not using three stars, that indicate the end of *bein ha-shemashot* and *Shabbat*, as the *gemara* in *Shabbat* 34b - 35a explicitly states. The problem results from Rabbeinu

antecedent to the method of calculation used by the calendar of Jerusalem, as well as other cities. The beginning and end of the daytime period specified in the verse are interpreted as *alot ha-shaḥar* and *tzait ha-kokhavim* (three stars) respectively, the endpoints used by many calendars like the one used in Jerusalem, which R. Tukitzinsky attacked.

Without a doubt, the method of calculation that Ramban endorsed, and any other practice normally assumed, divides the daytime period between *alot ha-shaḥar* and its evening equivalent⁷¹ into twelve parts to derive the length of an hour.⁷² However, despite the theory articulated by Ramban, the practical process even for those that followed Rabbeinu Tam may have

Tam's equating that time for three stars, *tzait ha-kokhavim*, to the period of the time to walk 4 *milin* after sunset and to the period between *alot ha-shaḥar* and sunrise; three stars appear significantly earlier than 72 minutes after sunset. That problem, like issues raised by the *Gaon* and others, led many to forcibly reject the opinion of Rabbeinu Tam, particularly because of its *erev Shabbat* implications. However, even if this opinion of Rabbeinu Tam is rejected, the arguments advanced against Rabbeinu Tam have no bearing on the *Gaon's* disagreement with the Magen Avraham on the method for calculating *shaot zemaniot*.

⁷¹ Ramban's position to use the point of *tzait kol ha-kokhavim*, when "all the stars" as opposed to just three stars are visible is provable as demonstrated in footnote 36 from his specification of the time of *plag ha-minḥah*, as the time to walk 1/6th of a *mil* before sunset.

⁷² I cite Ramban because the *gemara* in Pesahim 94a itself is not conclusive. The *gemara* can be interpreted as not necessarily providing a normative opinion to be used in calculating *shaot zemaniot* but merely an astronomical observation; Ramban unquestionably calculates *shaot zemaniot* using the endpoints implicit in the *gemara*.

been implemented differently (particularly prior to the era of clocks.) I would conjecture that:

- ❖ *Hatzot* was not calculated; time was estimated by angles that were approximated from the high point in the sky (*ḥatzot*) to a point in the morning and the evening.⁷³
- ❖ The evening point was not *tzait kol ha-kokhavim*⁷⁴ but *tzait ha-kokhavim*, the appearance of three stars that was also assumed to

⁷³ See Ravyah, volume 2, page 64 who apparently calculated using such an approximation. Note that using *ḥatzot* as an anchor, has no impact when using symmetric endpoints. Either by using *ḥatzot* as suggested or by calculating as we normally do by dividing the length of time between symmetric endpoints provides the identical result. However, when using asymmetric endpoints, the two methods give differing results for both *ḥatzot* and afternoon *zemanim*, while the morning *zemanim* remains the same. This will become clear as we proceed.

⁷⁴ *Tzait kol ha-kokhavim* as the counterpoint to *alot ha-shaḥar* is a logical consequence of Rabbeinu Tam's position, as formulated explicitly by Ramban when specifying the time of *plag ha-minḥah*. As mentioned in footnote 55, *tzait kol ha-kokhavim* as the position of Rabbeinu Tam was formulated even more sharply in R. Soloveitchik's *yarzeit shiur, yom ve-lailah*. However, despite its compelling logic, I have found no reference in the *halakhic* literature to where *tzait kol ha-kokhavim* was ever used **in practice** prior to recent times. In fact, many *posekim*, including those cited in footnote 66, who claimed to be following Rabbeinu Tam, used the appearance of three (small, adjacent) stars to define the end of *Shabbat*, before 72 minutes after sunset and well before *tzait kol ha-kokhavim*. How such a position might comport with Rabbeinu Tam's conceptual position is problematic; this dissonance between concept and practice is one of the most difficult issues in the study of *zemanim*. While making this change from *tzait kol ha-kokhavim* to three stars, *tzait ha-kokhavim*, in the opinion of Rabbeinu Tam is

equate to *alot ha-shaḥar* by many who followed the view of Rabbeinu Tam.

To be concrete assume sunrise and sunset are at 6:00AM and 8:00PM. Assume further for computational simplicity that *alot ha-shaḥar* is observed at 4:40AM and three stars are observed at 8:40PM, with the evening equivalent of *alot ha-shaḥar*, *tzait kol ha-kokhavim*, occurring at 9:20PM. The problem encountered was deriving *ḥatzot*, which can be observed at exactly 1:00PM. Taking the midpoint between 4:40AM and 9:20PM one gets 1:00PM, as expected. However, some of the calendars to which we alluded would calculate *ḥatzot* at 12:40PM, (the midpoint between 4:40AM and 8:40PM) 20 minutes too early, the issue R. Tukitzinsky raised. Assume now that we have no widely available clocks and associated measurements of time and we need to observe the hours of the day. It is hardly likely that we would calculate *ḥatzot*; we would simply assume it occurs when the sun is directly overhead. The morning hours would be calculated / approximated between *alot ha-shaḥar* and *ḥatzot*, the afternoon hours between *ḥatzot* and *tzait ha-kokhavim*, the appearance of three stars. Of course, those mathematically oriented will immediately note

challenging conceptually because Rabbeinu Tam equates the endpoint of the day in *Shabbat* 34b and *Pesaḥim* 94a, it need not create challenges when calculating the hours of the day. It should be noted that the more common use of a fixed 72 minutes for both the calculation of *shaot zemaniot* per the Magen Avraham and Rabbeinu Tam's end to *Shabbat* maintains consistency with Rabbeinu Tam's conceptual position, as well. It may well explain its popularity even though it is strongly challenged by observation both in the morning (when the interval is too short) and in the evening (when the interval is too long.)

that afternoon hours are (slightly) shorter than morning hours. As surprising as that may be, it is not contradicted by any principle I have ever found in *zemanim*⁷⁵ and is certainly less troublesome than miscalculating *ḥatzot*. If one follows the verse in Nehemiah precisely using *alot ha-shaḥar* and *tzait ha-kokhavim*, which are not equidistant from *ḥatzot*, while also insisting on an accurate point of *ḥatzot*, then one is forced to calculate the morning and afternoon hours separately.

My suspicion is that this approximation of *shaot ha-yom* may have been used in practice before the widespread use of clocks. As far as I know, it is the only methodology that uses non-equidistant endpoints for the calculation of *shaot zemaniot* that does not generate an inaccurate time of *ḥatzot*.⁷⁶

⁷⁵ R. Feinstein in *Igrot Mosheh O.H.* 1:24 and *Igrot Mosheh O.H.* 2:20, for dissimilar reasons and in conjunction with his view that *ḥatzot* is always at the same time at a given location, writes “*she-shenei ḥatzaei ha-yom einam shavim.*” Even though his rationale is entirely different, his view is nonetheless supportive, to a degree.

⁷⁶ One cannot rule out another alternative that also computes *ḥatzot* accurately – the use of a point of *alot ha-shaḥar* symmetric to *tzait ha-kokhavim*, as opposed to an earlier point symmetric to *tzait kol ha-kokhavim*. Such a point would preserve the equivalence between the length of the interval of time between sunset and *tzait ha-kokhavim* and between *alot ha-shaḥar* and sunrise. That equivalence is mandated by the conceptual position of Rabbeinu Tam; however, I do not know if two such points were ever used to calculate *shaot ha-yom*. In any case, since

- ❖ such a definition of *alot ha-shaḥar* is not supported by any accepted interpretation of *Pesaḥim* 94a, and

With the invention of clocks, this method centered around observation from *ḥatzot*, may have ultimately changed into a calculation from *alot ha-shaḥar* to the appearance of three stars, creating uniform hours throughout the daytime period but a slightly miscalculated time of *ḥatzot*. Were this argument adopted, *posekim* could re-establish its use by taking *ḥatzot* as observed and then separately calculating the morning and afternoon *zemanim* by calculating from *ḥatzot* to *alot ha-shaḥar* and from *ḥatzot* to the appearance of three stars, respectively.

While many will prefer the calendar of R. Tukitzinsky⁷⁷ that uses an equivalent point to *alot ha-shaḥar* in the evening, I strongly suspect that a calendar that uses the appearance of three stars, *tzait ha-kokhavim*, versus *tzait kol ha-kokhavim* may be more consistent with tradition from at least the 15th to 19th century, and perhaps, as we have hypothesized, even in more ancient times.⁷⁸ The *Gaon* differentiates the *gemara* in *Pesaḥim* from

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- ❖ any calculation using that point as *alot ha-shaḥar* would give inaccurate morning *zemanim*,

it will not be considered further.

⁷⁷ It is a more mathematically appealing calculation, supported by and consistent with the conceptual approach and calculations that follow from Ramban in *Torat Ha-adam* and accepted, at least conceptually, by many *ḥakhmai seforad* who followed Ramban's approach.

⁷⁸ For completeness, with respect to the latest morning times for *kriat shema* and *tefillah*, an approach that creates an early time for *ḥatzot* also creates the earliest morning *zemanim* for *kriat shema* and *tefillah*, clarifying why R. Sonnenfeld did not want to abandon an established practice. Except during certain seasons of the year in areas closer to the equator, an approach using depression angles produces earlier *zemanim*

the *gemara* in *Shabbat* by asserting that the time of *tzait kol ha-kokhavim*, the evening equivalent of *alot ha-shaḥar*, occurs much later than *ḥashekhah / tzait ha-kokhavim*. I would suggest that the discussion in both the *bavli* and *yerushalmi* in *Berakhot* that relates to the end of the day and *plag ha-minḥah* should be read similarly, referring to a point in time occurring well before *tzait kol ha-kokhavim*. Only the *gemara* in *Pesaḥim* is referring to *tzait kol ha-kokhavim*.

Interestingly, this approach completely removes any interdependency between the four alternatives for setting the time of *alot ha-shaḥar* and the various alternatives for *tzait ha-kokhavim*. Thus, this isolates the derivation of morning *zemanim* from those in the afternoon. When calculating the approach of the Magen Avraham from a known point of *ḥatzot*, morning *zemanim* depend only on *alot ha-shaḥar* while afternoon *zemanim* depend only on *tzait ha-kokhavim*. This isolation of impact has fundamental appeal.

When using asymmetric endpoints, there is still the need to choose between two primary options for *tzait ha-kokhavim* either

for *kriat shema* and *tefillah* than a similar approach that uses a fixed 72 or 90 minutes. Any approach which does not calculate but only observes *ḥatzot* has no impact on morning *zemanim* producing the same *zemanim* in the morning as the more standard approach that uses an equidistant evening endpoint; I leave that exercise to the reader. All approaches differ with respect to the calculation of *plag ha-minḥah*, a topic that is rarely covered. As we will demonstrate, several the approaches happen to create a practical time for *plag ha-minḥah* in the New York area for those wishing to *daven* early in the summer, but with *minḥah* and *maariv* before and after *plag ha-minḥah*, respectively.

- ❖ the earlier time at which three medium stars appear, or
- ❖ the later time when three small stars appear.

The latter is used as a stringency for determining the end of *Shabbat* while the former is used for some other evening obligations. The appearance of three medium stars is preferable for setting *shaot zemaniot*, absent the need for stringency.⁷⁹ Despite a preference for the use of three medium stars, both options are maintained going forward.

While we utilize the colloquial language of three medium and small stars, many recognize that the appearance of stars is a result of increased levels of darkness; stars are always present, but are unseen depending on the degree of illumination.⁸⁰ Thus, the appearance of three medium stars indicates that a certain level of darkness has been achieved. The appearance of three small stars occurs at a yet later time when the setting sun provides even less illumination. Currently, using depression angles to

⁷⁹ Of course, where stringency is desired for any reason, using the later *zeman* may be preferred.

⁸⁰ There is a dispute going back to the times of the *geonim* whether stars or darkness defines the end of the day of the week, with the other just being an indication. Even in the times of the *geonim* it was understood that stars are always present; our ability to observe them is a consequence of attaining a specific level of darkness. Thus, darkness was considered as defining. On the other hand, the Vilna *Gaon*, for example, considered stars as the actual definition and darkness only serves as an indicator to be used when stars are not visible or because of the difficulty in determining what is a medium versus small star. Since the two points in time are relatively close to each other, the practical impact of this dispute is minimal.

formalize the degree of darkness, the appearance of three medium stars is typically equated to a depression angle between 5.5 and 6.5 degrees; the appearance of three small stars typically equates to a depression angle between 7.5 and 8.5 degrees. This complex and critical area of *halakhah* is not pursued further; in the table and examples we use 6 degrees and 8.5 degrees for the appearance of three medium stars and three small stars, respectively. While both points are still disputed, there is significantly less consensus on the former. Use of 8.5 degrees, particularly as a stringency for the appearance of three small stars, has gained almost universal acceptance.

Thus, there are four approaches to implementing these asymmetric alternatives:

- ❖ Two fundamental points for *tzait ha-kokhavim* approximately at the time of the appearance of three medium or small stars.
- ❖ Two methods for calculating, either from *ḥatzot* or from one of the alternatives for *alot ha-shaḥar*.

One method, calculating from *ḥatzot* until the appearance of three medium stars, is strongly preferred. Calculating from *ḥatzot* until the appearance of three small stars provides a later point for all afternoon *zmanim* and may be preferred as a stringency. Approaches calculating from *alot ha-shaḥar* until either three medium or small stars miscalculate *ḥatzot* and should be avoided. Computationally, there are multiple views on how to specify the time of the appearance of either three medium or small stars. Coupled with the four fundamental alternatives used

consistently in this monograph for the time of *alot ha-shaḥar* results in multiple prospective calculations.

Tentative support for using an asymmetric evening point, *tzait ha-kokhavim* versus *tzait kol ha-kokhavim*, can be derived from several sources. However, those sources are not precise enough to imply a specific computational alternative. On a day in the Middle East near the spring or fall equinox, the difference between those modes of calculation for the length of a *halakhic* hour is only about 6 minutes.⁸¹ Additionally, the practiced times for three medium versus small stars, is itself variable enough to make any verification of which of the alternatives was employed highly unlikely. In every case, using an earlier time for *tzait ha-kokhavim*, results in an earlier time for *plag ha-minḥah*.

Using that earlier time for *tzait ha-kokhavim* to calculate afternoon *zemanim*, we can (at least partially) attend to issues in several areas:

- ❖ R. Pimental's challenges to the approach of the *Magen Avraham*.
- ❖ R. Isserlein's *pesak* for an early reading of the Megillah.

⁸¹ On a canonical 12-hour day, the largest difference is using the shortest interval for the time of *tzait ha-kokhavim*, approximately 18 minutes after sunset, and the longest interval for *alot ha-shaḥar*, 90 minutes before sunrise. Using *ḥatzot* as an anchor, the hours in the afternoon are 63 minutes. Using a 90-minute interval for *alot ha-shaḥar* and an early *tzait ha-kokhavim*, derive a 69-minute hour. A later time for *tzait ha-kokhavim* would reduce the difference. Of course, in locations further from the equator and during the summer, when using adjusted versus fixed times, that difference will grow.

- ❖ R. Feinstein's *pesakim* concerning *ḥatzot* and unequal *shaot zemaniot* in the morning and afternoon.
- ❖ Two *minhagim* of Jerusalem associated with *zemanim*.

R. Pimential's challenges to the approach of the *Magen Avraham*: In *Minḥat Kohen, ma'amar sheni*, chapters 7 and 8, R. Pimential summarizes three challenges to the opinion of R. Isserlein.⁸² For each, R. Pimential suggests a somewhat forced answer, and concludes by adopting the position of the author of the *Levushim*, R. Yaffe.⁸³ To enhance clarity, the discussion below omits much of the detail; however, I believe, reading the texts in their entirety reinforces the position taken.

First, R. Pimential attempts to show that specific *sugyot* including a primary *sugyah* for *shaot ha-yom*⁸⁴ regarding the latest time for *minḥah*, either *ad ha-erev* or until *plag ha-minḥah*, are more reasonably understood according to the position of the *Levushim*. The time of *erev* seems inconsistent with the time of *tzait kol ha-kokhavim*, when there is no longer any visible light from the sun, and more consistent with times when the light of the sun while subsiding is still observable. If the time of *erev* is close to the earlier time of *tzait ha-kokhavim*, when three medium stars appear and

⁸² Living during the lifetime of the *Magen Avraham*, that is how R. Pimential and other early authorities referred to what we refer to as the opinion of the *Magen Avraham*.

⁸³ Living approximately two generations after R. Yaffe, this is how R. Pimential referred to what we call the position of the *Gaon*.

⁸⁴ The *sugyot* are found in beginning of *Berakhot* in both the *bavli* and *yerushalmi*.

residual light from the setting sun is still apparent,⁸⁵ the issues that R. Pimential raises are fully addressed. While the language of the *sugyot* might seem inconsistent with times around *tzait kol ha-kokhavim*, they are entirely compatible with the earlier point around *tzait ha-kokhavim*, and perhaps even more so than a point yet closer to sunset.

Both of R. Pimential's next two challenges are based on the fact that even assuming 72 versus 90 minutes, as he tacitly does, *plag ha-minḥah* and sunset still occur exceeding close to each other.⁸⁶ His second challenge relates to the time to light candles on *erev Shabbat*, even according to the opinion of Rabbeinu Tam. In R. Pimential's view, unlike that of Ramban, the time to light candles on *erev Shabbat* is **ideally** before sunset, particularly on a cloudy day, but also after *plag ha-minḥah*. He therefore questions the reasonableness of a requirement to light candles within so short an interval.

His third and most challenging problem recounts the story in the *gemara* about a dark, overcast day when a *tzibbur* mistakenly prayed the *maariv* service thinking it was after *plag ha-minḥah* only for the sun to then

⁸⁵ At that time, sun rays are still visible on the western horizon.

⁸⁶ Both of R. Pimential's arguments are not entirely precise particularly given that he adopts 72 versus 90 minutes as *tzait ha-kokhavim*. Assuming not 72 but 90 minutes, *plag ha-minḥah* is, as previously discussed, within 3 "adjusted" minutes of sunset. However, even at 72 minutes, when *plag ha-minḥah* is 18 minutes before sunset, his observations remain valid, albeit less so. In any case, R. Pimential's question is much stronger according those who maintain a 90-minute interval.

reappear. How all that activity took place in so short an interval is somewhat implausible.⁸⁷

Numerous answers have been proposed by R. Pimental and others to these questions. However, calculating *plag ha-minḥah* from an earlier point, *tzait ha-kokhavim* versus *tzait kol ha-kokhavim*, completely eliminates the assumptions underlying these questions.⁸⁸ While the challenges raised by R. Pimental to the position of R. Isserlein probably assumed symmetric endpoints, asymmetric endpoints provide a straightforward resolution.⁸⁹

⁸⁷ Another highly plausible answer is that the sun reemerging does not mean that the sun itself reappeared but that significant illumination from the sun was present as occurs only directly after sunset proper when it is still too early for the *maariv* service.

⁸⁸ Note that regardless of the precise implementation of an asymmetric option, throughout the year in the Middle East there will be more than 30 minutes between *plag ha-minḥah* and sunset. Why R. Pimental did not suggest this alternative is uncertain. However, while he did not define such an earlier point conceptually, practically speaking, he resolved the issue analogously. Citing the practice that he supported, the appearance of three small stars, regardless of how early they appear, is to be used rather than (Rabbeinu Tam's) 72 minutes. Using this earlier endpoint for the end of *Shabbat* for the calculation of *shaot zemanot* as well, lengthened the interval between *plag ha-minḥah* and sunset by at least an additional 20 minutes.

⁸⁹ Again, Ramban and other *ḥakhmai seforad* endorsed symmetric endpoints and must avail themselves of answers like those provided by R. Pimental.

R. Isserlein's pesak for an early reading of the Megillah: *A teshuvah* from R. Israel Isserlein⁹⁰ as quantified by his student R. Yosef ben Mosheh in *Leket Yosher*⁹¹ is rather challenging. R. Isserlein allows one who is slightly ill on *Taanit Esther* to read the Megillah as early as *plag ha-minḥah* and then end their fast. R. Isserlein specifies the time only as *plag ha-minḥah*. However, in *Leket Yosher* the case was further described as having occurred in Neustadt during a leap year, **with *plag ha-minḥah* occurring slightly before 5:00PM**. Without such a specification, **which is assumed going forward**, it would be impossible to draw any further conclusions about the method by which R. Isserlein calculated.

How “slightly before 5:00PM” is to be further quantified is not clear. Nonetheless, it should be assumed that R. Isserlein estimated carefully as is illustrated in the first *teshuvah* in *Trumat Ha-deshen* where he approximated to 1/8th of an hour.

A full assessment of this topic must include a brief study of medieval clocks. If like current clocks where a specific time is of no astronomical consequence, little can be inferred from knowing that *plag ha-minḥah* occurred a few minutes before 5:00PM. While details about the clock in use are uncertain, one can safely assume that, unlike modern clocks where 6:00PM has no astronomical significance, 6:00PM may have been either

⁹⁰ *Trumat Ha-deshen* 109.

⁹¹ *Leket Yosher*, p. 157-158.

close to sunset,⁹² (similar to Arab clocks in use in the Middle East until the 20th century,) or more likely, 6:00PM was set to dusk, approximately 30 minutes after sunset, like some early mechanical / astronomical clocks from the late 14th century that existed for many centuries.⁹³ We will examine three *halakhic* alternatives assuming an astronomically aligned clock where 6:00pm was close to **either sunset or dusk**.

First, the normal assumption that R. Isserlein was calculating from a time for *tzait ha-kokhavim*, 72 minutes or more after sunset cannot be sustained. The earliest time for *plag ha-minḥah* results from 6:00PM equating to dusk, 30 or even 40 minutes after sunset. At the earliest, *tzait ha-kokhavim* is at 6:32PM, 72 minutes after sunset and 32 minutes after dusk. Given that Purim occurred in mid-March when the daytime period is at most 12 hours, a *halakhic* hour is approximately 72 minutes and *plag ha-minḥah*, 90 minutes before 6:32PM, is slightly **after** 5:00PM. If *tzait ha-kokhavim* was assumed to occur more than 72 minutes after sunset, or if 6:00PM and dusk were only 30 minutes after sunset, *plag ha-minḥah* occurs yet later.

⁹² Since the daytime period is approximately 12 hours on days close to the equinox, on those European clocks of that era where 12:00PM was set to midday, 6:00PM would be close to sunset. Given that R. Yosef ben Mosheh writes that the question addressed by the *teshuvah* took place in a leap year, we can assume that Purim occurred on a day close to the equinox. At other times of the year when the day is not close to 12 hours long, unlike Arab clocks, 6:00PM would not be close to sunset.

⁹³ See http://www.math.harvard.edu/~shlomo/docs/bein_ha-shemashot.pdf, where Prof. Sternberg describes this type of Italian clock. He asserts that this type of clock was likely what was intended by R. Yoel Sirkes in *Baḥ, Tur O. H.* 261.

Thus, assuming an astronomically aligned clock, this normally assumed alternative can be eliminated.

Second, given the failure to align R. Isserlein with his commonly assumed position of calculating *shaot ha-yom* from *alot ha-shaḥar* until *tzait ha-kokhavim*, some have argued that in this case or perhaps even more generally, R. Isserlein used or relied on the position later to be known as that of the *Gaon*.⁹⁴ In addition to being at odds with how R. Isserlein was understood by R. Pimental and others (probably including the *Shulḥan Arukh*), the use of the term *lailah* as the point from which 1.25 hours are subtracted to calculate *plag ha-minḥah* is difficult to associate with sunset proper. This conjecture is also implausible given the first *teshuvah* in *Trumat Ha-deshen*, where R. Isserlein struggles with and is unable to justify an early start to *Shabbat* three to four hours before *tzait ha-kokhavim*. If R. Isserlein was aware of calculating from sunset, on a long summer day with 16 hours of sunlight, 1.25 hours before sunset is 100 minutes before sunset⁹⁵ and approximately three hours before *tzait ha-kokhavim*. I suspect that R. Isserlein would likely have mentioned this justification of starting *Shabbat* close to three hours before *tzait ha-*

⁹⁴ See *Ha-zemanim Be-halakhah*, chapter 13, footnote 1, where R. Benish references this conjecture.

⁹⁵ Note that in the area addressed near Wiener – Neustadt, a summer day, from sunrise to sunset, is about 16 hours long. A *shaah zemanit* is $((16 * 60) / 12 =)$ 80 minutes. *Plag ha-minḥah* is $(1.25 * 80 =)$ 100 minutes before sunset.

kokhavim were he aware of it.⁹⁶ Furthermore, if R. Isserlein was in fact using the *Gaon's* calculation, if 6:00PM is around sunset, *plag ha-minḥah* is approximately 15 minutes, not a few minutes, before 5:00PM. Had the day been 24 minutes shorter, *plag ha-minḥah* would be approximately 10 minutes before 5:00PM, certainly a possibility, were it not for the fact that the *teshuvah* took place in a leap year, when Purim occurs closer to the equinox and the daytime period is approximately 12 hours. If 6:00PM on the clock was approximately dusk, something that is more likely, *plag ha-minḥah*, when calculated from sunset, is over 30 minutes before 5:00PM.⁹⁷

⁹⁶ In *Sefer Levushim O. H.* 267, R. Yaffe questions why R. Isserlein did not offer this explanation.

⁹⁷ *Teshuvah* 121 in *Trumat Ha-deshen* concerning the latest time to eat *ḥametz* on an *erev Pesah* that occurred towards the end of April is relevant as well. Again, some try to demonstrate that R. Isserlein was using the *Gaon's* calculation from sunrise that was approximately at 5:00AM that day. In my monograph on *Bein Ha-Shemashot*, I argue that that approach is unsupportable as it requires the entirely implausible assumption of not using *shaot zemaniot*. In fact, I show that the *teshuvah* strongly supports the view that R. Isserlein was calculating from *alot ha-shaḥar*. It should be noted, that some have argued that *teshuvah* 123, the assumed basis for the “18-minute *mil*,” in fact supports 22.5 minutes as the time required to walk a *mil*. The expression used in the *teshuvah*, $\frac{1}{4}$ of an hour + $\frac{1}{20}$ th of an hour, equals 18 minutes if one assumes a 60-minute clock hour. However, if the hour in question is a *halakhic* hour, $\frac{1}{12}$ th of a day counting from *alot ha-shaḥar* until *tzait ha-kokhavim*, or $(720 + 90 + 90 =) 75$ minutes, then $\frac{1}{4}$ of an hour + $\frac{1}{20}$ th of an hour is $(18.75 + 3.75 =) 22.5$ minutes. While *Leket Yosher* states that R. Isserlein assumed that 40 *milin* were walked from *alot ha-shaḥar* until *tzait ha-kokhavim*, the *teshuvah* states that the time in which 40 *milin* were walked was 12 hours. Both cannot be correct. Assuming both were correct, leads to the

This leads to the third, and most plausible, alternative: R. Isserlein was calculating from an earlier time for *tzait ha-kokhavim*, well before 72 minutes after sunset.⁹⁸ Knowing precisely the type of clock in use (and the exact length of the daytime period) would be very helpful. However, assuming as we have that

1. 6:00PM on the clock in question was set to dusk, 30 (not 40) minutes after sunset,
2. R. Isserlein was calculating from a point 15 - 20 minutes after 6:00PM, 45 - 50 minutes after sunset,
3. and that the length of the daytime period was 12 hours,

then the length of the afternoon from *hatzot* until, for example, 48 minutes after sunset is 408 minutes. A *shaah zemanit* is 68 minutes and *plag ha-minḥah* is 85 minutes earlier **at 7 minutes before 5:00PM**.⁹⁹ Assuming the

refutation of R. Isserlein's position raised by Gaon's in *O. H.* 459. As the Gaon notes, around the equinox there are 12 hours from sunrise to sunset, **not** from *alot ha-shaḥar* until *tzait ha-kokhavim*. The Gaon then understands R. Isserlein differently, if the 40 *milin* were walked from sunrise until sunset, the standard interpretation described earlier. That position differs from the view expressed in *Leket Yosher*, a view that when combined with a time of 12 hours in which 40 *milin* were walked produces a contradiction with observation and differs with R. Isserlein's normally assumed position. If we dismiss such an obvious conflict with observation, unlike our text of *Trumat Ha-deshen*, *Leket Yosher* must assume that only 32 and not 40 *milin* are walked in the 12-hour period from sunrise to sunset, thus supporting 22.5 minutes as the time required to walk a *mil*.

⁹⁸ The reader should note that in Neustadt at approximately 48-degree latitude, *tzait ha-kokhavim* is approximately 20 minutes later in the summer than in the spring.

⁹⁹ At 54 minutes after sunset, again calculating from *hatzot*, a *shaah zemanit* is $((360 + 54) / 6 =)$ 69 minutes and *plag ha-minḥah* is 86 minutes and 15 seconds earlier, slightly

length of the daytime period is almost 12 hours, there are multiple options based on *tzait ha-kokhavim* between 42 and 48 minutes after sunset that are all reasonable.¹⁰⁰ All of those options are more likely than assuming a clock where 6:00PM is around sunset. Under that scenario, if for example R. Isserlein was calculating from 24 minutes after sunset, hardly a reasonable time for the appearance of even three medium stars, even on a day around the equinox, *plag ha-minḥah* would still be 4 minutes after 5:00PM. To assume a yet earlier time for *tzait ha-kokhavim*, given his northern latitude, is entirely implausible.¹⁰¹

more than 2 minutes before 5:00PM. If one were calculating from the observed time for *alot ha-shaḥar* approximately 96 minutes before sunrise, each of the times would be approximately 4 minutes earlier.

¹⁰⁰ Note that the time is reflective of the appearance of three small versus three medium stars. If the length of the day in question were shorter, then three medium stars at 36 to 42 minutes after sunset is a possibility.

¹⁰¹ To get to even 5:00PM, the time used for *tzait ha-kokhavim* would have to be about 18 minutes after sunset, which is highly implausible unless R. Isserlein was using the time to walk $\frac{3}{4}$ of a *mil*. The table at the end of the paper shows three medium stars at approximately 36 minutes after sunset. I suggest paper and pencil to verify these calculations. However, if R. Isserlein was calculating not from *ḥatzot* but from *alot ha-shaḥar* then this option is slightly more plausible since that would add approximately another 8 minutes. However, given the northern latitude, it is unlikely that a time for *tzait ha-kokhavim*, occurring before 30 minutes after sunset would be used. Using 30 minutes after sunset and a time for *alot ha-shaḥar* 96 minutes before sunrise, places *plag ha-minḥah* $1.25 * (720 + 126) / 12$ approximately 88 minutes earlier at 2 minutes after 5:00PM. These calculations and potential scenarios are speculative but are

Thus, calculating from an earlier time for *tzait ha-kokhavim* is most likely, assuming 6:00PM was set to dusk, 30 minutes after sunset, as was the case for some clocks of that era.

R. Feinstein's *pesakim* concerning *hatzot* and unequal *shaot zemaniot*

in the morning and afternoon: R. Mosheh Feinstein's position (*Igrot Mosheh O. H.* 2:20) is that at each location, *hatzot* is always at the same time and is not calculated daily. He asserts this practice without a clear explanation and attributes it to his father. Astronomically, (in the New York area) *hatzot* varies by approximately 20 minutes during the year. As one travels further from the equator the variation increases; closer to the equator the variation decreases. Perhaps R. Feinstein was following a tradition that preferred the use of a simplifying constant approximation. However, I suspect that his position may be partially related to this conjecture. His assertion that *hatzot* is not calculated, unlike his assumption that it is always the same time, may derive from *hatzot* only being observed and hence not calculated, as has been proposed.¹⁰² As well his support for *shaot zemaniot* varying between morning and

certainly more likely than more standard interpretations that R. Isserlein is calculating from a full 72 minutes (or more) after sunset.

¹⁰² It is less plausible that the problematic position of R. Shlomo Zalman Auerbach (*Minhat Shlomo* 1:91 and 2:58) on the time for *hatzot ha-lailah*, which deviates from the precise astronomical point, also derives from this type of approach. He seems to argue strongly for sunset and *alot ha-shahar* as the endpoints to be used in calculating *halakhic* midnight, before which time one must eat the *afikomen*. It is ironic that both recognized *posekim* would both maintain unexplained, ostensibly untenable positions on *hatzot*.

afternoon, although differently conceptualized, is also intriguing and somewhat supportive.¹⁰³

Two *minhagim* of Jerusalem associated with *zemanim*: Two customs of Jerusalem provide powerful support for calculating using asymmetric endpoints.¹⁰⁴

First, as indicated, despite the presence and customs of the students the *Gaon*, Jerusalem followed the opinion of the Magen Avraham in calculating *shaot zemaniot* but always ended *Shabbat* in accordance with the opinion of the *geonim*.^{105 106}

¹⁰³ While Rabbi Feinstein asserts either morning or afternoon hours could be longer, consistent with a fixed time of *hatzot*, the argument advanced supports **only** longer morning hours.

¹⁰⁴ Using three stars as the evening *zeman* for ending *Shabbat* (and likely, calculating *shaot zemaniot*,) was also customary in many communities in Europe, further south and west of Poland as recorded in *Ha-zemanim Be-halakhah*, vol. 2 chapter 46. The history of this asymmetric option is unclear, beyond the explanation proposed. However, it was clearly practiced even according to the view of Rabbeinu Tam, in conjunction with an earlier end to *Shabbat* using the appearance of three stars, regardless of whether that event occurs before 72 minutes after sunset.

¹⁰⁵ From many 19th century *tshuvot* concerning *brit milah*, it is likely that the precise beginning of *Shabbat* in Jerusalem (absent extensions due to *tosefet Shabbat* and practical needs to be *maḥmir*) was approximately 5 - 10 minutes after sunset. R. Kapach suggests that Rambam position was 15 minutes after sunset. *Shabbat* would conclude approximately 30-40 minutes after sunset, depending on season.

It is entirely likely that the *minhag* R. Sonnenfeld was defending was closer to the ancient custom than the calculation proposed by R. Tukitzinsky and adopted in the official calendar. While entirely logical and consistent with the opinion of Ramban and other *ḥakhmai seforad*, there is no evidence that anyone ever calculated *shaot zemaniot* using *tzait kol ha-kokhavim*, particularly when ending *Shabbat* so much earlier at *tzait ha-kokhavim*. As well, since many in Jerusalem followed a practice that assumed *alot ha-shaḥar* preceded sunrise by (an adjusted) 90 minutes, were that equivalent interval used in the evening as well, **then there should have been some mention of *plag ha-minḥah* occurring more than 10 minutes after sunset on occasion in the winter,¹⁰⁷ an issue that was, to the best of my knowledge, never raised.** More than likely, the position of R. Sonnenfeld reflected traditional practice in Jerusalem. Assuming the need to calculate *ḥatzot* accurately, the approach outlined is the only one that I know of that can simultaneously address both the origin of the practice and the objections raised.¹⁰⁸

¹⁰⁶ Those who now wait 72 minutes in Israel are following European custom that was imported back into Israel and reintroduced. Note that (even) R. Y. Karelitz waited only 40 - 45 minutes after sunset to end *Shabbat* in Israel.

¹⁰⁷ See the table at the end of the monograph. Note that *plag ha-minḥah* occurs 13 minutes after sunset when adjusting 90 minutes to 96 minutes in the winter based on depression angles. Even if a fixed 90 minutes to be used, *plag ha-minḥah* would still occur approximately seven minutes after sunset.

¹⁰⁸ The practice of Jerusalem and other areas in the Middle East, where the impact of latitude need not be considered, both with respect the end of *Shabbat* and *shaot zemaniot*, is of significant value in understanding many aspects of *zemanim* without

Second, the custom of Jerusalem to light candles 40 minutes before sunset may also be a consequence of this prior mode of calculation.¹⁰⁹ Assume that one would want to calculate *plag ha-minḥah* for Jerusalem using the approach outlined. For simplicity assume that one chose some point between 21 - 30 minutes¹¹⁰ after sunset, depending on the time of the year, for *tzait ha-kokhavim* and then calculated *plag ha-minḥah*. On the average day, afternoon hours would be approximately 64 minutes, (6 hours and 24 minutes) / 6. *Plag ha-minḥah* would occur about 80 minutes earlier, 56 minutes before sunset. In the summer *plag ha-minḥah* is yet earlier¹¹¹ even counting back from a time for *tzait ha-kokhavim* 30 minutes after sunset. However, on the shortest days of the year, *shaot zemaniot* during the afternoon are approximately (5 hours and 27 minutes) / 6 or 54.5 minutes with *plag ha-minḥah* occurring approximately (1.25 * 54.5 =) 68 minutes earlier, about 41 minutes before sunset. Thus, lighting candles 40 minutes

having to deal with the complexities that latitude introduces; those complexities were often not dealt with accurately and very likely misunderstood.

¹⁰⁹ A paper I wrote on this topic with significantly more detail can be found on www.seforimblog.com on 1/3/2019.

¹¹⁰ Not the point at which *Shabbat* ends because of the various stringencies extending its length, but an earlier time used for various rabbinic laws and evening prayers. See, among many such examples, the current calculation in the Jerusalem calendar attributed to the *Ben Ish Hai*, or R. Tukitzinsky's time for ending a rabbinic fast. Note that 6 degrees gives a smaller range, 24 – 27 minutes.

¹¹¹ On a 14-hour day, an afternoon hour is ((7 hours + 30 minutes) / 6 =) 75 minutes and *plag ha-minḥah* is approximately 94 minutes earlier, more than an hour before sunset.

before sunset, guarantees that candles are always lit after *plag ha-minḥah*. The latest time for *plag ha-minḥah* occurs is in the winter; however, even then *plag ha-minḥah* is 41 minutes before sunset. Coincidence, perhaps; but this might explain the origin of the custom of Jerusalem.¹¹²

For those who follow the *Gaon's* time for *plag ha-minḥah*, an early time to light candles *erev Shabbat* is entirely understandable. However, calculating using the opinion of the Magen Avraham is a prevalent custom in Jerusalem. Absent this suggestion, there does not appear to be any justification for candle lighting 40 minutes before sunset for the many in Jerusalem who follow the approach of the Magen Avraham while insisting that candle lighting must also occur after *plag ha-minḥah*.¹¹³

Summary of options:

Summarizing the options covered, disregarding secondary opinions and disputes over the precise time of *tzait ha-kokhavim*, 20 options remain. For illustration throughout this summary, assume a 12-hour daytime period at

¹¹² Other explanations of 40 minutes as $\frac{1}{2}$ of a *shaah zemanit* appear neither accurate nor relevant. Note that if we were calculating using asymmetric endpoints as opposed to *ḥatzot* and were using an observed time for *alot ha-shaḥar* at 96 minutes before sunrise then *plag ha-minḥah* would occur approximately 6 minutes earlier; this is similarly consistent with a 40-minute custom.

¹¹³ *A fortiori*, in other cities further from the equator as indicated by one of the questions of R. Pimental this is an issue at various times for those following the Magen Avraham, absent this type of suggestion.

the latitude of Washington D.C.,¹¹⁴ with sunrise at 6:00AM and sunset at 6:00PM. There are three types of endpoints might be used in calculating the opinion of the Magen Avraham:

I - Assuming equidistant endpoints, we have the **four** options:

1. From 100 minutes, 90 minutes adjusted, before sunrise at 4:20AM to 100 minutes after sunset at 7:40PM: *Sof zeman kariat shema* is at 8:10AM and *plag ha-minḥah* is at 6:04PM.
2. From 90 minutes unadjusted before sunrise at 4:30AM to 90 minutes after sunset at 7:30PM: *Sof zeman kariat shema* is at 8:15am and *plag ha-minḥah* is at 5:56PM.
3. From 80 minutes, 72 minutes adjusted, before sunrise at 4:40AM to 80 minutes after sunset at 7:20PM: *Sof zeman kariat shema* is at 8:20AM and *plag ha-minḥah* is at 5:48PM.
4. From 72 minutes unadjusted, before sunrise at 4:48AM to 72 minutes after sunset at 7:12PM: *Sof zeman kariat shema* is at 8:24AM and *plag ha-minḥah* is at 5:42PM.

While options 1 and 3 are clearly preferred, option 4 has many adherents.

II – Assuming the calculations suggested in this monograph that use asymmetric endpoints but use *ḥatzot* as the anchor, there are **eight** possible combinations using any of the four points of *alot ha-shaḥar*

¹¹⁴ Washington is at approximately 39 degrees north latitude where the impact of adjusting based on latitude is already apparent.

calculated above and either of two evening endpoints, (the depression angle equivalent of) the appearance of either three medium or three small stars. As noted, *ḥatzot* isolates the calculation of morning and evening *zemanim* and calculates morning *zemanim* identically to those above. Using 24 and 39 minutes (approximately depression angles of 6 and 8.5 degrees¹¹⁵ for the appearance of three medium and small stars in Washington D.C.) gives two options for *plag ha-minḥah*:

- ❖ Using 24 minutes, *plag ha-minḥah* is at approximately 5:05PM.
- ❖ Using 39 minutes, *plag ha-minḥah* is at approximately 5:16PM.

Any of the four previous options for morning *zemanim* can be paired with any of the two options for *plag ha-minḥah*, for a total of **eight** possibilities.

Again, options 1 and 3 and using three medium stars is clearly preferred, assuming a *posek* was not consciously opting for a stringency by use of three small stars. This adds 2 more preferred options.

III - If we calculate using asymmetric endpoints, but as is normally assumed and calculate from the four options for *alot ha-shaḥar* until the two alternatives for *tzait ha-kokhavim*, each of the four points of *alot ha-shaḥar* is again paired with (the depression angle equivalent of) the appearance of either three medium or three small stars as above, again yielding **eight** additional methods of calculation. Note that these options calculate a time for *ḥatzot* before the point at which *ḥatzot* is observed, something viewed

¹¹⁵ Those two angles, with a range between 5.5 and 6.5 and 7.5 to 8.5 cover the clear majority of normative *pesak*.

by most as problematic. As with the original four options, the length of a *shaah zemanit* is identical in the morning and afternoon, but in all cases, a few minutes shorter. To illustrate consider calculating from 72 minutes before sunrise to 39 minutes after sunset. Relative to a symmetric calculation a *shaah zemanit* is now $(72 - 39) / 12$ or $(33 / 12 =)$ 2 minutes and 45 seconds shorter.¹¹⁶ Compared to the original calculation using an unadjusted 72 minutes, *sof zeman kariat shema* is 8 minutes and 15 seconds earlier.¹¹⁷ Afternoon hours differ from all the previous alternatives. Relative to symmetric endpoints calculating from either 39 or 24 minutes after sunset will always make *plag ha-minḥah* much earlier regardless of the slightly shorter length of a *shaah zemanit*. However, relative to the asymmetric alternatives, *shaot zemaniot* are longer by an identical amount, depending on the time for *alot ha-shaḥar* used. This makes *plag ha-minḥah* slightly earlier than using the computation above that calculates *plag ha-minḥah* from *ḥatzot*.¹¹⁸

¹¹⁶ Instead of $((720 + 72 + 72) / 12 =)$ 72 minutes, the length of a *shaah zemanit* is $((720 + 72 + 30) / 12 =)$ 69 minutes and 15 seconds.

¹¹⁷ A *shaah zemanit* can be as much as $((100 - 24) / 12 =)$ 6.25 minutes shorter making *sof zeman kariat shema* almost 19 minutes earlier.

¹¹⁸ In the example above using 39 minutes and *ḥatzot* gives an afternoon hour that is $(399 / 6 =)$ 66 minutes and 30 seconds. Using 39 minutes and *alot ha-shaḥar* 72 minutes before sunrise, is identical to the previous case and a *shaah zemanit* is 69 minutes and 15 seconds, exactly 2 minutes and 45 seconds longer. *Plag ha-minḥah* is earlier by 3.4375 to 7.91666 minutes. The reader is welcome to verify these calculations.

In total, we have **twelve** alternatives for morning *zemanim* and **fourteen** in the afternoon. If one were to insist that:

- ❖ *hatzot* must be accurate and
- ❖ *alot ha-shahar* must be defined to account for variation correlated to observation

then we are left with only **two** alternatives for the morning *zemanim* and **four** in the afternoon, yielding **six (not 8)** legitimate combinations:

- ❖ 72 minutes adjusted in the morning (i.e. a depression angle of approximately 16 degrees) coupled with a depression angle of 6, 8.5 or 16 degrees in the evening all using *hatzot* as the anchor.¹¹⁹
- ❖ 90 minutes adjusted in the morning (i.e. a depression angle of approximately 20 degrees) coupled with a depression angle of 6, 8.5 or 20 degrees in the evening all using *hatzot* as the anchor.

While the morning *zemanim* are widely available on many websites, the afternoon *zemanim* are rarely found. The table at the end of the monograph gives three¹²⁰ of the four alternatives listed above for *plag ha-minhah* at different latitudes / cities.

¹¹⁹ When calculating from 72 minutes before sunrise to 72 minutes after sunset, as well as with any other symmetric calculation, the standard mode of calculation is identical to using *hatzot* as an anchor.

¹²⁰ As noted, widespread use of 8.5 degrees or three small stars, that is common for the end of *Shabbat*, makes less sense in the context of calculating *shaot zemaniot* than an angle of approximately 6 degrees or three medium stars. That column was not included

Final Observations:

There are many *halakhic* ramifications; I will point out a few. Obviously, one should consult a competent *posek* regarding any unique situation.

- ❖ In the morning, one of the uses of *alot ha-shaḥar* is to specify the beginning of a fast day. Use of an unadjusted 72 minutes, allowing one to eat on the morning of a fast until an unadjusted 72 minutes before sunrise would appear to be an unwarranted leniency.
- ❖ While the *Gaon* is widely followed, on occasion, the times of the Magen Avraham are given, as, for example, on *erev Pesah*. If one wants to be stringent, then use of an adjusted (i.e. depression angle based) 72 (or even 90) minutes, versus the Magen Avraham *kenahug*, is more appropriate.^{121 122}

in the table. As well, the use of 6 degrees is meant to be illustrative; the varied rulings of *posekim* use (or more normally imply) slightly different angles.

¹²¹ Why this area is chosen for greater stringency than *sof zeman kariat shema*, particularly given the famous *heter* of R. Isserlein who allowed eating until two clock hours before *ḥatzot*, is undoubtedly related to the cautions / *ḥumrot* that the possibility of *chametz* has engendered.

¹²² In general, if one assumes that the time to walk 4 *milin* is a conservative approximation, then it is likely that *alot ha-shaḥar* occurs a few minutes earlier. Where it is a stringency, other *zmanim* should adjust accordingly, making *sof zeman kariat shema*, for example, slightly earlier.

- ❖ The afternoon *zemanim*, particularly the time of *plag ha-minḥah* is significantly more complex. A set of afternoon *zemanim* for various cites are provided in the table below.¹²³
- ❖ **Note the issue with a 90-minute interval even in Jerusalem.**
Given that Jerusalem had a 90-minute *minhag*, the plausibility of using an asymmetric earlier time for the evening endpoint is significantly increased.
- ❖ While many wait 72 or more minutes after sunset before ending *Shabbat*, 72 minutes after sunset as the latest time for *minḥah* is rather questionable. For other reasons, R. Belsky in a recent OU publication suggested at most 35 minutes in New York.¹²⁴ I would suggest that even 6 degrees (earlier than 35 minutes) that is used in

¹²³ The table uses 6 degrees for illustration. Using a depression angle between 5.5 and 6.5 degrees implies that it would be sufficiently dark for three medium stars to be visible. **Bold** entries highlight instances of *plag ha-minḥah* occurring before sunset. **Underlined, bold italic** entries highlight *zemanim* in Jerusalem around the winter equinox that lend support to the use of asymmetric endpoints, particularly given that a 90-minute interval was used in Jerusalem.

¹²⁴ *OU Daf Ha-Kashrus*, No. 5, SHEVAT 5769 / FEBRUARY 2009, footnote 27. Note that R. Belsky's approach also uses R. Moshe Feinstein's 50 minutes for defining the end of *Shabbat* and then subtracts, which used in this context, creates a leniency. This calculation, from a time that contains various stringencies can often create an unwarranted leniency; such a calculation can be found in various *tshuvot*. See my article in the spring issue of *Hakirah* 2019 where this approach, though like approaches by R. Moshe Sofer and R. Moshe Feinstein, is shown to be a logical error.

the table maybe a very slight leniency for establishing the latest time for *minḥah*.

❖ In New York and at similar latitudes in the summer time, both 72 adjusted minutes after sunset and the earlier time around the appearance of three (medium) stars provide a practical time for use in calculating *plag ha-minḥah* for those wishing to *daven* early on *erev Shabbat*, but with *minḥah* and *maariv* before and after *plag ha-minḥah*, respectively. Given the length of *kabbalat Shabbat*, one could even use three medium stars (~ 6 degrees) and three small stars (~8.5 degrees) to set 2 points of *plag ha-minḥah*, a point by which *minḥah* will be completed and a slightly later point after which to *daven maariv*. To illustrate, assume 14.5-hour daytime period, with three medium stars appearing 36 minutes after sunset, three small stars 12 minutes later and that 72 minutes adjusted is 100 minutes. Assuming sunset at 8:15PM, *plag ha-minḥah* is:

- 98.125 minutes before 8:51PM, or around 7:13PM, using three medium stars,
- 100.625 minutes before 9:03PM, or around 7:23PM, using three small stars and
- about 111.5 minutes before 9:35PM or around 7:44PM, using 72 minutes adjusted.

This is meant to illustrate that *davening* around 7:00 or 7:30PM can be implemented with some care, adjusting the time by a few minutes, assuming a longer *kabbalat Shabbat* or a short *dvar torah* before

maariv, etc. Both are preferable to what is often given as the Magen Avraham *kenahug* that uses an unadjusted 72 minutes.¹²⁵

- ❖ There are four alternatives for the morning zemanim. If forced to choose one, **72 minutes adjusted appears most logical**. 72 minutes is the position of Rambam and Saadyah *Gaon*, who both lived in the Middle East, as well as the *Shulhan Arukh*. The case for not adjusting is exceedingly difficult to sustain. The reasons have more to do with the reasons for not adjusting 72 minutes after sunset according to Rabbeinu Tam, to which this issue is conceptually linked.

- ❖ In the afternoon, there are four alternatives – the nighttime equivalents of the two adjusted times for *alot ha-shahar* in the morning and the two earlier points of *tzait ha-kokhavim*. Of those four alternatives, the use of either 72 minutes adjusted or a depression angle of approximately 6 degrees is preferred.

¹²⁵ The times given in a recent sefer, *Shaarai Zemanin* by R. Dovid Heber, as the *pesak* of R. Heinemann suggests calculating *plag ha-minhah* using both 72 unadjusted minutes as well as 50 minutes, based on R. Feinstein's *pesak* for the end of *Shabbat*. Both are similar to but hardly as precise or as well-grounded as the suggestions above of an adjusted 72 minutes or a depression angle corresponding to three medium stars. Using an adjusted versus a fixed time is a classical disagreement. In addition, when using 50 minutes, R. Heber uses the more classical method of calculation that does not calculate *hatzot* accurately.

Personal Observations:

- ❖ Why did *gedolai ha-mesorah* use endpoints that calculated *ḥatzot* imprecisely? My suspicion is that they had a *mesorah* that specified the endpoints to be used. While the endpoints were retained, over time the method of calculation changed subtly. What is proposed may be consistent with what may have been practiced for some period. It is certainly consistent with the verse in Nehemiah as it is almost universal interpreted, defining a period from *alot ha-shaḥar* until *ḥashekhah* / the appearance of three medium stars.
- ❖ The position of the *geonim* versus Rabbeinu Tam was practiced in many communities. Combined with a second group who while conceptually agreeing with Rabbeinu Tam nonetheless practiced using an earlier end to *Shabbat*,¹²⁶ it is evident that many if not most Jews throughout some / much of our history used *tzait ha-kokhavim* as the evening endpoint, which occurs significantly earlier than *tzait kol ha-kokhavim*. For both groups, the use of a different and later endpoint for calculating *shaot zemaniot* was never ever mentioned prior to the 20th century.¹²⁷

¹²⁶ That group includes R. Pimental, R. Loerberbaum, R. Sofer, R. Feinstein, among many others.

¹²⁷ It is of course **indirectly** mentioned by those like Ramban in *Torat HaAdam* who implemented the conceptual position of Rabbeinu Tam and ended the day of the week at *tzait kol ha-kokhavim*, a point as far from sunset as *alot hashaḥar* is from sunrise.

- ❖ Given the migration of Jews to Europe well before the impact of latitude was understood, I remain partial towards the practices of communities in the Middle East with a history of unchanged practice with respect to *zemanim*. Not requiring a change based on latitude and having only a limited change required due to season, places additional significance on Mideast practice. The practice in Jerusalem followed both the *geonim*'s end to the day of week and clearly used asymmetric endpoints for the calculation of *shaot zemaniot*.
- ❖ The consequences of the standard equidistant version of the Magen Avraham's position in Europe may have contributed to the adoption of the position of R. Yaffe and the *Gaon*, particularly in northern Europe. Assume that the daytime period is only eight hours, then the *halakhic* day is minimally $(480 + 2 * 72 =)$ 624 minutes and the *halakhic* hour is $(624 / 12 =)$ 52 minutes. (Using depression angles latitude and seasonal adjustment adds an additional 36 minutes (108 versus 72 minutes) making the problem yet more apparent.) *Hatzot* is at noon, *tzait ha-kokhavim* is at 5:12PM and *pelag ha-minḥah* occurs $((1 + \frac{1}{4}) * 52 =)$ 65 minutes before *tzait ha-kokhavim* at 4:07PM, **7 minutes after sunset**. Once one goes further than 48 degrees from the equator, these problems are magnified during the winter. Either
 - abandoning the Magen Avraham's normally assumed position either in favor of the *Gaon*'s position, or
 - modifying the Magen Avraham's position and using a calculation between *hatzot* and each of two non-equidistant endpoints

avoids the problem.

- ❖ Similarly, given the necessity according to Rabbeinu Tam to find one number that equals both the length of the period from *alot ha-shaḥar* until sunrise and the time from sunset until *tzait ha-kokhavim*, the preference for 72 minutes over 90 minutes is understandable. Even as far north as Lithuania, except perhaps in the summer, one can see stars by 72 minutes after sunset. It is easier to simply assume that at the time of *alot ha-shaḥar* the level of darkness equals that observed at *tzait ha-kokhavim*. It is hardly surprising that *poskim* from the Middle East tended to favor an earlier point of *mi-she-yakir*.
- ❖ The language used by *ḥazal* in both *Berakhot* and *Shabbat, ad ha-erev* and *ḥashekhah*, would suggest an evening endpoint that is neither sunset nor *tzait kol ha-kokhavim*. An earlier point around *tzait ha-kokhavim*, **though asymmetric from both sunrise and *alot ha-shaḥar*** appears most plausible.
- ❖ I would speculate that the reason that the use of asymmetric endpoints was never advanced in the recent past, is a consequence of the fact that in the century since the problem was identified we appear to have all been fixated on the use of clocks and long arithmetic. Understandably, the thought of not calculating *ḥatzot*, especially the resulting unequal *halakhic* hours in the morning and the afternoon, appears strange.
- ❖ Using asymmetric endpoints avoids the need for use of a depression angle to determine *tzait kol ha-kokhavim*. *Alot ha-shaḥar* was only

known to occur earlier than the significantly easier task of determining *mi-she-yakir*. No such discussion of physical approximations exists in the evening, making it difficult to explain how *tzait kol ha-kokhavim* would be estimated without the use of depression angles.

- ❖ Given the issues in the winter with both
 - 90 minutes in Jerusalem, and
 - the standard mode of calculation for the Magen Avraham's position in locations where the day's length is less than 8 hours,

I am biased towards using asymmetric endpoints in calculating the opinion of the Magen Avraham.

TABLE