

CANTILLATION: SOME OBSERVATIONS – PART 2

By **William Gewirtz** - February 17, 2020

פִּשְׁטָא מִנַּח זִרְקָא מִנַּח סְגוּל מִנַּח
 מִנַּח רְבִיעֵי מִהֶפֶךְ פִּשְׁטָא זְקַף־קִטְוִן
 זְקַף־גְּדוּל מִרְכָּא טְפַחָא מִנַּח אֲתַנְחַתָּא
 פִּזְר תְּלִישָׁא־קִטְנָה תְּלִישָׁא־גְּדוּלָה קְדָמָא־
 וְאַזְלָא אֲזַלָּא־גְּרֵשׁ גְּרֵשִׁים דְּרָגָא תְּבִיר
 יְתִיב פְּסִיק | סוּף־פְּסוּק: שְׁלֵשֶׁת קַרְנֵי־
 פָּרָה מִרְכָּא־כְּפוּלָה יְרַח־בֵּן־יוֹמוֹ:

WILLIAM GEWIRTZ

Introduction

Part 1 of this essay briefly introduced the *trop*, followed by a study of its significance in some **local contexts**, concluding with some evidence of *trop*'s rabbinic origin. Part 2 looks at *trop* in its **global context**, structuring the two parts of most *pesukim*, until and after the word containing an *etnahta*. The process by which the *trop* operates demonstrates its recursive nature, providing a very early example of recursion in a musical context.

Trop contains 4 levels of separators (*mafsikim*) and a single set of connectors (*meshartim*). All *trop* symbols are either separators or connectors. The first level separators (often referred to as *keisarim*, Caesars) are the *sof pasuk*, which ends the sentence, and the *etnahta*, which divides the *pasuk* into two parts, analogous to a semi-colon. Both parts of the sentence, before and after the *etnahta*, are treated identically by the rules of *trop*. The second level of separators (often referred to as *melakhim*, kings), the *zakeif katan*, *zakeif gadol*, *segol*, *shalsholet* and *tipha*, define the major structure of the *pasuk*. *Pashta*, *revi'i*, and *tevir*, an

additional level lower, are common third level separators, while *darga*, *pazeir*, and *telisha gedolah* are common fourth level separators.

Munah, *merha*, *mahapah*, and *kadma* are common connectors; there should not be an apparent pause between the reading of words where they appear and the following word.

Trop identifies the *pasuk's* structure both at a global / macro level (the entire *pasuk*, or its two components divided by the *etnahta*) and at a local / micro level (each individual phrase).

Trop is Recursive

Recursion is primarily a mathematical notion which operates on an entity, dividing that entity into parts where at least one part is operated on by the identical process. One can think of this as an arbitrary number of Russian *matryoshka* (often called Babushka) dolls, each embedded in another.

In a brilliant book, *Gödel, Escher, Bach: An Eternal Golden Braid*, Douglas Hofstadter shows that recursion, which in mathematics was brought to its ultimate use by Kurt Gödel, was also present in painting (such as by Escher) and music (such as by Bach). In music, recursion involves a (completely or partially) identical pattern that repeats (iteratively) within a pattern. Recursion was present in the *trop* 1,000 years before its occurrence in Bach's music, albeit with *trop's* much less intricate musical scope.

Trop's global operation

Except for short *pesukim*, the vast majority of *pesukim* contain one *etnahta* that divides the *pasuk* into its two principal parts.^[1] Going forward, we refer to either a short *pasuk* or to either of the two parts of a longer *pasuk* as an **initial segment**. *Trop* operates independently on each individual segment. Note that all initial segments end with a first level *mafsik*, either a *sof pasuk* or an *etnahta*. The lower level *mafsikim* (listed above) further divide the *pasuk* into smaller segments.

Trop's operation on a segment is governed by the following rules:

1. Read the segment (from right to left) until the first *mafsik* one level lower than the *mafsik* on which the segment ends is encountered.
2. If such a *mafsik* is encountered, divide the segment into two, with the *mafsik* acting as the separator. Those two segments are then operated on again by the rule.
3. If a *mafsik* one level lower is not found, the segment is not further divisible, and no further operation is performed.

Since all *pesukim* are of finite length, part 3 of the rule will eventually occur either because

- the *mafsik* at the end of the segment is at level 4 (and there are no *mafsikim* of a lower level), or
- even though the segment ends with a *mafsik* of levels 1, 2, or 3, no *mafsik* one level lower is present.

One of the fundamental rules of *trop* forbids the presence of a *mafsik* of lower level than the level being sought. For example, if a segment ends with a second level *mafsik* and there is no third level *mafsik* earlier in the segment, one can be certain that a fourth level *mafsik* will also not be present.

When operating with the rules of *trop* on any segment, the rule will divide that segment into two parts, providing it finds a *mafsik* one level lower; the part to the right ends on the word containing the *mafsik*, and the part to the left is the remainder of the original segment. This pattern repeats on any segment, regardless of length.^[2] The rule's identical repetition on both segments demonstrates its recursiveness.^[3]

Consider the second *pasuk* in Ki Tavo (**Deuteronomy 26:2**). The first part of the *pasuk*

וְלִקְחֶתָּהּ מֵרֵאשִׁית אֶל־פְּרֵי הָאֲדָמָה אֲשֶׁר תִּבְיֵא מֵאֲרָצְךָ אֲשֶׁר יְהוָה אֱלֹהֶיךָ נָתַן לָךְ וְשָׂמֶתָ בַּטֶּנָּא

encounters its first *melekh*, a *tipha*, on the word לָךְ. Note that this symbol accurately divides the first section into two parts; the first part tells us what should be taken, and the second part tells us where it should be placed. The second part of the *pasuk*,

וְהִלַּכְתָּ אֶל־הַמְּקוֹם אֲשֶׁר יִבְחַר יְהוָה אֱלֹהֶיךָ לְשָׁכֵן שָׁמָּה שֵׁם

encounters its first *melekh*, a *zakeif katan*, on the word הַמְּקוֹם. The *pasuk* tells us to travel to the place, and then provides a further description of the place.

The segment comprising the second half of the *pasuk* succinctly illustrates a critical detail that can cause some confusion when separating a *pasuk* into its constituent parts. Consider the two subdivisions of this half-*pasuk*, one up to and including the word **הַמְקוֹם** and one after it. The second subdivision can be further divided by a **second level mafsik**, the *zakeif katan* on the word **אֶלְהֵיךָ**. However, the first subdivision is further divided by a **third level mafsik**, the *pashta* on the word **וְהִלַּכְתָּ**. Note that it is not the level of a *mafsik*, but its role in the *trop's* division of a segment, that determines a *pasuk's* syntax.

Syntax only, not semantics

As noted in Part 1, since *trop* provides only syntax, it can

1. provide likely support for a specific interpretation or
2. be conclusively inconsistent with a specific interpretation.

The following examples, more complex than those covered in Part 1, all contain a separator / connector where the other might be expected, and therefore support dramatically different interpretations.

Consider the semantically ambiguous reply that occurs when a pregnant Tamar confronts Yehudah (Genesis 38:26). Yehudah responds:

וַיִּכַּר יְהוּדָה וַיֹּאמֶר צְדָקָה מִמֶּנִּי כִּי־עַל־כֵּן לֹא־נִתְתִּיחַ לְשִׁלָּה בְּנִי

The first part of the *pasuk* ends on the word **מִמֶּנִּי**, which contains a *zakaif katan*. The word **צְדָקָה** has a *munah*, linking it to the word **מִמֶּנִּי**. The *trop* is seemingly in accordance with the interpretation given by those such as **Rashbam** where Yehudah admits that "she is more righteous than I." On the other hand, the *trop* is inconsistent with an alternative interpretation, "she is righteous; the child is mine," which is the interpretation given by **Onkelos, Rashi**, and others. For that interpretation to be tenable, the word **צְדָקָה** would require a *mafsik*.

Often the syntax can provide (nearly) equal support for two alternative interpretations.

Consider the brief *pasuk* in Genesis (49:18) with which Yaakov ends his *berakhah* to Dan:

לִישׁוּעַתְּךָ קִנִּיתִי יְהוָה:

An interpretation like: "I await for Your deliverance, O Lord," as translated by JPS, is inconsistent with the *trop*. Such an explanation would require placing the *tifha* one word further, at קניתי. This interpretation is also hard to reconcile with the context, unless God's deliverance is awaited not on behalf of Yaakov but on behalf of Dan. However, as written, the *trop* is consistent with various semantic alternatives. The sentence can mean "For **deliverance by You**, I have prayed to the Lord," without stating explicitly for whom deliverance is prayed for. Again, the context more likely implies that Yaakov is praying for Dan's (or his descendant's) deliverance. Alternatively, directly addressing Dan, Yaakov tells him that he prays to the Lord for **his deliverance**. This explanation is given by **Rashbam**.^[4]

On occasion, dramatically different semantic interpretations are both possible given the *trop*. In both of the following *pesukim* the *trop* is consistent with either interpretation. First let's consider **Exodus 8:19**:

וּשְׁמַתִּי פְּזֹת בֵּין עַמִּי וּבֵין עַמֶּךָ לְמַתַּר יְהִיָּה הָאֵת הַזֶּה:

Does פְּזֹת mean a separation or a salvation? Both interpretations likely agree that God will create a **separation** between the Israelites, who will receive **salvation**, and the Egyptians, who will be **afflicted**. The argument is about the meaning of the word פְּזֹת, either a separation or a salvation, making one word explicit and the other implied. **Onkelos** interprets פְּזֹת as salvation, more consistent with its typical meaning; most commentators prefer separation, more consistent with the context of this *pasuk*.

Next, let's look at Exodus **17:16**:

וַיֹּאמֶר כִּי־יָד עַל־כִּסֵּי יְהוָה מִלְחָמָה לִיהוָה בְּעַמְלֵק מִדֶּרֶךְ דָּר:

Are we taking an oath, or referring to a time when there is a monarchy? The term יָד עַל־כִּסֵּי יְהוָה is ambiguous. It could mean that one's hand is on God's throne, as might happen as one is holding a religious object while taking an oath. This explanation is given by Rav Saadyah Gaon, and likely **Onkelos** as well. Alternatively, as posited in **Sanhedrin 20b**, it could be indicating that the command to obliterate Amalek refers to an era when a king is leading a religious monarchy. Which explanation is correct is disputed by the classical commentaries, some proposing both possibilities.

Dealing with lists

In numerous places, the *trop* deals with the individual elements in a list of items.

Let us first give two examples that comport with what one might sense as the expected case. Numbers (30:6) and Exodus (6:3) given below are representative.

- וְאִם־הִנִּיא אָבִיָּהּ אֶתְהוֹ בַּיּוֹם שֶׁמָּעוֹ כָּל־נִדְרֵיהָ וְאֶסְרֶיהָ אֲשֶׁר־אָסְרָה עַל־נַפְשָׁהּ לֹא יִקְוֹם
- וְאִלֹּא אֶל־אֲבֹרְהֶם אֶל־יִצְחָק וְאֶל־יַעֲקֹב בְּאֵל שַׁדַּי

The first example divides the segment on the word שֶׁמָּעוֹ – if the father objects on the day when he first hears. The second segment then lists two types of restrictions:

1. vows; and
2. self-imposed restrictions.

The second example divides the segment first on the word יַעֲקֹב and then on the word וְאִלֹּא. God declares he appeared, and then lists the three people to whom He appeared. In both examples, the action applies to all items on the list.

The next example from Numbers (30:3) contains a similar pattern but in reverse, with the list occurring first.

אִישׁ כִּי־יִדַּר נֶדֶר לַיהוָה אִז־הִשָּׁבַע שְׁבַעַה לְאָסֵר אֶסֶר עַל־נַפְשׁוֹ לֹא יִחַל דְּבָרָו

The segment divides on the word נַפְשׁוֹ, with the first part listing vows and restrictions and the second half admonishing the listener not to profane them.

In the following three examples, only certain elements of the list link to the verb in the opening phrase.

1. Numbers (6:14): וְהִקְרִיב אֶת־קָרְבָּנוֹ לַיהוָה כֶּבֶשׂ בֶּן־שָׁנָתוֹ תְּמִים אָחֵד לְעֹלָה וְכִבְשָׂה אַחַת בֵּת־שָׁנָתָהּ תְּמִימָהּ לְחִטָּאת וְאֵיל־אֶתֶד תְּמִים לְשָׁלְמִים
2. Exodus (1:6): וַיָּמָת יוֹסֵף וְכָל־אָחָיו וְכָל־הַדָּוָר הַהוּא
3. Exodus (1:14): וַיִּמְרֹדוּ אֶת־חַיֵּיהֶם בַּעֲבֹדָה קָשָׁה בְּחֹמֶר וּבַלְבָּנִים וּבְכָל־עֲבֹדָה בַּשָּׂדֶה

In each case, one can assume the verb applies to all elements of the list, despite being syntactically linked **only** to the first element. In Numbers (6:14) the opening phrase וְהִקְרִיב אֶת־קָרְבָּנוֹ לַיהוָה presumably applies to the two other elements in the list, even the element

occurring in the next segment, **after** the *etnahta*. The *pasuk* may be read as if the phrase is implicitly assumed to be repeated.

The reasons for this syntactic choice may often be semantic or stylistic.^[5] In the second example above, it is highly plausible that the *pasuk* is ranking the people mentioned: Joseph is most important, followed by his brothers, and finally other members of his generation. There are many other examples, sometimes with a less compelling assumed ranking among list members. The last example may link to the most prevalent work performed. Many other examples that occur in the Torah are less clear.

Conclusions

The formality introduced is necessary to guide a beginner trying to parse a sentence following the rules associated with the *trop*. Fortunately, almost anyone experienced with how the *trop* operates can look at a *pasuk* and directly observe the implied levels of division implied. My late father went a step further, claiming that if he assumed a particular interpretation, he could normally deduce the associated *trop*. I inherited my mother's mathematical skills and not my father's literary prowess; on occasion, I still make embarrassing errors studying and teaching *trop*.

[1] Of the 5,853 *pesukim* in the Torah only 372 do not contain an *etnahta*; see <https://quantifiedcantillation.nl/>.

[2] The book of Esther has particularly long *pesukim*, providing the most involved examples.

[3] A detailed recursive algorithm and an example is available [here](#).

[4] These alternatives would be clearer if there was a ֿ before Hashem.

[5] Considerations based on length, potential rhythm, dramatic impact, etc. might influence the sequence of *trop* symbols chosen. A semantic reason can also on occasion be linked to a midrashic source, as the genre of *seforim* like *Ve-Yavinu ba-Mikre* by R. Naftali Tzvi Yehudah Gettinger on occasion attempts to demonstrate.



William Gewirtz

Dr. William Gewirtz, a former CTO of AT&T Business, is a consultant in the technology and communications sector. He maintains a strong interest in halakhic areas where mathematics, science and/or logic play an important part, including all aspects of zemanim and kinim. He can be reached at wgewirtz@att.net.
