

A Categorization of Errors Encountered in the Study of Zemanim

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Introduction

To begin, I feel obligated to address my motivation for writing an essay focusing on the errors regarding *zemanim* one encounters throughout rabbinic literature. My motivation is at least three-fold:

1. For theorists in the development of *halakha*, these examples provide valuable information for analysis.
2. Without errors identified and accounted for, mastery of this vast and critical area of rabbinic literature will remain difficult even for accomplished *halakhists*. One need only read the revealing *teshuvah* in *Minhat Yitzhok*¹ to observe the (self-reported) challenges faced by one of the last century's celebrated *poskim*.
3. On occasion, the erroneous reasoning provided in a *teshuvah* provides the basis on which others construct their rulings. This has caused practical errors usually, but not always, by lesser authorities.

Throughout, I do not cite multiple *teshuvot* where an error occurs; with a few exceptions, one example suffices. Normally, a footnote will specify a source where an error can be found. Most illustrated errors were made by *poskim* living after the 19th century, when almost all needed science was widely available.

The errors that I list are focused on the rationale or structure of the arguments regardless of whether the resulting *psak* may still be reasonable. As a result of the errors, however, many *pesakim* are not well-founded. Furthermore, many who study and then apply those arguments may use the rationale as a basis for an erroneous *psak*.

¹ *Minhat Yitzhok* (4:53).

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The area of *zemanim* is one where elementary mathematics and not-so-elementary science play a key role. While there are examples in other areas of *halakha* as well, I do not believe that other areas have been responsible for such an extensive list of issues.² Also, attempts to justify erroneous prior opinions or render consistent conflicting views may themselves result in errors. These may even include rare logical errors.

My suspicion is generations of Talmudic masters and *poskim* perceived as disrespectful addressing the erroneous reasoning behind various positions, identifying fundamental changes in positions that have occurred across centuries, or identifying and adjusting for an error made by a prior *posek*. As a result, a variety of errors have become difficult to isolate; some have become institutionalized and even given canonical status. Further adding to the complexity, the nature of the errors themselves changed, sometimes appreciably, over time.

One final introductory remark. Anyone who subsequent to reasonably accurate examination of observable events acts accordingly has little reason to worry. However, if one extrapolates from a given day at a specific location to another location and/or another season, then knowledge about how *zemanim* are impacted by changing location or season becomes critical. Many *poskim* did not consider the dependency of all *zemanim* on both location and season; that, coupled with the inability to calculate their effects correctly even when the dependence is recognized, is characteristic of many errors.

But this gets worse; the feedback effect of poor *psak* and other factors, such as clock use, reduced dependence on the very thing that was required—accurate observation.³ In fact, as we will illustrate, the very ability to observe knowledgeably has probably declined, perhaps appreciably. There are positions maintained by current *poskim* that are contradicted by observation, most often concerning the time of *alot ha-shahar*.⁴

This essay assumes basic familiarity with the two fundamental areas in *zemanim*.

² This includes other areas like *Kinnim*, which require more complex logical and mathematical reasoning.

³ Telling someone to go outside and observe frequently engenders this sort of response: “Who should I believe: the *psak* received, or my lying eyes?” I find it impossible to understand many relevant *sugyot* without accurate observation of Middle Eastern skies.

⁴ The *psak* of various *gedolim* on *alot ha-shahar* and the times for the end of *Shabbat* provided in various publications are given as examples in Category 5 below, which addresses fixed time intervals.

1. The argument⁵ between the *geonim* and Rabbeinu Tam about the transition point between days of the week.
2. The argument currently referred to as being between R. Avraham Gombiner, the Magen Avraham, and R. Eliyahu from Vilna, on the endpoints between which the hours of the daytime are calculated. According to the *Gaon*, *halakhic* hours are calculated from sunrise to sunset. According to Magen Avraham *halakhic* hours are calculated with *alot ha-shahar* as the morning endpoint.⁶

Six Error Categories

I will classify errors into six categories and illustrate representative errors occurring in each category. It appears to me that the list of categories is both necessary and sufficient to capture all primary areas of error. The categories used are:

1. **Latitude:** Explicit mention in rabbinic literature of latitude and its impact on the length of the twilight periods did not occur until the 17th century. Despite being explained in detail in *seforim* written during the first half of the 20th century,⁷ the impact of latitude is not widely understood.

⁵ The *geonim*, the *Gaon* and the *Ba'al Ha-Tanya* have slightly different positions with respect the start of *Shabbat*; there does not appear to be any dispute over when *Shabbat* ends. R. Posen in *Ohr Ha-Meir* strongly differentiates the position of the *geonim* **who he believes started *Shabbat* 15 minutes after sunset** from that of the *Gaon* who wrote clearly that *Shabbat* **started at sunset**. The entire text of the *geonim* is printed in an appendix to Vol. 2 of *Ha-Zemanim Be-Halakha* by R. Hayyim Benish. The *shittab* of the *Ba'al Ha-Tanya*, entitled *Seder Hakbnosot Shabbat* is found towards the end of every *siddur* published by *Habad*. It says explicitly that *Shabbat* begins **4 minutes after sunset**. Rabbeinu Tam's position is found in *tosefot* on both *Shabbat* 34b and *Pesahim* 94a; it has a more elaborate (and arguably slightly different) presentation on pp. 251–255 of *Torat Ha-Adam* by Ramban, in the R. Chavel edition.

⁶ Surprisingly, the evening endpoint in the calculation of Magen Avraham, which is after sunset, is in dispute. The complex topic of Magen Avraham's evening endpoint is not addressed further in this paper. Both of their positions are in their respective commentaries to *O. H.* 58. Of course, this *mabloket* has a more ancient origin.

⁷ Perhaps the most important *sefer* of the twentieth century on *zemanim* is *Bein Ha-Shemashot* by R. Yehial Miḥal Tukatzinsky (1929) where depression angles are broadly discussed. The *sefer* contains comments by R. Isser Zalman Meltzer, *Rosh Ha-Yeshiva* of *Yeshivat Eitz Hayyim* where R. Tukatzinsky taught, who

2. **Season:** Season was also first acknowledged in rabbinic literature as a factor in the length of the twilight periods in the 17th century. Unfortunately, the proposed method for calculating seasonal impacts was faulty, contradicting accurate observation. That error, in addition to other unrelated errors, persists to this day.
3. **Mathematics:** Although *zemanim* does not have a monopoly on mathematical errors, errors in this area have a tendency to be more significant *halakhically*.⁸
4. **Logic:** Logical errors are the most disturbing and fortunately remain relatively rare. My suspicion is that dealing with the complexities of trying to uphold conflicting or unsupported positions is the primary stimulus for these isolated errors.
5. **Fixed time intervals:** The use of fixed intervals of time should always trigger suspicion. In ancient times before the advent of clocks, it is entirely implausible that fixed time intervals as opposed to observable events played a defining role. This area remains controversial as many current leading *poskim* insist on using fixed intervals of time even when they conflict with observable events, at times dramatically.
6. **Assumed or required symmetry:** In many areas of *zemanim*, symmetry may occur naturally. However, on rare occasion, assumptions of required symmetry may lead to incorrect conclusions. This category is not always clear-cut, with several areas that still require further careful study.

Attention to examples of erroneous reasoning in each of these categories can aid in developing competence in this complex area. To be helpful as opposed to formally elegant, the list of errors is pragmatic and partially repetitive to allow the reader to better identify errors as they are encountered. I also omit some very technical errors that would require significantly greater background to address; the existence of a 96-minute interval between *alot ha-shahar* and sunrise is often repeated, but is, nonetheless, an example of such an error.⁹ Arguing that a 96-minute interval

disagrees on how altitude and visibility should be addressed. I assume his lack of commentary on depression angles signifies his acceptance of their legitimacy in a *halakhic* context.

⁸ The examples all draw from areas where no more than competency in (junior) high school algebra is required.

⁹ *Melamaid Le-Ho'el* 30 and *Iggerot Mosheh O. H.* 4:62 both mention a 96-minute interval.

cannot be reconciled with the *gemara* in *Pesahim* 94a is rather straightforward, but requires familiarity with the *sugyah*. Some of the errors briefly noted in both the fixed intervals and symmetry categories border on errors of this type.

We avoid, as well, critical analysis of *sugyot* possibly conflicting with positions taken in the *halakhic* literature that require familiarity with astronomy at a level likely known to several, but not all, *tannaim*, *amoraim*, *geonim*, *rishonim*, and *aḥaronim*.¹⁰ Another avoided area are theoretical inconsistencies within a given *posek*'s position. Often, discovering the motivations that might have driven that position and its practical impacts is of great value.¹¹

The origin of errors: latitude and season and the use of clocks

Latitude and season

In the entire period of the *rishonim*, instead of time-based measures, most *mitzvyot* dependent on *zemanim* were performed based on the observation of natural events. **The effects of latitude and season were incorporated implicitly via observation.** The occurrence of darkness or the appearance of stars varied naturally between locations regulated by a yet unknown science. How *zemanim* differed at distinct locations was largely immaterial; as far as I know, there is no discussion in the *halakhic* literature comparing *zemanim* at different locations prior to the 17th century.

Beyond question, the most impactful ruling concerning *zemanim* in the entire period of the *rishonim* is that of Rabbeinu Tam. The normative opinion of the *gemara* in *Shabbat* 34a states that the duration of the *bein ha-shemashot* period equals the time needed to walk $\frac{3}{4}$ of a *mil*. One would therefore expect *Shabbat* to end (i.e. for three stars to be visible) between 13.5 and 18 minutes after the beginning of *bein ha-shemashot*, depending on the assumed time it takes to walk a *mil*. Rabbeinu Tam,

¹⁰ The statements in *Shabbat* 35a and 34b of *Shmuel* relative to the appearance of stars and *Rava* and *Abaye* looking in different directions are both complex but relevant examples. Both appear in my *Seforim Blog* entries from 2010 and will appear in a more elaborate future article focused on those *sugyot* and related sections of *Mishneh Torah* in *Shabbat* and *Kiddush Ha-Hodesh*.

¹¹ The views of R. Yaacov Loberbaum in the section of *Derekh Ha-Hayim* about *hadlakat ha-neirot* and the *bein Ha-Shemashot* period is a good example. The *Derekh Ha-Hayim* was reprinted several times in the period immediately following R. Loberbaum's death with a few significant emendations to those sections, undoubtedly to better conform with local practice. In one community where practice was radically different, the relevant section was deleted entirely.

living in France, was undoubtedly unaware of latitude's impact on the appearance of stars and as a result likely assumed the *gemara* was discussing all regions of the world uniformly. As such, if sunset defines the beginning of the *bein ha-sbemasbot* period precisely, Rabbeinu Tam would not have been able to observe the appearance of three stars within so short a period after sunset. For that matter, even those living in the Middle East can rarely see three stars 15 to 18 minutes after sunset, and then only under the most ideal conditions, often aided by a telescope in the hands of an expert observer knowing exactly where in the skies to look. Rabbeinu Tam, living at a more northern latitude in France (especially during the summer months), might not have seen what he considered three medium stars until approximately an hour or more after sunset, depending on his interpretation of the term "medium."

However, the *gemara* in *Pesahim* 94a, in apparent contradiction to the *gemara* in *Shabbat*, but perhaps closer to what Rabbeinu Tam observed, discusses the end of the day occurring at the time needed to walk **not $\frac{3}{4}$ of a *mil*, but 4 *milin*, 72 or 90 minutes, after sunset.** That *gemara* in *Pesahim* 94a equates the interval between *alot ha-shahar* and sunrise with the interval from sunset to the end of the day. Rabbeinu Tam assumed the *gemara* in *Pesahim* was defining the end of *Shabbat* when sunset is understood in its usual sense. The *gemara* in *Shabbat*, however, which calculated that *Shabbat* ends at the time to walk only $\frac{3}{4}$ of a *mil* after sunset, was referring to a second sunset, which occurs about an hour after sunset in its usual sense, closer to the time when residual light from the sun begins to disappear completely. (As anyone can observe, the western sky still maintains a reddish glow one hour after sunset.) Explaining this critical area in greater detail is unnecessary for our purposes.

Knowledge of the impact of **latitude and season** on various *zmanim* cannot be assumed even to this day; it was certainly absent when Rabbeinu Tam equated the two twilight intervals, the one preceding sunrise and the one following sunset. Rabbeinu Tam's position was vigorously opposed and, in the opinion of many, was eliminated as a legitimate *halakhic* option by the *Gaon* of Vilna.¹² It was also opposed by R. Shneur Zalman of Liadi,¹³ whose criticism and rejection, while unequivocal, was differently reasoned.

¹² In his commentary on multiple sections of the *Shulhan Arukh*, particularly O. H. 261.

¹³ R. Shneur Zalman of Liadi's opinion reverses the position he took in *Shulhan Arukh Ha-Rav*, which supported Rabbeinu Tam.

However, all this took place approximately 600 years after the opinion was first formulated. In the interim, Rabbeinu Tam's opinion, while challenged on occasion, ruled the *halakhic* world. As Rabbeinu Tam's position was regarded as applicable without regard to latitude or season, as it was indeed intended, the study of *zemanim* started down a path that has proven difficult to alter.

Rabbeinu Tam's position would have to address three fundamental challenges, all derivable from or explicit in the *Gaon's* criticism:

1. In the Middle East, which is clearly the location the *gemara* addresses, three stars are observed to appear after sunset but well before the time needed to walk 4 *milin* has elapsed. Assumedly, Rabbeinu Tam read the *gemara* as if it was describing the appearance of 3 stars in all locations, including his location in northern France, which he likely assumed was not different from that of the Middle East. Because Rabbeinu Tam applied an interval of the time needed to walk 4 *milin* without any dependence on location and season, even to this day, almost all who follow Rabbeinu Tam rarely adjust his interval to **more** than the time needed to walk 4 *milin*.
2. By observation, the time to walk 4 *milin* is a strict lower bound on the length of the interval between *alot ha-shahar* and sunrise for all locations in the Middle East or further north; as one moves north, *alot ha-shahar* occurs even more than 72 minutes before sunrise. However, Rabbeinu Tam's opinion stipulated that the length of the period between *alot ha-shahar* and sunrise equals the length of the period between sunset and the end of *Shabbat*. As noted, those following Rabbeinu Tam would typically wait **at most** 72 minutes everywhere before ending *Shabbat*. By logical necessity, *alot ha-shahar* occurs the same number of minutes before sunrise. Thus, *poskim* ended up postponing the practiced point of *alot ha-shahar*, sometimes considerably, to only 72 minutes before sunrise, well after its actual occurrence. Unlike the first point, which is based on conditions **observable only in the Middle East**, this point was **directly observable in central and northern Europe**.
3. Most fundamentally, how could the time of the appearance of only three stars and the time of *alot ha-shahar*, when almost all the stars are still visible, be identically separated (as specified by the *gemara* in *Pesahim* 94a) from sunset and sunrise respectively? The pre-dawn parallel to the time after sunset when only three

stars are first visible **cannot be** *alot ha-shahar* when almost all stars are still visible.

Attempts to tackle any of the three challenges above often led to errors in one or more of the afore-mentioned six categories.

Clocks:

At the turn of the 16th century, almost two centuries after the introduction of mechanical clocks in Europe, clocks entered the *halakbic* literature. This was in an era when knowledge of the impacts of latitude and season was non-existent. Throughout almost the entire period of the *rishonim*, time intervals were referred to not as the number of minutes on a clock, but primarily as approximate intervals. Clocks allowed opinions previously specified in terms such as “the time required to do X” to be translated into a precise, easily specified time of day or time interval.

Clocks began to proliferate almost 200 years before the first recorded reference in *halakbic* literature to either the impacts of latitude or season. Those impacts were incorporated by R. Avraham Pimental in his *sefer Minbat Kohen* in the 17th century.¹⁴

The first mention of a clock in the *halakbic* literature was in *Leket Yosher*,¹⁵ by R. Yosef ben Moshe, a student of R. Yisroel Isserlein. It was written around the turn of the 16th century. During the 14th through 16th centuries, clock making rapidly accelerated. The significantly early advent of clocks was a likely trigger for many subsequent errors.

Additionally, the precision that clocks provide may have resulted in their increased prominence at the expense of observation. Precision and accuracy are often confused. Clocks provide precision for measurements that may or may not be accurate *halakbically*. Saying that *Shabbat* ends at a specific time, may be very precise but totally inaccurate. Clocks provided a level of precision that may have been overly seductive. Even more dis-

¹⁴ R. Pimental was acknowledged as an expert in *zemanim* by R. Avraham Gombiner, the author of Magen Avraham. *Minbat Kohen* was carefully organized and argued; unfortunately, two significant errors appear in *ma'amar sheni*, chapters 4 and 5 (discussed below) which haunt us to this very day. *Ma'amar rishon*, organized in the style of Abarbanel, presents the arguments for and against the major positions in *zemanim*. Given his *halakbic* mastery and his unique role in introducing the important notions of latitude and season, his errors are minor compared to his organized analysis. Oddly and regrettably, the persistence of his errors is a testament to his monumental impact.

¹⁵ On pp. 157–158 at the very end of his commentary on *O.H.*, which contain *derashot* and *pesakim* from R. Isserlein with respect to *Purim*.

concerting, clocks allowed *psak* to be rendered independent of observation. With an assumed reduced reliance on observation, critical *halakhic* definitions became more subject to dispute. Examples abound in the *halakhic* literature: distinguishing between levels of darkness, differentiating between medium and small stars, and determining the amount of illumination necessary to recognize a friend after dawn are three plausible illustrations.

Centuries later, as personal timepieces proliferated and greater uniformity became necessary with the growth of railroads, time became even more important in many *halakhic* contexts.

Even if one were to doubt the combined impacts of clocks, latitude and season, which I put forth as the probable origin of many errors, the errors that occur in the *halakhic* literature cannot simply be denied.

Depression Angles¹⁶

Before the 16th century, astronomical events were approximated with basic methods for estimating intervals of time, such as the time to walk a specific distance. We will assume, as has been scientifically established, that **depression angles**, a term explained below, provide a precise measurement of illumination, akin in accuracy to the way a watch measures time.¹⁷ Before it rises and after it sets, when the sun is only a few degrees below the horizon, illumination from the sun is still observable. The level of illumination is measured by a depression angle.

A depression angle¹⁸ measures how far below the horizon the sun appears to have descended; a larger angle indicates that the sun is further

¹⁶ I am not certain of the earliest reference to depression angles in the halakhic literature. *Melamaid Le-Ho'el* 30 contains a brief discussion. *Ha-Zemanim Be-Halakha* by R. Benish, p. 453, shows a depression angle chart constructed for R. Naftali Tzvi Berlin; I do not know if or how it was used by him. As mentioned, throughout *Bein Ha-Shemashot* R. Tukatzinsky covers various aspects of depression angles. In *Halakhic Times* by Prof. Leo Levy, a formula to compute depression angles is given. That book, published 12 years after the death of R. Tukatzinsky attributes the formula to R. Yehial Mikhal Schlesinger, an author of several *hiddushim* on *zemanim*; I am not aware of his role in the development of depression angles in *halakha*. He died in 1948.

¹⁷ However, unlike clocks, depression angles have no apparent downside. Those who doubt the connection between depression angles and *Hazal's* use of more visual definitions for levels of darkness may not find much of this essay useful.

¹⁸ Similarly, albeit without the precision, *Hazal* used terms like *mi-she-yakkir*, *hikhsif ha-elyon*, the appearance of small/medium stars, etc., all of which relate

below the horizon, which would mean less visible light from the sun. If a depression angle of, say, 12 degrees occurs before sunrise at 4:30 AM in London and 4:50 AM in New York, one can be certain that the amount of illumination from the sun is the same at those two times.¹⁹

A small depression angle corresponds to a significant amount of illumination still coming from the sun even though the sun is below the horizon. After sunset, the level of illumination decreases in mirror image to illumination approaching sunrise; these levels of illumination can be accurately measured by depression angles.

At a depression angle of around 5–6 degrees, the *halakhic* end of a day as specified in the Talmud occurs;²⁰ a depression angle of around 11–12 degrees corresponds to the point of *mi-she-yakir*. In between, at a depression angle of 8.5 degrees, as is typically practiced currently, *Shabbat* concludes.²¹

Translating *zemanim* into a depression angle is neither always straightforward nor undisputed. For certain *zemanim*, *alot ha-shahar* for example, the only basis available is the level of illumination at the beginning of the daytime period that normally corresponds to an average time to walk 4 *milin*. What that average time is, 72 and 90 minutes being the two most prevalent opinions, corresponds to depression angles of approximately 16 and 20 degrees, respectively.²²

It is a complex matter to determine the transition point between days of the week and the end of *Shabbat* according to the *geonim*, either at a biblical level or in current practice incorporating various *humrot*. Fortu-

to the degree of darkness or equivalently the amount of residual illumination from the sun.

¹⁹ As mentioned, depression angles were popularized by R. Tukitzinsky in his work *Bein Ha-Shemashot* and by Leo Levi in his book *Halakhic Times* (Jerusalem, 1967). In recent times, most online internet sites that provide *zemanim* (as well as many printed calendars) use this methodology extensively, albeit on occasion use of depression angles is disguised by use of terms involving *kokhavim*.

²⁰ That point is relevant according to many *poskim* to determine the end of a rabbinic fast.

²¹ The important depression angles, the end of *Shabbat* and *alot ha-shachar* were set in the calendar of *Yerushalayim* under the direction of R. Tukatzinsky. Dispute was rather minimal, amounting to less than 1/2 of a degree. Other depression angles were established by converting previous *pesakim* into depression angles, *mi-she-yakir* being a good example. *Mi-she-yakir* and the end of *Shabbat* on a biblical level, absent any stringencies is still broadly disputed, as can be seen in chapters 23 and 50 respectively of *Ha-Zemanim Be-Halakhab*.

²² Interestingly, modern scientific instruments can only detect light from the sun at a depression angle of about 18 degrees or less.

nately, following R. Yeḥial Miḥal Tukatzinsky's calendar for Jerusalem, the practiced end of *Shabbat* is almost universally accepted as 8.5 degrees by those who rely on depression angles.²³ Very few *poskim*²⁴ following the *geonim* are more *maḥmir*, the practice of the vast majority of 19th century *poskim* for whom we have calendars (from which a depression angle can be inferred) were more *maikil*. However, the earlier point absent any *ḥumrot* of *ḥashekba* or 3 medium stars is still disputed.²⁵

Given the earth's circular shape, tilt, and rotation, computing depression angles involves spherical trigonometry, which fortunately is not needed for our purposes. Similarly, albeit without the precision, *Ḥazal* used terms like *mi-she-yakkir*, *hikhsif ha-eyon*, the appearance of small/medium stars, etc. all of which relate to the degree of darkness or equivalently the amount of residual illumination from the sun. There is a long-standing *halakbic* dispute pitting the primacy of **darkness** against the **appearance of stars**; which is defining, and which is just a useful indicator?²⁶ I am strongly biased in the direction of darkness as defining, something that was already recognized in *geonic* writings as the cause for the visibility of stars. Since the two are strongly correlated, the dispute is not consequential in the examples that follow.²⁷

²³ MyZemanim.com and every internet site of which I am aware that uses depression angles uses 8.5 degrees as the end of *Shabbat* according to the *geonim*.

²⁴ Even the *Ḥazon Ish* used a depression angle equivalent of 9.4 degrees, approximately 10 additional minutes. See *Ha-Zemanim Be-Halakhab*, chapter 51, p. 534.

²⁵ Remember that we benefit from a significant amount of artificial illumination at night. In areas where artificial illumination is entirely absent, the above depression angles will appear more reasonable.

²⁶ The *geonim* recognized that that darkness enabled stars to be visible. R. M. ha-Meiri and R. E. Wasserman all considered darkness as defining. Most *poskim*, however, including the *Gaon*, viewed stars as defining. See Benish, vol. 2, beginning on p. 496 for additional detail.

²⁷ In my mind, the following represent the strongest arguments in favor of darkness:

1. Early *tannaic* literature speaks almost exclusively of darkness.
2. Darkness causes the appearance of stars that are present but not visible during the daytime period.
3. The *sugyah* about *Teveryah* and *Tzipporri* (Shabbat 117a) strongly implies that darkness is defining. (I found a visit to *Tzipporri* extremely helpful in understanding why the *sugyah* did not choose an elevated location closer than *Tzipporri*, over thirty miles from *Teveryah*.)

One side benefit of relying on darkness is that unlike the number of stars, measuring the darkness of the eastern horizon versus the top of the sky is less subject to light pollution.

Before discussing latitude and season, more details are required on the effect of both latitude and season on when various levels of darkness are reached.

Latitude, Season and Depression Angles

Halakbic zman requires three variables to specify times, namely:

1. The location's latitude,
2. The date of the year, and
3. The desired level of darkness (depression angle)

An intuitive grasp of the impact of each is important. The former two inputs are indisputable facts; the latter requires a *halakbic* determination.

Those mathematically inclined should think of this as a **function of three variables: 1) latitude, 2) date, and 3) darkness level**, where those inputs generate the value of the function, a **number**. That **number is the length of time before sunrise or after sunset**

- 1) at that latitude,
- 2) on that day,
- 3) when the degree of illumination expressed by that depression angle is attained.

Both latitude and date play a critical role. However, until latitudes beyond 40 degrees are reached, maximal seasonal variation is only about 10 minutes. (Seasonal variation will be covered separately.)

To better understand the impact of latitude alone, the following discussion focuses on an arbitrary but **specific** day. For that specific day, the two inputs and a depression angle calculator will tell you how many minutes before sunrise or after sunset that degree of darkness is achieved.²⁸ Achieving that level of darkness will take longer further from

Nonetheless, absent light pollution, about 30 minutes after sunset in Israel there is little practical difference. Given the larger number of *poskim* promoting stars as defining, including the *Gaon* of Vilna, it is hard to be obstinate in maintaining an unrestrained bias for darkness as defining. Nonetheless, support for darkness as defining traces as far back as the times of the *geonim*, who explicitly mention the second point above, several *rishonim*, and R. Elhanon Wasserman in *Kovetz Shiurim Pesachim*, 2.

²⁸ With respect to depression angles one will often hear that the sun **appears**, as opposed to **is**, X degrees below the horizon to incorporate accurately the critical importance of the position, i.e. latitude, of the observer. An observer at different latitudes will perceive the sun differently based on both 1) their distance

the equator. What takes 42 minutes in Jerusalem will take 50 minutes in New York.

Calculations are complicated. Mathematicians will describe the result as non-linear, something that equates to: "It is not simple." It takes 8 minutes longer to reach an equivalent level of darkness in New York with a depression angle of 8.5 degrees than in Jerusalem, situated about 9 degrees further north. If things were simple, i.e. linear, one might guess that it takes about 8 minutes more for every 9 degrees we travel further north from the equator. However, 18 degrees further north in Prague, for example, it takes about **26 - not (a linear) 16** minutes longer.

Prague is further south than the locations of most European Jews living in Poland and Russia. Two observations:

- 1) The time required to reach a level of darkness begins to accelerate even faster as you travel above the 50th parallel.
- 2) Most of European Jewry lived from about 48 to 56 degrees north latitude where change began to accelerate.

Additionally, depression angles have a second complicating factor. Instead of varying latitude, let us hold latitude fixed, at say, 50 degrees, the latitude of Prague. Compare, for example, the numbers of minutes after sunset that it takes to reach depression angles of 8.5 and 16 degrees, the latter number being less than twice the former. On an average day in early May the numbers of minutes for Prague are 58 and 130 respectively, the latter being more than twice the former; a second non-linearity.

As both latitudes and desired level of darkness change, either very careful observation or scientific knowledge is required. It is not at all surprising that such precision was not always exhibited in the *halakhic* literature. Note that at latitudes further from the equator and at greater levels of darkness, the degree of seasonal variation increases as well, as we will see in section 2.

In summary: proper understanding of the impact of latitude and season, and special attention to all attempts to address the three challenges to the position of Rabbeinu Tam are often all that is required to detect most of the errors that occur.

from the equator and 2) whether they and the sun on the same or opposite sides of the equator.

Category 1: Properly measuring the impact of latitude

Errors concerning the effect of latitude date back to latitude's inaugural appearance in rabbinic literature in *Minbat Kohen*²⁹ by R. Avraham Pimental. Despite recognizing that Amsterdam was further from the equator than Jerusalem, R. Pimental concluded nonetheless that at the time of the equinox, even according to the position of Rabbeinu Tam, one waits only 48 minutes after sunset for the end of *Shabbat* in Amsterdam.

How can this be reconciled with the fact that he correctly took for granted that in Jerusalem one waits 72 minutes after sunset according to Rabbeinu Tam? Accurate application of depression angles in Amsterdam would have required that one wait 102, not 48, minutes after sunset around the fall or spring equinox before a level of darkness equivalent to 72 minutes in Jerusalem is met.

Acknowledging this problem, R. Pimental tried to attribute the difference to elevation; Dutch lowlands are commonly emphasized (despite their insignificance relative to sea-level). While elevation plays a role, the effect of elevation is only a rounding error compared to the impact of latitude; Amsterdam is more than 20 degrees north of Jerusalem. Despite that, R. Pimental and many others assumed they were following Rabbeinu Tam's position and nonetheless ended *Shabbat* based on the appearance of the requisite number of stars, something that occurs before 72 minutes in most locations in Southern and Central Europe during almost all seasons of the year.³⁰ Even currently, *poskim*³¹ still make a similar error; while acknowledging the effect of latitude they simultaneously equate the opinion of Rabbeinu Tam to observation of stars as

²⁹ *Maamar sheni*, chapter 4.

³⁰ *Shut Hatam Sofer*, 80 that deals with a baby born 27 minutes after sunset and 25 minutes before the end of *Shabbat*. Thus, despite a claim of maintaining Rabbeinu Tam's position, in Pressburg they waited only (27+25=) 52 minutes after sunset, not 72. R. Yaacov Lorberbaum gives an almost identical *psak*, although his precise wording has been edited in the different *siddurim* where *Derekh Ha-Hayim* was included, a topic I covered in the TuMJ, 2013.

³¹ Most prominently R. Moshe Feinstein and those follow his *pesakim* in this area. See *Iggerot Moshe O.H* (4:62). This specific issue was raised previously by R. Mordechai Willig in Am Mordechai on *Berakhot*, chapter 2, at the very end of the chapter and by R. Dovid Heber in Sha'arei Zemanim, chapter 10, p. 90 about a decade ago.

they perceive it, well before 72 minutes.³² This troubling approach is discussed again in the section on symmetry.

A similar issue arises in situations where the wrong basis for comparison is chosen. *Poskim* who choose Lithuania or Germany as opposed to the Middle East as a base for Rabbeinu Tam's 72-minute position implicitly assume that the Talmud was also describing astronomical conditions in Europe versus the Middle East, something that cannot possibly be correct. This argument is often implicit in the way a *posek* addresses a *sheailah*. Thus, you will find *poskim* arguing that:

- We waited 72 minutes after sunset in Lithuania and we were following Rabbeinu Tam (something often assumed but incorrect).
- At 50 minutes after sunset in New York, the sky is as dark or starry as it was 72 minutes after sunset in Lithuania, (a reasonable observation given that New York is well south of Lithuania).³³
- Hence, in New York those following Rabbeinu Tam need only wait 50 minutes after sunset.³⁴

The above error, as well as that of R. Pimental, occurs consistently in rabbinic literature. Even after acknowledging the importance of latitude, *poskim* go on to apply it incorrectly.³⁵ Another example of this error is R. Feinstein's *psak* that the *geonim's* period of *bein ha-shemashot* ends less than ten minutes after sunset, when 3 stars are not visible under any circumstances.³⁶

³² Note that R. Moshe Feinstein's *teshuvah* in *Iggerot Moshe O. H.* (4:6) during the gas crisis of 1973 allowed early davening, 90 minutes before sunrise; he even considered allowing a yet earlier time based on the use of depression angles, but chose not to follow that path. In contrast his position in *O. H.* (4:62) that *meikar badin* according to Rabbeinu Tam, one need wait only 50 minutes after sunset for the end of *Shabbat* in and around New York. This is another example of problematic reasoning, albeit coupled with broad support in practice, a topic well beyond this essay.

³³ This is a perfect example of indefensible rationale leading to an accurate *psak*, but one based on the position of the *geonim*, not Rabbeinu Tam as claimed.

³⁴ R. Moshe Feinstein in *O.H.* (4:62).

³⁵ Often this can also be considered a logical error.

³⁶ Interestingly, R. Feinstein's 3 applications of this opinion in actual *pesakim* are justified but using a very different conceptual approach. My opinion is that great *poskim*, who have absorbed *halakhic* practice and thought, have an intuitive sense of the correct answer, a unique gift that perhaps results from a special *siyattab di-Shemayah* as well.

For *poskim* who were not aware of or did not acknowledge the impact of latitude, their positions will likely contain errors. Unfortunately, there are many current instances, including websites of famous Jewish organizations,³⁷ where such errors still exist.

In fact, the fundamental position of Rabbeinu Tam must face this issue. It led to the *Gaon's* total rejection of his approach, citing almost no rabbinic sources, basing his arguments primarily on latitude and logic. Ironically, R. Shneur Zalman of Liadi, who also attacks Rabbeinu Tam's position, reasons very differently, citing observation and a host of prior rabbinic sources.³⁸

Efforts to defend Rabbeinu Tam while incorporating the impact of latitude were attempted by R. Dovid Shapiro³⁹ (and several others who developed variants of his position) and by R. Joseph Soloveitchik.⁴⁰ Despite their very different but brilliant approaches, I find both of their unique accounts of Rabbeinu Tam's position highly problematic, a topic well beyond our scope and worthy of its own analysis. Ultimately, given that 72 (or 90, according to other versions of Rabbeinu Tam's position) was meant to apply in the Middle East, at locations throughout the northern United States and central and northern Europe, all further from the equator than the Middle East, those following Rabbeinu Tam should have waited over 100 minutes before ending *Shabbat*. Such practice, with rare exceptions, never occurred.⁴¹

³⁷ The OU website has changed multiple times and slowly improved over the last decade. However, this error still exists. On sites like the OU, dailyhalacha.com and others, a fixed 42 minutes after sunset alternative is still given. Fortunately, such sites appear to be disappearing. However, as reported by R. Hoffman on the *Hirburim* blogsite 9/12/18, 42 minutes after sunset is still used in many calendars.

³⁸ Independent of our subject, but in his epistle, he gives advice on how and when to reproach others on this subject; his advice is well worth following more broadly.

³⁹ R. Dovid Shapiro in *Shut Benei Tzzyyon* (2:16). This complex topic is not pursued further.

⁴⁰ In a *yartzeit shiur* by R. Soloveitchik entitled *Yom Ve-Lailah in Shiurim Le-Zekher Avi Mori*, volume 1. The *shiur* disregards the challenges to Rabbeinu Tam from the *sugyah* on *Shabbat* 34b. What R. Soloveitchik may have been implying is beyond the scope of this article.

⁴¹ As has been widely reported, R. Soloveitchik waited a latitude- and season-adjusted 90 minutes after sunset before performing a biblical level violation of *Shabbat*. See *Am Mordechai Berakhot*, chapter 2.

Category 2: Improperly dealing with seasonality

Poskim can deal properly with seasonality in two fundamentally different ways:

1. A *posek* can use a simple upper bound for a *zman* where too large a number does not create any concerns. Some⁴² treat R. Moshe Feinstein's 50-minute *zman* for the conclusion of *Shabbat* that way.
2. Alternatively, a *posek* can use depression angles; R. Yisroel Belsky adjusted R. Feinstein's 50-minute *zman* using depression angles, as is attested to in his approbation for the website www.myzemanim.com.

To begin with, it is important to recognize that the magnitude of seasonal variation increases both for:

1. Locations further from the equator.
2. Increased levels of darkness. (Thus greater variation in *mi-she-yakir* than in the end of *Shabbat*. The average depression angle for *mi-she-yakir* is approximately 3 degrees larger than the currently prevalent depression angle used to compute the end-time for *Shabbat*.)

For example, the variation in the end of *Shabbat* in Jerusalem is only 6 minutes, from about 36 minutes after sunset near the spring or fall equinox to about 42 minutes after sunset around the summer solstice. On the other hand, the variation in *alot ha-shahar* in Lithuania is infinite. *Alot ha-shahar* is 102 minutes before sunrise at the spring equinox, 120 minutes before sunrise at the winter solstice, and set to *halakhic* midnight during periods of the summer. In periods during the summer, the requisite level of darkness equating to a depression angle of 16 degrees **never occurs**; it **never** gets that dark during the night, something the *Gaon* observed.⁴³ Said differently, illumination from the sun never diminishes to that level neither in the evening nor equivalently in the morning.⁴⁴

The impact on the point of *mi-she-yakir* provides another interesting topic for study. *Psakim* from the Middle East tend to have an earlier

⁴² His yeshiva, *Tiferet Yerushalayim* and my local *Va'ad* of Raritan Valley among others.

⁴³ O.H. 459.

⁴⁴ The extent to which this was neither recognized by *poskim* prior to the *Gaon* nor followed even after the times of the *Gaon* would require its own (lengthy) essay to illustrate.

point of *mi-she-yakir*, often equating to a depression angle of between 13 and 11.5 degrees; *psakim* from European *poskim* are usually less than 11.5 degrees.⁴⁵ It suffices to say, *poskim* from northern Europe need to be read with care in their discussions of this issue. Their views on *alot ha-shahar* and *mi-she-yakir* are clearly linked; a delayed point of *alot ha-shahar* will obviously impact the point of *mi-she-yakir* as well.

Those following the 72-minute position of Rabbeinu Tam should behave equivalently with respect to the end of *Shabbat*, again a practice rarely observed. It is alleged that R. Chaim of Brisk made *havdalah* Sunday morning, recognizing that *Shabbat* ends at midnight, coincident with *alot ha-shahar*, and after he had already gone to bed. Such practice was rare. Interestingly, in Vilna, using a depression angle of 8.5 degrees to compute the end of *Shabbat*, a prevalent practice today, even the position of the *Gaon* requires waiting 95 minutes after sunset to end *Shabbat*, around the summer solstice.

Unfortunately, many incorrect alternatives remain widely prevalent. Two primary *zemanim*,

1. the interval between sunset and the end of a day (including *Shabbat*, currently assumed to be a depression angle of 8.5 degrees) and
2. the interval between *alot ha-shahar* and sunrise (a depression angle of either approximately 16 or 20 degrees)

are used to illustrate. Proper use of depression angles confirms that the shortest intervals for either occur in the spring and fall close to either equinox. The longest intervals occur around the summer solstice. Surprisingly, the interval around the winter solstice is longer than the spring or fall interval, but shorter than the summer interval. Because this was not properly understood, another error going back to R. Pimental⁴⁶ persists until today; it has been repeated in rabbinic literature published in the last 20 years.

While acknowledging that intervals vary by season, instead of variation determined by a depression angles calculator, the error links variation in the interval with variation in the length of the period between sunrise and sunset. With this mistaken approach, the summer interval is

⁴⁵ See the various *pesakim* quoted in R. Benish, *Ha-Zemanim Be-Halakha* chap. 23.

⁴⁶ Without a wintertime observation R. Pimentel assumed the period was 1/15th of the sunrise to sunset period, assuming a linear relationship that conformed to his two points of observation at the spring equinox and summer solstice. See especially *ma'amar sheni*, chapter 5.

lengthened as it should be, but in an imprecise manner. In the winter, the interval is shortened as opposed to lengthened, a very consequential error.

Interestingly, and for reasons I can only suspect, R. Pimental himself advised **against** using the implied winter time reduction when it creates a leniency;⁴⁷ perhaps the observed result did not conform to expectations or, as some might suggest, his counsel is another example of *siyattah di-Shemayah*. To my surprise, the website of an iconic synagogue⁴⁸ upends sage rabbinic advice in a unique and troubling way. It chooses not to lengthen the summertime interval. Instead, the synagogue maintains a slightly stringent spring and fall interval throughout the entire period between the spring and fall equinox, resulting in a minor (and untroubling) leniency during some parts of the summer. However, between the fall and spring equinox, the interval between sunset and the end of *Shabbat* is adjusted **downwards** apparently in a manner proportional to the reduced length of the period between sunrise and sunset. This results in a noteworthy error, perhaps even bordering on a potential biblical violation around the winter solstice according to some opinions.

A large and well entrenched group chooses not to make any seasonal adjustment. If done to promote simplification, as noted, that is a reasonable approach where implemented with care (particularly for the end-time of days of the week, in locations where the variance is not that large).

Often the implementation is entirely indefensible (most often for *alot ha-shahar*), very often in combination with an equally poor approach to latitude, and normally challenged by observation. The clearest and most prevalent example are those who insist that *alot ha-shahar* is **always** 72 minutes before sunrise.⁴⁹ This approach has widespread support and has been defended by known *Roshei Yeshivot*. Using this approach, one can easily end up with *mi-she-yakir* occurring before *alot ha-shahar*, a *halakhic* absurdity of the first order. This was displayed about 10 years ago on the now improved OU website. In years when *shivah assar betammuz* is close to the summer solstice, as it was in 2018, those maintaining an un-

⁴⁷ *Minbat Kohan ma'amar sheni* 5. See *Ha-Zemanim Be-Halakha*, pp. 164–166 for pictures of calendars that violated R. Pimental's advice.

⁴⁸ On the Spanish Portuguese Synagogue website, the time for *habdala* around the winter solstice is listed as 26 minutes after sunset.

⁴⁹ Rabbi Reisman in his lecture "A Dawn's Early Night," October 13, 2007 provides justification and support for this viewpoint that is held by numerous *poskim*.

adjusted time for *alot ha-shahar* and allow eating until *alot ha-shahar* end up allowing one to eat after the time that *mi-she-yakir* may have already occurred in some of the locations where American Jews reside.

Issues involving latitude and season are again addressed in the discussion on fixed intervals and symmetry.

Category 3: Incomplete knowledge of elementary mathematics

We begin with several clearly derivable mathematical facts that are largely overlooked in rabbinic literature. Ramban⁵⁰ as well as many *hakbmai seforad* from his school assert that *plag ha-minhab* occurs at the time to walk 1/6th of a *mil* before sunset.

It is mathematically provable that anyone who asserts that *plag ha-minhab* occurs the time to walk 1/6th of a *mil* before sunset must also maintain a time to walk a *mil* of 22.5 minutes, and **cannot** simultaneously maintain 18 minutes as the time required to walk a *mil*. Nonetheless, the assertion of both

- an 18-minute time to walk a *mil* and
- Ramban's claim about when *plag haminhab* occurs

is made repeatedly in rabbinic literature.⁵¹

Linear equations have only one solution. Ramban's assertion can be expressed as a linear equation, as explained in the long footnote below.⁵²

⁵⁰ *Torat Ha-Adam*, Chavel edition, pp. 251–255.

⁵¹ In fact, R. Hayyim Druk in his sefer *Orot Hayyim* (see chapter 6) tries to prove that an 18-minute time to walk a *mil* and Ramban's claim about when *plag ha-minhab* occurs are consistent, in glaring conflict with the next footnote. The *haskamot* to *Orhot Hayyim* include a short note from R. Menachem Mendel Schneerson and a very lengthy response from R. Tukatzinsky who both disagreed with him. His "proof" is addressed in the next paragraph.

⁵² The only solution to

$$(720 + 8 * X) / 12 * 1.25 = 4 * X + 1/6 * X$$
 Is $X = 22.5$. Note the left side of the equation specifies the time between *plag haminhab* and the end of the day as it is defined in the *gemara*—a *sha'ah zemanit* ((720 + 8 * X) divided by 12) multiplied by 1.25. The right side of the equation specifies the time between *plag ha-minhab* and the end of the day given by Ramban—the time to walk 4 *milin* between sunset and *tzeit ha-kokhavim* added to the time to walk the additional 1/6th of a *mil* by which *plag ha-minhab* precedes sunset. However, if the time to walk a *mil* is 18 minutes as opposed to 22.5 minutes, then *plag ha-minhab* is ((720 + (8 * 18)=)144) / 12 * 1.25 (=) 90 minutes before *tzeit ha-kokhavim* or 18 minutes before sunset, the time to walk

There is no **clear** Talmudic source that specifies by how far *plag bamincha* and sunset are separated; the time to walk a *mil* **or** $1/6^{\text{th}}$ of a *mil* before sunset are **both** reasonable but associate **only** with a time to walk a *mil* of 18 or 22.5 minutes, respectively.

In addition, Ramban and his followers clearly calculate *shaot hayom* following what we currently refer to as the opinion of the Magen Avraham, and not the *Gaon* of Vilna. The unavailability of *seforim* of *hakbemei sforad* may have been a factor before the 19th century; why this observation about calculating *shaot ha-yom* is rarely recognized even today is a mystery. While sins of omission are puzzling, sins of commission are troubling.

There is a flawed “proof”⁵³ that attempts to maintain both the time to walk a *mil* of 18 minutes and Ramban’s position on when *plag baminhab* occurs. The proof mixes use of times of day from sunrise to sunset with times calculated from *alot ha-shahar* until night, *tzait ha-kokhavim*. Thus, an hour is set to 60 minutes on a canonical 12-hour day, counting the time between sunrise and sunset and dividing by 12. However, the time of *plag baminhab* is derived **inexplicably** by subtracting $(1.25 * 60 =) 75$ minutes from *tzait ha-kokhavim* at 7:12 PM, not sunset at 6:00 PM as required. This ostensibly “proves” that 72 minutes and a time to walk a *mil* of 18 minutes is consistent with the opinion of Ramban—*plag baminhab* is 75 minutes before 7:12 PM, at 5:57 PM, which is the time to walk $1/6^{\text{th}}$ of a *mil* (3 minutes) before sunset. Of course, the hour should be $((720 + 144) / 12 =) 72$ minutes long and *plag baminhab* is derived by subtracting $(1.25 * 72 =) 90$ minutes from *tzait ha-kokhavim* at 7:12 PM. It occurs at 5:42 PM, the time to walk a *mil* before sunset.

The reader can verify that this approach does not even allow the calculation of *hatzot*. Subtracting 6 hours from *tzait hakokhavim* sets the time of *hatzot* at 1:12 PM, while adding 6 hours to *alot ha-shahar* sets the time of *hatzot* at 10:48 AM. In addition to its being a mathematical error as shown above (and a logical error as mentioned below), R. Willig⁵⁴ also notes that this “proof” contradicts a *gemara* asserting that the interval from *minhab gedolah* to *minhab ketanah* is 3 hours.

a **full** mil. Only a time to walk a *mil* of 22.5 minutes’ results in *plag baminhab* occurring the time to walk $1/6^{\text{th}}$ of a *mil* before sunset.

⁵³ *Orot Hayyim* by R. Hayyim Druk, chapter 6, both in the very beginning of the chapter and in (6:6).

⁵⁴ See *Am Mordechai, Berakhot* chapter 2.

Beware that what is unique is the attempt to defend a widely made error of the past. The existence of such errors necessitates carefully checking calculations for their consistency.

In addition to not recognizing that Ramban's specification of the time of *plag ha-minhah* implies maintaining a time to walk a *mil* of 22.5 minutes, a $(4 * 22.5 =)$ 90-minute versus a $(4 * 18 =)$ 72-minute interval, is also not widely recognized as the opinion of Ramban. The very limited availability of *seforim* written by the *hakbmai seforad* is probably again a major contributing factor.

Inexplicably, however, even well after the literature of almost all *hakbmai seforad* became widely available by the middle of the 19th century, 90 minutes, 1/8th of a 12-hour day, is often called a “*Brisker ahtel*,” somehow attributing a period of 90 minutes not to Ramban and his school, but to *Brisker* insight. While the *Shulhan Arukh* and most *ahronim* adopted both 18 and 72 minutes as opposed to 22.5 and 90 minutes, several major *ahronim* from the 17th through 20th century argued strongly for 22.5 and 90 minutes.⁵⁵

Additionally, Ramban and *hakbmai seforad*, living in the 12th and 13th centuries, were also staunch supporters of what is known currently as the opinion of Magen Avraham (17th century) and previously attributed to R. Israel Isserlein (15th century), based on several comments in his *sefer Trumat Ha-Deshen*.⁵⁶ Why that trivial observation is also largely absent from rabbinic literature, even currently, remains a mystery.

Category 4: On rare occasions, even the logical reasoning of important *poskim* can be impenetrable

The mathematical error attempting to uphold both Ramban's time for *plag haminha* and a time to walk a *mil* of 18 minutes outlined above is a logical error as well; the examples that follow are subtler.

⁵⁵ Those include R. Yaacov Reicher's commentary *Hok Yaacov* on *O. H.* 459 and R. Moshe Sofer in his famous *teshuvah* 80 on *brit*. The position of the *Gaon* is disputed. Both R. Yisrael Meir Kagan and R. Yehoshua Karelitz support 72, not 90 minutes, while both R. Feinstein and the R. Soloveitchik support 90 minutes. I have read but not verified that the *Gaon* explicitly writes 72 minutes in a *midrashic* context while 90 minutes is (strongly) **implied** in various sections of his commentary on the *Shulhan Arukh*. The extent that either source records the words of the *Gaon* accurately and further analysis of the *Gaon's* position is beyond the scope of this essay.

⁵⁶ Throughout *Minhat Kohan*, R. Pimental refers to what we call the opinion of Magen Avraham as the opinion of *Trumat Ha-Deshen*.

There is a subtle error, one that might produce an erroneous ruling, that has occurred multiple times in *teshuvot* of world famous *poskim* over the last 200 years. Typically, the practiced time to end *Shabbat* includes several stringencies making it many minutes after the biblical end of *Shabbat*. We typically await 3 smaller, adjacent stars versus the 3 medium stars mandated at the biblical level in the Talmud. We wait until $X + Y$ minutes after sunset, where X is the time *Shabbat* ends biblically, and Y minutes are added to be cautious, given the stringencies associated with *Shabbat* violations.

If we subtract a reasonable length for the period of *bein ha-shemashot* from the time that 3 medium stars appear, X minutes after sunset, we can argue cogently that we are at the beginning of the *bein ha-shemashot* period, well before the point of transition to the next day. However, subtracting the length of a period of *bein ha-shemashot*, from the time that three small stars appear, $X + Y$ minutes after sunset, results in a time that is certainly past the beginning of *bein ha-shemashot*.

In those cases where a short period of *bein ha-shemashot* is used, we may even be past the biblical point of transition to the next day, particularly during the spring and fall.⁵⁷ The structure of the argument is what is not logical; whether or not it results in an errored *psak* is not our current focus. A reader should examine *teshuvot* using such a construct carefully; if a stringency is desired, using an earlier time closer to the biblical end of *Shabbat* and/or a lengthened time for the period of *bein ha-shemashot* is required. Discovering errors of this type often require extensive familiarity with rabbinic literature; such opinions exist with respect to various subjects like *Shabbat*, *zemanai tefillah* and *brit milah*.⁵⁸

⁵⁷ See for example, R. Dovid Ribiat, 39 *Melakhot*, who uses 40 minutes after sunset repeatedly in Volume 1 Section 8, well past the biblical start of *Shabbat* in the weeks surrounding the fall or spring equinox. His basis, *Iggerot Moshe* (4:62) combines a *humrah* with respect the practiced end of *Shabbat* with a short/truncated time to walk a *mil*.

⁵⁸ R. Moshe Sofer allowing a *milah* for a baby born 25 minutes after sunset on Shabbat afternoon in *teshuvah* 80. R. Moshe Feinstein allowed *amira leikum* forty minutes after sunset in *Iggerot Mosheh O. H.* 4:62. R. Yisroel Belsky discussed his objections a late *minhag* in *OU Daf Hakeshrus*, No. 5, SHEVAT 5769 / February 2009, footnote 27. All three *poskim* used an argument similarly structured. In my opinion, R. Feinstein's *psak* is the most questionable, particularly in the spring and fall when the end of *Shabbat* is a significant *humrah*. This is then coupled with his subtraction of a shortened time of only 10 minutes for the time to walk $\frac{3}{4}$ of a *mil*. R. Sofer's *psak* given

1. for a date very close to the summer solstice,
2. using a very lenient point for the end of *Shabbat*, and

Another logical error is of the form: If one maintains A, then he must also maintain B. Normally such assertions are supported by compelling *halakbic* reasoning, but not always. On occasion, someone wanting to demonstrate B demonstrates A and assumes the implication is valid, particularly when those maintaining A normally maintain B as well. If not a logical at least a *halakbic* dependence must be proven. Each instance of an argument so structured requires careful evaluation.⁵⁹

We disregard the fixed calendar despite the existence of some logical and/or mathematical issues. However, without having to delve into the complexities of the calendar *per se*, we can highlight one issue concerning not the calendar, but how time was specified: **the choice of 1,080 *halakim* in which to divide an hour**. The reason provided for 1,080 is that 1080 is divisible by 2, 3, 4, 5, 6, 8, 9, 10, and 12. That explanation is questioned⁶⁰ given that a smaller number, 360, already has all those same divisors.

Of course, there is no need for an answer. Today we divide an hour into 3,600 seconds; in Talmudic times they were already able to divide an hour into 1,080 parts, called *halakim*. What the existence of all those divisors explain is why 1,000, 1,100 or 1,200 were not chosen in place of 1,080. Why 360 was not chosen is patently obvious—it is not as accurate; 360 divides a minute into 6 parts while 1,080 divides a minute into 18 parts. We ought to admire as opposed to question the remarkable precision *hazal* exhibited.

3. coupled with a time to walk $\frac{3}{4}$ of a *mil* of almost 17 minutes is arguably reasonable, though hardly practiced currently. R. Belsky's *psak* requires its own essay to analyze.

⁵⁹ A version of this type of reasoning is deriving the time to walk one *mil* from a position on the length of the interval between *alot ha-shahar* and sunrise, assuming that that interval equals the time to walk four *milin* without proving that the individual quoted considered that ratio to be four. According to many *rishonim*, four is a definitive conclusion of the *gemara* in *Pesahim* 94a. However, some *rishonim* may have considered the ratio not to be four, but five or even three, 90 minutes = 5* 18 minutes or 72 minutes = 3 * 24 minutes. Unlikely for sure; but something that must be demonstrated and not assumed.

⁶⁰ See for example footnotes on *Kiddush Ha-Hodesh* (6:2) in the *Rambam Le-Am* published by Mossad Harav Kook, which mentions *Rutx Le-Mishne* by a R. Tzvi from Ektzin who raises this issue on p. 36 of his commentary on *Arikkhin*. While R. Tzvi (from Ektzin) leaves the question unanswered, the proposed answer given in the footnote that 1080 but not 360 is divisible by $(6*9) = 72$, while true, is also in my mind irrelevant. Other editions of *Mishneh Torah* by R. Kapach or the one published by Moznaim provide additional sources which question the need for 1080.

Category 5: Fixed time intervals and the decline of observation

There are intervals that are self-evidently fixed, as well as intervals for which a fixed value can be argued cogently. The length of time to walk a *mil* or eat a quantity of food are clearly fixed intervals, although their precise value is subject to dispute. Whether the length of the *bein ha-shemasbot* period (as strictly defined in Talmudic literature, not colloquially) is fixed or dependent on latitude and season is a representative example of a debatable issue, and beyond the scope of this essay.

Despite my conviction that almost all intervals tied to astronomical events are unfixed and vary by latitude and season, in a short paper I cannot seek to convincingly confront the many prominent rabbis committed to fixed intervals.⁶¹ I will cover only two examples, both based on *zemanim* in Krakow where observation patently challenges a fixed alternative—*babush maibish* in rabbinic terms.

While the understanding of latitude and season is more widespread, for reasons that require a historical study, the commitment to fixed intervals remains strong. (Some even claim that a commitment to fixed intervals increased in the last century. If I had to guess, the proliferation of watches may be a contributing factor.) From Chassidic groups to *chareidi yeshivot* to modern orthodox communities, adherence to a fixed interval of 72 minutes for either Rabbeinu Tam's position for the end of *Shabbat* or for the time of *alot ha-shahar* is widespread.

Migration to North America, where seasonal variation and the latitudinal adjustment required is less apparent than in central Europe, may be a contributing factor. Consider New York and Krakow, approximately 40 and 50 degrees north latitude, (versus Jerusalem at approximately 30 degrees north latitude). In New York⁶² *alot ha-shahar* varies between 82 and 110 minutes before sunrise, while in Krakow that variance is 97 to 195 minutes before sunrise. A fixed 72 minutes in New York is clearly less challenging. A possible explanation of why this reality in Krakow

⁶¹ A recent book length attack on fixed intervals, *Dvar Yom* by R. Dovid Braunschweig chose not to name the rabbinic icon being challenged.

⁶² Assuming the commonly used *halakhic* time of 72 (as opposed to 90) minutes around the equinox in the Middle East as the time of *alot ha-shahar*. If instead of using *halakhic* times like 72 minutes we were to measure scientifically, the point of first light would be about 10 minutes earlier. Given that *Hazal* often disregarded miniscule quantities not visible to the human eye, this discrepancy is perfectly well understood. The first point of light from the rising sun is visible scientifically before the later point, *alot ha-shahar*, when the light can first be perceived by humans; at a yet later point, *mi-she-yakir*, the light becomes useful.

and in much of Europe generally did not eliminate the use of fixed intervals will be proposed in the next section on symmetry.

Using Krakow, let us look at two examples:

3. 72 minutes before sunrise as the time of *alot ha-shahar*, and
4. 42 minutes after sunset as the time *Shabbat* ends according to the *geonim*.

Near the summer solstice, sunrise and sunset are at 5:24 AM and 7:52 PM, respectively. Even the strictest (European) view of *mi-she-yakir*, equating to a depression angle of 10.2 degrees, occurs at 4:16 AM, only 4 minutes after a fixed 72 minutes calculation of *alot ha-shahar*. Of course, further north the error increases dramatically; we will soon again encounter our favorite *halakhic* absurdity, *mi-she-yakir* before *alot ha-shahar*. Such an outlandish error was made in the 21st century on a major orthodox organization's website.

Around the summer solstice in Krakow, 42 minutes after sunset occurs at 8:34 PM is 2 minutes after a reasonable time for the end of the day absent any stringencies at 8:32 PM. Travel further north to Warsaw and maintain 42 minutes, you are almost definitely violating *Shabbat* prohibitions at a biblical level. By the time you get to Vilna, any remaining doubt over biblical violation has long since faded.⁶³

It is unlikely one can find a coherent rationale that would use fixed intervals in New York but not Krakow. Similarly, while many use depression angle-based intervals to determine the end of *Shabbat* according to the *geonim*, they then revert to using fixed intervals for *alot ha-shahar*. Such behavior is scientifically incomprehensible; the variation around *alot ha-shahar* is far greater than the variation around the *geonim's halakhic* end of a day. I can find no rational basis for such behavior; the next section suggests an errored *halakhic* basis.

Category 6: Symmetry: The good, the bad and the questionable

Note that both the terms “day” and “*yom*” refer to both the daytime period, as in “during the day,” as well as the day of the week, as in “*yom ha-shishi*.”

⁶³ The OU calendar has exhibited various errors over the years as they appear to be moving slowly to a more supportable position. Note their still troubling use of a fixed intervals for *alot ha-shahar* in their *erev Pesah zemanim* where the opinion of Magen Avraham is given as **always 36 minutes** prior to that of the *Gaon*.

The basic text in *Pesahim* 94a establishes explicit symmetry between *alot ha-shahar* and an evening endpoint. Those two points delimit the daytime period. The significance of that evening endpoint to the day of the week, has been **the central dispute dominating zemanim for the last 900 years**. For those who follow the *geonim*, that evening endpoint likely has **no** or at best minimal significance.⁶⁴ For the *geonim*, the transition to the next day happens well before the end of the daytime period; **for the *geonim*, the transition point between days and the end of the daytime period are emphatically not the same**.

On the other hand, Rabbeinu Tam unequivocally **equates** the **end** of a **day** with the **end** of the **daytime period**. Thus, the interval between *alot ha-shahar* and sunrise equals in length the interval between sunset and the end of *Shabbat* or any other day. This undeniable symmetry in the position of Rabbeinu Tam has been implicitly denied for at least the last 400 years, beginning again with R. Pimental and including many of our most venerated *poskim*.⁶⁵ In one form or another, as we have seen, it is not uncommon for *poskim* to maintain a morning interval between *alot ha-shahar* and sunrise that is longer than the period between sunset and the end of the day, while, inexplicably, also insisting their practice follows Rabbeinu Tam.

Oddly, this clear symmetry may have caused an even more obvious problem even when it was acknowledged. Those who maintained a fixed 72-minute interval in the evening felt forced to maintain a corresponding fixed 72-minute interval in the morning; that resulted in a very late point of *alot ha-shahar* (and as a likely result, a delayed point of *mi-she-yakir* as well). Thus, as mentioned earlier, the very early **observable** point of *alot ha-shahar* in Northern Europe, often **hours before sunrise**, was rarely acknowledged; *alot ha-shahar* often began a fixed 72 or 90 minutes before sunrise. (Perhaps confusion over an increased level of street illumination played a role as well, but I assume the combination of symmetry and fixed intervals were the primary culprits.)

⁶⁴ Only in the last century have some 20th century *Brisker* conceptual Talmudists proposed some potentially meaningful *halakhic* significance to the end of the daytime period; the *Gaon* did not refer to any.

⁶⁵ *Poskim* who like R. Pimental adjusted Rabbeinu Tam's end of *Shabbat* to less than 72 minutes nonetheless maintained *alot ha-shahar* as 72 or 90 minutes. These include, for example, R. Lorberbaum in *Derekh Ha-Hayyim*, in the chapter entitled *zeman hadlakat ha-ner ve-kabalat Shabbat* and R. Sofer in *teshuvah* 80. I know of no instance when the practiced time of *alot ha-shahar* anywhere north of the Middle East was less than 72 minutes before sunrise.

Ramban, a follower of Rabbeinu Tam's opinion, goes one step further; he calculates the 12 hours of the day using the two endpoints of the daytime period. Like those who calculate the 12 hours of the day from sunrise to sunset, each hour of the day is of the same duration. Not a surprising insight or even a noteworthy one, many will assert.

Nonetheless, there is a (mysterious) tradition, maintained explicitly by R. Feinstein⁶⁶ and strongly implied by several other *poskim*,⁶⁷ that afternoon hours can be **shorter** than morning hours, something that clearly differs from both those calculating hours between sunrise and sunset and those following Ramban's method of calculating the hours of the day from *alot ha-shahar* to its evening equivalent. Surprisingly, there is explicit support for this view from major *aharonim*. This is often overlooked.

Often related, but **not** logically equivalent, is *hatzot* time, both during the day and the night. Here again, almost everyone will assume that symmetry as well as direct observation forces *hatzot* to the day's midpoint. But that assumption about *hatzot* is inexplicably challenged by two of last century's most followed *poskim*.⁶⁸

The existence of symmetry in some cases and the possibility that it may not exist in others is a topic that is often treated intuitively rather than rigorously in rabbinic literature. Readers beware: This complex topic impacts several (primarily technical) areas in *zemanim* and will not be covered further.⁶⁹

⁶⁶ To be entirely accurate R. Moshe Feinstein also claims in *Iggerot Mosheh* O. H. 2:20 that afternoon hours can be **longer** than morning hours, something for which I cannot find **any** support.

⁶⁷ The method of any *posek* who miscalculated *hatzot* (including R. Nosson Adler, the Ben Ish Hai and R. Yosef Chaim Sonnenfeld, among others) slightly adjusted to calculate *hatzot* correctly, would result in shorter *halakhic* hours in the afternoon. For more details, see my entry on the *Seforim Blog* on lighting candles 40 minutes before sunset in Jerusalem, recently published.

⁶⁸ R. Moshe Feinstein with respect to *hatzot bayom* and R. Shlomo Zalman Auerbach with respect to *hatzot ba-lailah*. There are many prior examples of similarly troubling positions, including R. Nosson Adler, whose early point of *hatzot* is still in use in Zurich, and the Ben Ish Hai. See as well any of the accounts of the famous dispute between R. Sonnenfeld and R. Tukatzinsky adjudicated about 115 years ago in *Eretz Yisroel*.


⁶⁹ This topic, including issues mentioned in the two previous paragraphs, is addressed more comprehensively in the afore-mentioned *Seforim Blog* article.

Conclusions

There is no area of *halakha* outside *zemanim* where both *psak* and its rationalization are subject to such **tergiversations**, borrowing a word used by R. Aharon Lichtenstein in this context.

Despite significant disputes going back to the time of Rabbeinu Tam, practice has often been less divisive than the options the theory might imply. We noted the numerous opinions which claimed to follow Rabbeinu Tam but did not wait 72 minutes or more to end *Shabbat*. More importantly, at no time do we have conclusive evidence of large groups of people starting Shabbat close to a full hour after sunset. Well after sunset, certainly; close to an hour after sunset, dubious.

Nonetheless, it is impossible to study this complex topic without realizing how auspicious our history has been. Today, while diversity still reigns, and the diverse positions mentioned in this essay are still practiced in different communities, the overall direction appears to favor convergence in the long term. It is worth repeating that the science around depression angles has strong support across all parts of the *halakhic* spectrum. More importantly, depression angles are believed by many to capture *Hazal's* notions of darkness and light akin to the way a watch measures time.

Changing direction and focusing on *psak*, I want to illustrate the ability of depression angles to provide practical guidance. In all such *halakhic* matters, I suggest you consult with your local/preferred competent *posek*. However, if the *posek* disagrees with what I advise below or argued above, I would appreciate hearing about it. 

APPENDIX: *Zemanim* to live by:

If the above explication seems reasonable, then the following *zemanim* represent preferred practice.

Alot ha-shahar. I assume that *alot ha-shahar* corresponds to the first light of the day that people can perceive, probably a bit later than can be observed by current technology. That logic makes the preferred *zman* 72 minutes in the Middle East around the spring and fall equinox, **adjusted** elsewhere by latitude and everywhere by season. Second choice is 90 minutes in the Middle East around the spring and fall equinox, similarly adjusted by latitude and season. The 90-minute *zman* might represent a point slightly before illumination from the sun is visible, when people may rise in anticipation of the day.⁷⁰ These two views equate to depression angles around 16 and 20 degrees, respectively.

Mi-she-yakir. A depression angle of around 11.5 degrees, although some Middle Eastern *poskim* were more lenient. The suggestion by R. David Tzvi Hoffman (a very early proponent of depression angles) to dispense with *mi-she-yakir* given our ability to specify *alot ha-shahar* precisely has received minimal support from *poskim*.

Sunrise. As per your local paper, and please no atomic clocks out of respect for tradition.

Hours of the morning. Either from sunrise or a depression angle calculation of *alot ha-shahar* until midday/*hatzot* divided by 6, what are currently termed the opinions of the *Gaon* and Magen Avraham respectively. What is sadly called Magen Avraham *kenahug*, which uses a fixed 72-minute interval, something I consider upsetting given his well-deserved reputation, should be avoided.

Midday/hatzot. as observed. The opinions of great *poskim* that differ would require extensive *halakhic* discussion; it is indefensible and likely to have originated in a clock inspired error.

Hours of the afternoon. Midday to sunset (the opinion of the *Gaon*) divided by 6 is provided in all calendars. Alternatives that calculate to a later point in the evening (the opinion of the Magen Avraham) require a more elaborate *halakhic* discussion.

⁷⁰ *Zemanim Ke-Hilkhasom* by R. Dovid Yehudah Boorstyn, p. 348 suggests that 90 minutes, the view of Ramban and used as a basis for a *beter* by R. Feinstein, ought not to be relied on *le'heter* because it conflicts with the first light of day as observed by science. I find this view disturbing for many reasons.

Sunset. As defined in your local paper.

The point of transition between days. The equivalent of three medium stars equates to a depression angle between 5 and 6.5 degrees. For biblical obligations, *poskim* typically add 1 to 1.5 degrees corresponding to the appearance of three small stars. The end of *Shabbat* is typically 8.5 degrees, a bit stricter than what was observed historically, intended to correspond with the appearance of three small, adjacent stars. Those who wish to follow Rabbeinu Tam ought to wait for a depression angle of about 16 degrees. This practice is almost never observed except by some *Briskers* including R. Joseph Soloveitchik, who waited yet later until 20 degrees (the so-called *Brisker ahtel*, something you now should call Ramban's *ahtel*) before commencing with any activity biblically forbidden on *Shabbat*.

Observance is no harder than that; study remains more challenging. Hopefully awareness of the errors that arise in each of the six areas will help guide readers as they attempt to decipher the challenging texts in this area's extensive *halakhic* literature.