

Preamble:

Several more modern works, including *Mishnayot Mevuarot* by R. Pinhat Kehati, a commentary and accompanying illustrations by Yisrael Yitzchak Bankier and a comprehensive mathematical treatment by Dr. Moshe Koppel,¹ make another review of the entire tractate of *Kinnim* unnecessary.² Instead, this monograph focuses on just four topics where a mathematical perspective is helpful:³

1. Rambam's suggestions concerning specific cases in the first chapter of *Kinnim* that result in more valid sacrifices.
2. Rambam's and Raavad's approach to the first three *mishnayot* of the second chapter.
3. A precise definition of the term – *merubah* - that is fundamental to understanding Mishnah (3:2).

¹ Moshe Koppel, *Sefer Kinnim: A Mathematical Commentary on Tractate Kinnin* (Jerusalem, Israel 1998.)

² These three works as well as other contemporary works largely follow the commentary of the Rosh and R. Ovadiyah of Bartenura.

³ This monograph purposely avoids mathematical notation unless unavoidable, and only when such notation adds precision that would be difficult to express otherwise. My preference is to concentrate on classical mathematical reasoning. I would like to believe that this approach is more consistent with the way *tannaim* and *rishonim* may have thought. Even the one short formal proof uses modern mathematics largely for structure not content.

4. The *maḥloket* Raavad and R. Zerahyah ha-Levi, the *Ba'al Ha-Moar*, on how to define the phrase - *ve-hi mishaleket beinaihem* - that occurs at the end of the fourth Mishnah in the third chapter, and the commentary of R. Yehoshua Heller who introduces modern combinatorics in his approach to that phrase.

All but the third topic focus on approaches to the tractate of *Kinnim* different from those normally given. The first two topics focus on Rambam's and Raavad's interpretations, and relate, primarily, to the first two chapters; those two topics will be considered concurrently. In those two chapters, while differing in many details, interpretations rooted in the approach of R. Ovadiah and earlier *rishonim* particularly the *Ba'al Ha-Moar* and *Rosh*, have been widely followed by *aḥronim*. Despite (minor) differences among these authorities, their collective approach is referred to as the **standard interpretation**, disregarding those (minor) differences. Differing fundamentally from this standard interpretation is the approach of Rambam; Raavad's approach, while agreeing on fundamental principles, differs primarily on assumptions vis-à-vis the cases in the second chapter.

In Rambam's case both the principles (the what) and their potential rationale (the why) are addressed. Otherwise, we avoid delving into rationale, concentrating only on the circumstances, principles and *halakhic* rulings.

The fundamental issues with both Rambam's unique approach to the first and (to a lesser extent) second chapters of the tractate,⁴ as well as the challenges to the standard interpretation of the third Mishnah of the second chapter are well known.

- In the first chapter, covering the **complete intermingling** of incompatible types of nests, **twice** Rambam provides a practical suggestion that creates additional valid sacrifices, suggesting an option that is not provided by the Mishnah. In addition to the prescribed *halakhic* approach provided, Rambam⁵ introduces an alternative option, one that Raavad and almost all commentators consider invalid. Prefaced by the phrase “*ve-yirah li*,” it would appear to me, Rambam provides an option that evokes this sharp criticism: “a major error...how could one who consulted be allowed to bring invalid sacrifices to the altar.”⁶ Raavad's approach to the first chapter follows the standard interpretation.

⁴ When lacking any indication to the contrary, Rambam's Mishnah Torah and his *Peirush Ha-Mishnayot* are treated as mainly consistent. Therefore, the ninth chapter of *Pesulei Ha-Mikdashin* in Mishnah Torah and the Rambam's explanation of the second chapter in *Kinnim* in his *Peirush Ha-Mishnayot* are assumed identical. In fact, rather than being repetitive in explaining parts of the ninth chapter of *Pesulei Ha-mikdashin*, R. Kapach refers readers of his commentary in *Peirush Ha-Mishnayot*. However, Rambam's innovations in the eighth chapter of *Pesulei Ha-Mikdashin* are introduced by the phrase “*ve-yirah li*,” which is normally assumed to indicate that Rambam did not have a firm source for his point of view. Not surprisingly, these innovative approaches do not appear in his *Peirush Ha-Mishnayot*. Thus, **when discussing Rambam's views on the first chapter of *Kinnin*, his point of view in the eighth chapter of *Pesulei Ha-Mikdashin* is assumed.**

⁵ *Pesulai Ha-Mikdashin* 8:3, 4, and 6.

⁶ Raavad on Rambam *Pesulai Ha-mikdashin* 8:3; his comment on 8:6, which contains another suggestion absent from the Mishnah, where interestingly the phrase “*ve-yirah li*” is absent, is yet more strident.

- The first three *mishnayot* of the second chapter deal with undesignated *Kinnin* of diverse sizes, which are only **partially intermingled**. This set of *mishnayot* culminates in a challenging Mishnah, *Kinnim* (2:3), where birds fly multiple times between adjacent *kinnim* in a precise sequence. All who follow the standard interpretation struggle with *Kinnin* (2:3.) **In one way or another, all conclude that the Mishnah is stating a general *halakhic* conclusion, which albeit slightly imprecise, provides a simplified, easy to remember *halakhic* rule.**⁷ Unlike the standard interpretation, we demonstrate that both the approaches of Rambam and Raavad to the 3rd Mishnah of the second chapter, while differing from each other relative to the principles each maintained and the assumed circumstance, are provably precise. A constructive proof demonstrates that the *halakhic* rulings of *Kinnim* (2:3) follow precisely from both Raavad's and Rambam's principles. As well, using mathematical induction, a generalization of Rambam's formulation of these *mishnayot* is formally proven based on his principles.⁸

⁷ The *halakhic* ruling given in the Mishnah is described as only a useful approximation. Others explain that the Mishnah is stating a *halakha* that while normally correct, invalidates certain sacrifices unnecessarily to articulate a rule that could be more easily understood and followed. Some go so far as to consider the result a *gezairah*. All major commentators who follow the standard interpretation recognize the imprecision.

⁸ The proof appears at the end of the section on Rambam and Raavad's approach to accommodate those with an unnatural fear of mathematics. Even the formal proof avoids excessive formalisms. The informal proof provided is close to the way classical commentaries on *Kinnim* expressed themselves and *tannaim* likely reasoned. While the statement of the theorem is important, skipping the formal proof will not detract from one's understanding.

It would be presumptuous to claim this paper provides a definitive rationale for Rambam's position. R. Moshe Reich succinctly comments⁹ that Rambam's unique rulings in the first chapter of *Kinnim* have never been completely explained.¹⁰ Nonetheless, the approach developed provides a comprehensive basis and plausible rationale to address Rambam's interpretation of the two initial chapters of *Kinnin* addressing the major challenges raised.

The third topic, the term *merubah* in *Kinnim* (3:2), was precisely defined and formally proven in its most general form by Dr. Phillip Riess;¹¹ I commented suggesting an alternative approach to conceptualize the term *merubah* and prove the correctness of the Mishnah.¹² This monograph includes an elaboration on that comment that further clarifies the term *merubah* in a more intuitive, but rigorous approach.

The fourth topic reviews the *maḥloket* between Raavad and the Ba'al Ha-Maor specifying what the term "*ve-hi miṣḥaleket beinaihem*" in *Kinnim* (3:4) entails in financial terms and then analyzes in depth the novel mathematical approach of R.

⁹ *Rambam Le-am*, Mossad Ha-rav Kook, *Pesulei Ha-Mikdashin* 8:3, Jerusalem, 1963.

¹⁰ However, R. Yehoshua ben Aharon Heller, Av Beit Din of Telz, in his commentary on the Mishnah, *Mei'aiyanei Yehoshua*, provides staunch support for Rambam's approach to the second chapter. He did not address the issues in Mishnah Torah concerning the first chapter of *Kinnim* as Rambam's commentary on the Mishnah does not raise any of those issues, as noted in footnote 2.

¹¹ Philip Riess, "A mathematical proof of *Kinnim* 3:2," *The Torah u-Madda Journal* 9 (2000) 58-75. This result is often illustrated, but not formally proven.

¹² *The Torah u-Madda Journal* 10 (2001) 189-192, where I used a bin packing model. Despite its modernity, bin packing provides a precise formulation of *Kinnim* 3:2, where an arbitrary number of intermingled sacrifices were brought without consultation.

Heller. In addition to a subtle mathematical error, R. Heller's innovative approach raises both logical and *halakhic* issues in his use revolutionary use of combinatorics. Only *Kinnim* (3:4) addresses how financial liability is shared between owners of intermingled nests, where additional birds must be acquired and sacrificed.

Acknowledging the mathematical gifts, I inherited from my mother A'H, this monograph is dedicated to my late father R. Yonah ben R. Yehudah A'H who died on the 21st of Elul; I started to study *Kinnim* during the year following his death and completed a complete draft on his 22nd *yartzeit*.

Introduction to *Kinnim*:

Kinnim discusses bird sacrifices that have become intermingled. Bird sacrifices are either *olot* or *hatta'ot*, each following a different sacrificial procedure.¹³

While there are instances of an obligation to bring only one or more *olot* to fulfill a *neder* or a *nedavah* or even to sacrifice one or more *hatta'ot*,¹⁴ they are more commonly brought in pairs. A *hovah*, an obligation, refers to a *ken* (a nest of birds) that consists of an **even** number of birds, half of whom must be sacrificed as *hatta'ot* and half as *olot*. The procedures for *hatta'ot* and *olot* are different and adherence to these rules is critical to fulfill the requirements of a proper

¹³ Where the blood of the *korban* is sprinkled, above or below a line around the middle of the *mizbayah* called the *hut ha-sikrah* differentiates the sacrifice of *hatta'ot* and *olot*.

¹⁴ This can occur if a *hovah* was partially sacrificed.

sacrifice.¹⁵ Not only does an improper sacrifice fail to fulfill one's requirement, under most or perhaps even all circumstances, it is disallowed as well.¹⁶

If a woman¹⁷ were to obligate herself to bring a *ken* of say 8 birds, she may designate each of two groups of 4 birds as *hatta'ot* and *olot* respectively forming what is called a *ken mefureshet*, a designated nest; the Kohen would then sacrifice them accordingly. Alternatively, she might give all eight birds to the Kohen, in what is called a *ken stumah*, an undesignated nest, and the Kohen can sacrifice each bird as he chooses as long as any 4 are sacrificed as *olot* and the other 4 as *hatta'ot*. In this case, sacrificing a fifth *olah* or *hattat* is not just ineffective, it is also disallowed. The standard interpretation assumes that once 4 of the birds have been sacrificed as say *olot*, the remaining birds are treated as having been **implicitly designated** as *hatta'ot*. Precisely how this principle of implicit designation is to be formulated is disputed in a limited sense by many interpreters following the approach of the *Ba'al Ha-Moar* and *Rosh*; the extent to which this principle exists at all, is the central issue of contention between Rambam and other commentators.

The *mishnayot* of *Kinnim* deal with (an arbitrary number of) nests of birds, *stumot* or *mefurashot*, that become intermingled and discuss two sets of rules for *kinnim* that have become intermingled, depending on whether consultation preceded the Kohen's sacrifice. The language of the Mishnah at the beginning of the third

¹⁵ This is explained clearly at the beginning of the tractate.

¹⁶ Despite this introduction, studying *masekhet Kinnin* in its entirety, as well as chapters 8 and 9 of *Pesulei Ha-Mikdashin* is strongly suggested.

¹⁷ It is normally women who are required to bring these sacrifices after childbirth.

chapter, introduces the chapter with “*ba-meh devarim amurim, be-Kohen nimlah.*” As normally understood, this introductory phrase states that while the first two chapters covered cases of prior consultation, this third chapter covers cases where there was no prior consultation. According to most commentators, throughout the first two chapters we deal with cases where the Kohen consulted, and **only the third** chapter addresses cases where he did not consult. It is this assumed circumstance that Raavad changes; for the first three *mishnayot* in the second chapter, Raavad also assumes a circumstance where the Kohen acted without consultation.¹⁸ The precise nature of consultation, both with whom or to what extent, is not covered.

Thus, the laws of *masekhet Kinnim* cover several types of intermingled nests under two scenarios:

- the Kohen is aware of the situation and consults prior to performing any sacrifice, or
- the Kohen is either unaware of the situation or, despite being aware, nonetheless sacrifices the birds without prior consultation.

The guiding principle where there is consultation is:

- Disallow any potentially incorrect or ineffective sacrifice.

¹⁸ In defense of Raavad and others who assumes no consultation already in the second chapter, they will read the phrase at the beginning of the third chapter as connected to a similar case in the first chapter, effectively rendering the second chapter as if it were a digression between the first and third chapter. While the first and third chapters largely discuss the **complete** intermingling of nests, the second chapter discusses individual birds that fly between nests, providing somewhat increased credibility to Raavad’s position.

The guiding principle where there is no consultation is simple to state but on occasion difficult to compute:¹⁹

- Construct the worst case, invalidating as many birds as possible; the remaining number of birds are deemed to have been sacrificed correctly.

Note that after the fact, we do not penalize a sacrifice done without consultation to allow only the same number of valid birds that can be sacrificed under the *ab initio* rule. Rather, after the fact, a greater number of birds are often credited as having been sacrificed effectively.

Throughout all *masekhet Kinnim* we are constructing two varieties of worst cases, either as the basis for disallowing sacrifices *ab initio* or calculating what is valid after the fact. While the former cases require some thought, the latter cases are more challenging to describe with precision and to formally demonstrate. To avoid confusion, the term “worst case” will apply only to the latter, after the fact, scenario. When there is prior consultation, we will refer to the principle as disallowing (even the most remote possibility of) an improper sacrifice.

A few simple examples help illustrate the principles. Assume that one *ken stumah* of 2 birds and another *ken stumah* of 4 birds are intermingled, creating an intermingled nest of 6 birds. With prior consultation only 2 birds may be sacrificed, one as an *olah* and one as *hattat*. If we were to sacrifice more than those 2 birds, then the 3rd bird is immediately problematic. That 3rd bird is the 2nd

¹⁹ I know of no instance where this rule is disputed. In those (very few) cases where a commentator appears to disagree, I believe it is more than likely that the *halakhic* rule is accepted but incorrectly computed.

bird sacrificed either as an *olah* or as a *hattat*. Both of the 3rd bird and one of the 2 sacrificed in the same manner as the 3rd bird might both come from the smaller nest resulting in both of its 2 birds being sacrificed identically (and hence incorrectly.) To avoid this possibility of an improper sacrifice, the sacrifice of a 3rd bird is disallowed.

Where both nests were of equal size, say each with 4 birds, then 4 of the 8 birds could be sacrificed, 2 birds as *olot* and 2 birds as *hatta'ot*. Regardless of whose birds are chosen, 2 birds may always be sacrificed as *olot* and 2 birds as *hatta'ot*. No bird can be sacrificed incorrectly. In general, $\frac{1}{2}$ of the intermingled nest can be sacrificed, $\frac{1}{4}$ as *olot* and $\frac{1}{4}$ as *hatta'ot*. Interestingly, as pointed out in the first Mishnah in the third chapter, in a case of equal sized nests that were intermingled, even proceeding without consultation does not improve that result. If all the birds in two equal sized nests that were intermingled are sacrificed, it is possible that all the birds in each of the two original nests are sacrificed identically, invalidating half of birds. Thus, in the case of equal sized nests, with or without consultation, half of the combined nest will always be valid.

However, when 2 unequal sized nests are intermingled, proceeding without prior consultation gives a different result. For example, in the case of two nest with 2 and 4 birds respectively, the Kohen, assuming that he is dealing with a normal *ken stumah* of 6 birds, would proceed to sacrifice 3 birds as *hatta'ot* and 3 birds as *olot*. Based the *halakhic* principle above, the worst case must now be determined. It is easy to see that the worst case occurs if both birds of the

smaller nest are sacrificed identically.²⁰ In that case, 3 of the birds from the larger nest are also sacrificed identically.²¹ Now let us compute what was correctly sacrificed. From the smaller nest, 1 bird was sacrificed correctly (and 1 was not.) In the larger nest 1 pair of birds (an *olah* and a *hattat*) as well as 1 of the other 2 birds was sacrificed correctly; therefore, 3 birds from the larger nest were sacrificed correctly. In total, 4 of the 6 birds (1 from the smaller nest and 3 from the larger nest) were sacrificed correctly.

This case of unequal sized nests that are intermingled generalizes easily. If 2 undesignated *kinnim* of $2*N$ and $2*M$ birds are intermingled where $N \leq M$, then with consultation $2*N$ birds can be sacrificed. However, without consultation, after the fact, the owners get credit for $2*M$ birds. This rule, which I believe is accepted without dispute, is stated explicitly by Rambam.²² This is a (simpler) special case of the term *merubah* that we will address more generally.

As an illustration of the rule, assume two undesignated nests, one consisting of 4 birds and the second of 6 birds ($N=2$ and $M=3$) are intermingled. When the Kohen consults, he is told to sacrifice only the number of birds in the smaller nest, 4 birds. Regardless of whose birds are chosen, it is impossible to sacrifice incorrectly if only 4 birds are chosen, with 2 birds sacrificed as *hatta'ot* and *olot*. On the other hand, where the Kohen acts without consultation and sacrifices all

²⁰ If the two birds in the smaller *ken* were not sacrificed identically but instead one was sacrificed as an *olah* and the other as a *hattat*, both *kinnim* are sacrificed correctly as well.

²¹ Without loss of generality, assume three *olot* from the larger nest, 1 *hattat* from the larger nest and 2 *hatta'ot* from the smaller nest. Replacing *hatta'ot* with *olot* and vice versa, provides an equivalent example.

²² Rambam in Mishneh Torah *Pesulai Ha-Mikdashin* 8:6.

10 birds, 5 as *olot* and 5 as *hatta'ot*, we need to construct the worst case. It is easy to demonstrate that in the worst case all 4 birds in the smaller nest along with a single bird from the larger nest are sacrificed as say *olot* and the remaining 5 birds, all from the larger nest are sacrificed as *hatta'ot*.²³ Under that scenario, a pair of birds of the larger nest are sacrificed correctly, one as a *hattat* and one as an *olah*. Of the remaining 8 birds, only half of each nest's 4 birds are sacrificed correctly. Thus, of the 4 remaining *olot*, 2 are sacrificed correctly as *olot* and of the 4 remaining *hatta'ot*, 2 are sacrificed correctly as *hatta'ot*. In total, the larger group of 6 birds ends up with at least 4 birds sacrificed correctly and the smaller group of 4 birds has at least 2 birds sacrificed correctly. The two owners must now bring 4 additional birds. How they might share the expense of the purchase of the 4 additional birds is not covered in the *mishnayot*;²⁴ commentary by several *rishonim* and *achronim* on the fourth Mishnah of the third chapter, another area of focus, addresses that issue.

Section 1: The standard interpretation, Rambam and Raavad's approach to the first 2 chapters of *Kinnim*

This first section is organized into 5 parts; a first and second pass through each of the two chapters is followed by some conclusions that complete this section.

²³ Reversing *olot* and *hatta'ot* does not impact the analysis in any way.

²⁴ In many cases where undesignated nests are intermingled, it would stand to reason that the expense would be shared proportionately, a topic partially addressed in the last section of the monograph.

Part 1: Chapter 1 - First pass

The first chapter of *Kinnin* discusses cases where (an arbitrary number of) undesignated nests are combined or a set of designated birds fly into an undesignated nest. In the first chapter, the Mishnah discusses cases where there is prior consultation; in the third chapter similar cases without consultation are discussed. However, Rambam in the 8th chapter of *Pesulei Ha-Mikdashin* deals with the first and third chapter of *masekhet Kinnim*, covering cases both with and without consultation in the same chapter. This leads to various opinions whether a recommended sacrifice results from consultation or whether it is only an after the fact occurrence.

Focusing first only on the first chapter of the Mishnah, four cases below are examined.

1) One or more *olot* fly into a nest of N *hatta'ot* or alternatively one or more *hatta'ot* fly into a nest of N *olot*. In this case no sacrifice is permitted; all the birds are lost. Clearly, any bird, regardless of how it is sacrificed stands a chance of being sacrificed incorrectly.²⁵ This Mishnah generates no disagreement.

2) A designated set of birds, either all *olot* or *hatta'ot*, fly into an undesignated nest. Assume, without loss of generality, 4 *olot* enter an undesignated nest of 10 birds. The Mishnah rules that while no *hatta'ot* may be brought, the women can bring half of the number of birds in the undesignated nest as *olot*. In our example, 5 *olot*, half of the 10 birds in the undesignated nest are sacrificed, losing

²⁵ Note that the principles of (even overwhelming) majority do not apply; the Mishnah does not differentiate based on the size of N.

9 of the 14 birds. Were one to sacrifice a 6th bird as an *olah*, then all 6 birds might have come from the undesignated nest from which only 5 (half of the 10 birds) may be brought as *olot*. Nine new birds must then be sacrificed:

- the two women must jointly offer 4 additional *olot*,
- while the woman owning the original nest of 10 birds must also bring 5 *hatta'ot*.

Notably, in this case, Rambam²⁶ adds a suggestion, which never appears in the Mishnah: all 14 birds may be brought as *olot*. Nine *olot* are valid, leaving the woman with the original undesignated nest with an obligation to bring 5 *hatta'ot*. Rambam's approach differs sharply and allows not 5 (or even 9 birds,) but all 14 birds to be sacrificed as *olot*, thereby increasing the number of valid sacrifices from 5 to 9.

Rambam repeats this suggestion in a parallel case where *olot* are replaced by *hatta'ot*, and in that parallel case where the phrase "*ve-yirahem li*," again appears. Clearly Rambam's suggestions apply even a priori, with consultation, where the Kohen is fully aware of the situation.

- First, and fundamentally, the phrase "*ve-yirahem li*," would make little sense if the sacrifices took place after the fact; by the principles of the 3rd chapter, were it after the fact, Rambam's ruling is not an intuitive leap but an entirely reasonable and expected conclusion if all the birds as *olot*.

²⁶ *Pesulai Ha-Mikdashin* 8: 3, 4.

- Second, Rambam appears to parallel the *mishnayot* in *masekhet Kinnim* where this case is only discussed in the first chapter.

3) Two women have undesignated nests of equal size, say 6 birds that are accidentally combined to form a single nest of 12 birds. The Mishnah states that one can sacrifice 6 birds, 3 as *olot* and 3 as *hatta'ot*, while 6 birds are lost. The logic of only 3 but not 4 is again straightforward. If all 3 *olot* sacrificed happen to belong to the same woman, it is possible that the 4th *olah* that would be sacrificed belongs to her as well. That sacrifice is certainly disqualified and according to most interpretations disallowed. As noted in the introduction, by sacrificing only 3 birds as *olot* and only 3 birds as *hatta'ot*, we guarantee that regardless of their ownership, there would be no possible violation. The two women must then bring another 6 birds, in partnership, to fulfill their obligations.

Where there is no consultation and all 12 birds are sacrificed, nothing is gained; only 6 birds are valid since the possibility that all the birds sacrificed as *olot* are from one woman and all the birds sacrificed as *hatta'ot* from the other woman.

Rambam's formulation in *Pesulai Ha-Mikdashin* 8:5 does not mention consultation explicitly, stressing only that if the nests are of equal size, each of following four alternatives:

1. sacrificing all the birds as *olot*,
2. sacrificing all the birds as *hatta'ot*,
3. sacrificing half of the birds, $\frac{1}{4}$ of the birds as *hatta'ot* and $\frac{1}{4}$ of the birds as *olot*,

4. sacrificing all the birds, $\frac{1}{2}$ of the birds as *hatta'ot* and $\frac{1}{2}$ of the birds as *olot* results in the same outcome – 6 birds are valid and 6 additional birds must be sacrificed. Note that unlike the case to follow, the number of valid birds does not change.

While the 3rd and 4th alternatives are given in the first and third chapter of *masekhet Kinnim* (depending on whether there was consultation), the 1st and 2nd alternatives are unique to Rambam's formulation and are not mentioned in *masekhet Kinnim*. As noted, those two alternatives do not increase the number of valid sacrifices. More fundamentally, the two alternatives are stated without the phrase "*ve-yirahem li*," implying that they are derivable directly. For primarily that reason, one can assume that those alternatives only occur without prior consultation. What Rambam thought might (possibly) be gained by this suggestion is addressed later.

In each alternative when two nests of equal size are intermingled, another sacrifice consisting of half the number of birds in the two nests will be required. This case differs from the next case where unequal sized nests are intermingled.

4) If two undesignated nests of varied sizes, say 6 and 14 birds, are intermingled, the law is more complex.²⁷ In this first chapter where we assume that the Kohen consults before sacrificing, only the number of birds in the smaller nest may be

²⁷ In the first chapter where consultation is assumed, cases where more than two nests of assorted sizes are completely intermingled do not add more complexity. However, the cases dealt with in the third chapter where there is no consultation, then the *halakhic* (and mathematical) complexity increases dramatically as Dr. Reiss's paper demonstrates and will be clear in the discussion of the term - *merubah*.

sacrificed, 6 birds in this case; 3 birds are sacrificed as *olot* and 3 birds as *hatta'ot*. The larger nest of 14 birds is lost and 14 birds must be brought again jointly. The logic is exactly as in the case of two equal-sized *kinnim*. A 4th bird sacrificed as say an *olah*, may belong to the same woman as the original 3 *olot* and that would not be permitted.

As in the previous case, Rambam again raises the option of sacrificing all the birds either as *hatta'ot* or *olot*. As already noted, in the case of equal sized nests, Rambam's suggestion had no apparent **practical**²⁸ benefit since under every option half of the nest is valid and the other half of the nest must be purchased anew and sacrificed jointly. However, in the case of nests of unequal size as for example, nests of 6 and 14 birds, Rambam's suggestion reduces the number of lost birds from 14 to 10; his approach allows half of the birds to be valid either as *olot* or *hatta'ot*. However, while in the cases of *olot* or *hatta'ot* intermingled with an undesignated nest, Rambam introduces his suggestion with the phrase "*ve-yirahem li*," in this Mishnah again, that phrase is absent. In addition, the verb is in the past tense – "*asah*", suggesting that in this case the sacrifice of all the birds is not a pro-active suggestion but an after the fact occurrence. Rambam suggests that if all 20 birds were sacrificed as say *olot*, 10 birds are valid, and thereby both women dispense with their obligation to bring *olot*. The two women then bring 10 birds as *hatta'ot*, separately for that matter, 3 by the women with the smaller nest and 7 by the other woman.

²⁸ There may be a *halakhic* benefit, a topic we return to later in the paper.

Disregard for a moment, that in the cases of *hatta'ot* or *olot* intermingled with an undesignated nest Rambam suggest that all birds can be similarly sacrificed, while when two undesignated nests are intermingled, the sacrifice of all the birds as *hatta'ot* or *olot* is only considered after the fact. This may suggest a principle that Rambam will apply throughout his discussion of *Kinnim*, one suggested by R. Yehoshua Heller, that where possible Rambam prefers sacrifices with clear ownership. Note that by sacrificing all the birds, each woman is certain that every *olah* for which she receives credit belonged to her originally.

The second case is in sharp dispute between Rambam and all other commentators. Consider a simple case with a nest of 4 birds. The standard interpretation would assume that one can designate 2 as *hatta'ot* and *olot* respectively or if originally undesignated, the sacrifice of 2 as *olot* implicitly designates the other 2 as *hatta'ot*. There is an assumed pairing: a bird sacrificed designates some remaining bird as its mate to be differently sacrificed. This Rambam apparently rejects, at least to some extent. Rambam undoubtedly admits that once you have brought 2 birds as say *olot*, and you are sacrificing to fulfill your requirement to bring a *hovah* you must obviously sacrifice the other 2 birds as *hatta'ot*. He also clearly states that if you were to bring a third *olah*, not only do you receive no credit, but that sacrifice is disallowed. However, Rambam also explicitly says designation can only occur initially or when the bird, and not its theoretical mate, is sacrificed.²⁹ Rambam apparently excludes, at the very least in

²⁹ Rambam is explicit on this point in *Pesulei Ha-Mikdashin* (8:8) that refers to *Pesulei Ha-Mikdashin* (5:11). He states: "...nests are designated only when taken by their owners or when they are sacrificed by the *Kohen*, as we explained." This statement made in the *gemara* as well, appears to be taken literally by Rambam. See footnote 45.

certain cases, an implicit form of designation that occurs at an intermediate point after some of the other birds are sacrificed. Before considering further what other principles and potential rationale may be helpful in clarifying Rambam's position on when and even if implicit designation is to be applied, the opening *mishnayot* of the second chapter are analyzed.

Part 2: Chapter 2 - First pass

Despite the added computational complexity of the third Mishnah in the second chapter, once the principles implicit in the first two *mishnayot* in the chapter are clarified, the third Mishnah follows logically.³⁰ We will therefore restrict attention to a simple case of two women, Rachel and Leah, each with an undesignated nest of 4 birds. Following the case of the second Mishnah, assume one bird flew from Rachel's nest to Leah's nest, leaving Rachel's nest with 3 birds and Leah's nest with 5 birds. Assume no more birds escape and we must decide the *halakha* in this simple case.

After analyzing this simple case, we will need to examine one further situation covered in the first two *mishnayot*, where 1 of the 5 birds currently in Leah's nest flies to Rachel's nest again leaving both nests with 4 birds.

The standard interpretation: First, we assume that we are dealing with a case of consultation that will specify how to proceed. The *halakha* would then be as follows. Rachel loses not just the bird who escaped to Leah, but she must put

³⁰ Demonstrating this in detail is tedious. For Rambam and Raavad, the proofs of their positions demonstrate this in detail. For the standard interpretation, either R. Ovadiah of Bartanura or R. Kehati's explanation makes this clear; the extent to which the Mishnah does not follow precisely is viewed as problematic.

aside one additional bird as well. Leah sacrifices 4 (of the 5) birds currently in her nest and fulfills her obligation; Leah loses no birds. Rachel, from whom one bird flew away and had to put another bird aside, loses 2 birds and must add 2 new birds. In total, only 2 birds are added, both by Rachel. Rachel sacrifices 4 birds; 2 of the 3 birds from her original nest are sacrificed as an *olah* and *hattat*, and separately the 2 new birds are sacrificed, one as an *olah* and one as a *hattat*.³¹

Two different rationales both imply this result. First, if Rachel's third bird is sacrificed, its mate, who is now a part of Leah's nest, is implicitly designated. However, when only 2 birds are sacrificed one as an *olah* and one as a *hattat*, then the bird that flew into Leah's nest remains undesignated. Assume alternatively, for example, that the third bird is sacrificed as say an *olah*. Were that allowed, then Leah has an undesignated nest into which a bird (implicitly) designated as a *hattat* has flown. By the principles of the first chapter, Leah can then bring only 2 of her birds as *hatta'ot*. Clearly, Leah cannot bring any *olot*. Were Leah to bring 3 *hatta'ot* in conjunction with Rachel, the possibility that all 3 are from Leah's original nest disallows that possibility as well.³² In that case, of the original eight birds, only 5 are sacrificed, 3 by Rachel and only 2 by Leah; 3 new birds must be added. Ostensibly, to maximize the result and limit the loss to only 2 birds, we penalize Rachel by forcing her to set aside the third bird in her

³¹ Why there must be two separate pairs of birds sacrificed, as opposed to just 2 birds added to Rachel's nest, is explained in footnote 33.

³² Leah would sacrifice 3 *hatta'ot* in conjunction with Rachel; Leah getting credit for 2 and Rachel for 1. Rachel would separately sacrifice 2 *olot* and one *hattat*, and Leah would bring 2 additional birds as *olot*. Such an option is clearly disallowed.

nest to avoid the implicit designation (by Rachel's third bird) of a bird that is now a part of Leah's nest.³³

Second, beyond minimizing the number of lost birds, there is a more fundamental reason Rachel's third bird must be disqualified. When Leah sacrifices her four birds, one of Rachel's is possibly included and we cannot know whether it is brought as an *olah* or as a *hattat*. It would then be possible that 3 birds from Rachel's original nest, 2 birds currently in her nest and her original bird now in Leah's nest, are being identically sacrificed, a possibility that must be avoided.

For either reason, both losses accrue to Rachel, who owned the nest from which the bird escaped. Leah likely receives credit for a bird (originally) owned by Rachel.^{34 35}

³³ Given a choice between Rachel's penalty of 2 birds or a penalty of one bird to Rachel and 2 birds to Leah, the former option is chosen. Of course, the situation could get worse if Rachel does not carefully note how the added bird is sacrificed. Were Rachel to add one bird and then sacrifice all four without regard to how the new bird is sacrificed, then Leah would be restricted from sacrificing any of the 5 birds in her nest, since her nest now contains an implicitly designated bird, that may be either an *olah* or a *hattat*.

³⁴ The probability that Rachel's bird is one of the 4 sacrificed is 80% -- each of the five birds has an equal 20% probability of being left out. The precise rationale behind this ruling is not designated and as stated is not our focus. Via some mechanism, Rachel may have gifted one of her birds to Leah or an act of the *Beit Din* gives the bird to Leah, perhaps to penalize Rachel for allowing her bird to escape. In any case, the principle is clear -- Rachel loses two birds.

³⁵ According to both bases for disqualification, Rachel cannot form a nest of 4 birds and then sacrifice, unless she first marked the original birds to guarantee they were sacrificed differently. If she were to sacrifice her original 2 remaining birds identically, as say *olot*, then she again has created a *hattat* in Leah's nest, according to the first reason, or 3 of her original nest may have sacrificed identically, according to the second reason. As noted, the 2 birds from the original nest and the 2 new ones must be treated as two separate nests, with each pair sacrificed one as an *olah* and one as a *hattat*.

Note that since we cannot identify which of the 5 birds in Leah's nest came from Rachel's nest, no penalty on Leah to set aside a bird is even effective. Leah has no way of knowing which of the 5 birds in her nest came from Rachel's nest; thus, having Leah set aside a bird accomplishes nothing. Slight variants of the two rationales are given in support of the standard interpretation.³⁶

In summary:

- the **circumstance** assumes consultation,
- the **principle** that supports the ruling is some variation of the concept of implicit designation and
- the **halakhic ruling** is a loss of 2 birds by Rachel.

Raavad's interpretation: Raavad largely assumes that the birds were sacrificed without consultation, applying the principles of the third chapter.³⁷ While Raavad discussion of the second Mishnah assumes a situation where there was no consultation, Raavad questions his approach and also considers a case of consultation when discussing the third Mishnah. **What follows posits that Raavad assumes there was no consultation in all three of the *mishnayot* in the second chapter, although he also offers alternative approach.** The principle articulated in the third chapter, is to establish the worst-case scenario; having determined that scenario, the women receive credit for all sacrifices that

³⁶ Some of these variants are necessitated by the third Mishnah.

³⁷ What may have driven Raavad to this explanation is covered at the end of the section. Note that Raavad considers the standard formulation as well.

nonetheless must have been sacrificed correctly. Raavad interprets the second Mishnah to say that each woman, who now has an odd number of birds in her nest, adds one bird to her nest and then sacrifices the entire nest, without consultation. All the original 8 birds and 2 additional birds are sacrificed; Rachel adds a bird and sacrifices 4 birds and Leah adds a bird and sacrifices 6 birds. Each woman then receives credit for 4 birds.

Let us examine the case in detail. After one bird has flown from Rachel to Leah, each woman has an odd number of birds in their nest, 3 in Rachel's and 5 in Leah's. Clearly, a bird must be added to each nest to create a *hovah* that can then be given to a Kohen who is not told about either *ken's* history. The Kohen proceeds to sacrifice both *kinnim* "correctly," half as *olot* and half as *hatta'ot*. Given Raavad's approach, we can assume³⁸ Rachel's sacrifices present no issue since she sacrificed 4 birds owned by her. Leah, however, has a problem because the unidentified bird from Rachel's nest that entered her nest may have been sacrificed incorrectly. If the new bird added to Rachel's nest and the bird now in Leah's nest that was originally from Rachel's nest happen to have been sacrificed the same way, then all 10 sacrifices were proper. **The mate of the new bird in Rachel's nest and the escapee to Leah's desk were sacrificed differently and hence correctly.** However, given that we cannot determine which bird now in Leah's nest came from Rachel's nest and we are therefore unable to determine how it was sacrificed, by the principle of the third chapter, we must assume the worst case. There are, in fact, two parallel worst cases, each leading to a disqualification of 1 of the 6 birds in Leah's nest. Assume the new bird added to

³⁸ The basis for this assumption is clarified below.

Rachel's nest was sacrificed as say a *hattat* and the bird that escaped from Rachel's to Leah's nest was sacrificed as an *olah*. Consider only Rachel's original nest: since the new bird in her nest was a *hattat*, 2 of her original birds **and** her original bird that escaped to Leah's nest were all 3 sacrificed identically as *olot*; 3 birds from Rachel's original nest of 4 birds, were all sacrificed as *olot*.³⁹ By the rules of the third chapter, one of the three *olot* is disqualified; for reasons of fairness⁴⁰ we choose to disqualify Leah's bird. Alternatively, consider a parallel case where 3 of Rachel's original 4 birds, the escapee and 2 original birds still in Rachel's nest, were sacrificed as *hatta'ot*. Again, 1 of the 3 birds sacrificed as a *hattat* is disqualified and for reasons of fairness we invalidate the bird that flew into Leah's nest. Thus, we must invalidate one *hattat* and one *olah* and only 4 of the 6 birds Leah sacrificed are valid. Unlike the standard interpretation where Rachel must add 2 additional birds, according to Raavad each woman adds one additional bird; Leah sacrifices 6 birds, Rachel sacrifices 4 birds and each fulfills their obligation getting credit for 4 valid sacrifices.

It is important to note that unlike the examples of worst case-based disqualification covered in the third chapter, in this Mishnah **two parallel worst**

³⁹ Raavad's explanation is expressed differently. He addresses the case before a bird is added to each nest, the nests having 3 and 5 birds, concluding that each woman must sacrifice an additional bird. The explanation of Raavad's position that is given, which first adds a bird to each nest, is slightly easier to follow. Note that the worst case that we must determine is a function **only** of what was done with the new bird in Rachel's nest and the escapee to Leah's nest. As with any application of the third chapter's rules of non-consultation, careful thinking is often required. This case is no exception; doing this case with pencil and paper is recommended.

⁴⁰ Perhaps because both women should have taken better care of their nests, each must now acquire one additional bird for sacrifice.

cases separately disqualify a *hattat* and an *olah* respectively. In the more typical cases, the worst case specifies how many pairs of birds are disqualified as opposed to an individual *hattat* or *olah*.

In summary:

- the **circumstance** assumes **no** prior consultation,
- the **principle** that supports the ruling is from the third chapter that assumes that in a case of no prior consultation, you construct the most disadvantageous scenario and give credit for all sacrifices that are still not able to be disqualified and
- the ***halakhic* ruling** is a loss of one bird by both Leah and Rachel.

Note Raavad differs from the standard interpretation in his explanation on all three parameters. Given Raavad's strong criticism of Rambam's interpretations of the first chapter, Raavad clearly maintains the principle of implicit designation. However, given his understanding of the language of the *halakhic* ruling,⁴¹ he was forced to a different circumstance and applicable principle. His disagreement with the standard interpretations is thus more technical than conceptual.⁴² Given

⁴¹ Raavad assumes that the straightforward meaning of the *halakhic* ruling is that Leah and Rachel each lose, and each must bring one additional bird versus the standard interpretation's reading that Rachel adds 2 birds, a reading that Raavad probably considered forced, particularly in the third Mishnah. Specifically, according to Raavad, the language implies that the bird that flew and its mate in Leah's nest are disqualified, since we cannot determine if a *hattat* or an *olah* were potentially sacrificed incorrectly. The standard interpretation assumes that the second disqualified bird was its old mate in Rachel's nest.

⁴² From his carefully reasoned explanation of the chapter, it is obvious that he agrees to the standard interpretation's principle of implicit designation. Raavad's commentary on the third Mishnah of the chapter formulates his acceptance of implicit differentiation definitively.

the circumstance of the standard interpretation Raavad would likely reach the same *halakhic* ruling, but he would express that opinion differently.

Rambam's Interpretation: Rambam's *halakhic* result is identical to Raavad's – each woman adds one bird. Leah sacrifices 6 birds and Rachel sacrifices 4 birds; each receiving credit for their original donation of 4 birds. There is almost no commentary on Rambam's position. That Rambam and Raavad agree on the *halakhic* result has not, to my knowledge, even been noted, though a careful reading allows no other conclusion. Unlike Raavad, although not explicitly stated, we assume that according to Rambam the cases involve prior consultation. While there is an attempt to explain Rambam assuming that he too is assuming no prior consultation,⁴³ the text particularly in *Peirush Ha-Mishnayot* that appears rather like his commentary in Mishnah Torah makes such a reading unlikely. Unlike Raavad, who when writing on the second chapter, states explicitly that he is assuming a case of no consultation, Rambam in his commentary in *Peirush Ha-Mishnayot* does not. However, it should be noted that, as hypothesized earlier, in the eighth chapter of *Pesulei Ha-Mikdashin* where Rambam addresses both the first and third chapter of *masekhet Kinnim*, it is very likely that in part of the sixth Mishnah, Rambam is addressing a case of no prior consultation without saying so explicitly. Nonetheless, this chapter in Mishnah Torah, which parallels the second chapter in *masekhet Kinnim*, is more likely to involve prior consultation.

⁴³ This position is maintained by R. Y. Heller; his disagreement involves another issue that is addressed subsequently. See footnote 46 and 47. Despite that position, he supports some of the major principles developed in this paper to address Rambam approach to the second chapter.

But if there is prior consultation, then Rambam must reject the notion of implicit designation, at least to the extent that it would apply in this instance.

Otherwise, consider the 3 birds left in Rachel's nest. If 2 birds are sacrificed as say *olot*, then the 3rd bird remaining in the nest and the escapee are **both** implicitly *hatta'ot*. In that case the laws of the first chapter, where a *hattat* flies into a *ken stumah*, Leah's nest, would apply. That would invalidate 3 of the 5 birds in the nest; only 2 *hatta'ot* could be brought. As Rambam explicitly validates 3 of the 5 birds, not just 2, he must reject the application of implicit designation, in at least this case.⁴⁴

Unfortunately, that alone would only explain why all the birds may be sacrificed. It would **not** explain why Leah must add a bird or why 2 of her birds are then disqualified when 6 are sacrificed. Simply rejecting all or some of the principle of implicit designation leaves us without a clear basis for disqualification; if Rambam rejects implicit designation in at least this instance, then why are any (of Leah's) birds disqualified?

Let us unpack Rambam's approach systematically. Two issues need to be addressed:

1. How might Rambam's rejection of implicit designation be circumscribed and explained?

⁴⁴ Rambam states clearly that were Leah to sacrifice 5 birds, 3 in one manner and 2 in the other manner, the escapee could end up on as 1 of the 2, versus 1 of the 3. In that case only 3 sacrifices are valid, 2 of the 3 and 1 of the 2. Adding another bird insures that regardless in which group of 3 the escapee is included; both groups have at least 2 of Leah's birds and 4 valid sacrifices.

2. What principle does Rambam employ as a basis for disqualification that can give the precise *halakhic* result that Rambam specifies in this case as well as in the more complex case of the third Mishnah in the chapter?

Rambam need not deny implicit designation *in toto*, despite his formulation in *Pesulei Ha-Mikdashin* 8:8 and 5:11⁴⁵ that limits designation to the point of original declaration and when the birds (and not their implied mates) are sacrificed by the Kohen. While Rambam in the first chapter adds modes of sacrifice, he clearly supports the conclusions of the Mishnah that must use either some form of the principle of implicit designation or some other principle that restricts sacrifice. However, Rambam also limits the scope of implicit designation; it is clearly not operative both in the cases addressed at the beginning of the second chapter and in some cases in the first chapter as well. At a minimum, the parameters used for limitation is what needs to be established. In the second chapter one potential rationale for limitation is to require that birds being implicitly designated (still) belong to the same nest. If a bird is no longer part of its original nest, sacrifice of birds in the original nest do not impact its status and do not render the bird as either a *hattat* or *olah*. R. Heller makes this point, arguing strongly in his introduction to the second chapter, that by sacrificing a bird in nest A, no bird currently in nest B can be impacted. In this and perhaps other cases, Rambam limits the application of implicit designation. Note however that this narrowly defined rationale is not applicable to the relevant cases in the first chapter.

⁴⁵ Rambam is quoting the *gemara* in *Chagigah* 28a. The meaning of “designation when sacrificed” may be in dispute; for Rambam it should be read literally, while those who maintain implicit designation probably read the *gemara* as also including designation by the sacrifice of its mate.

The more complex issue is Rambam's basis for disqualification. Rambam in both the ninth chapter of *Pesulei Ha-Mikdashin* as well as *Peirush Ha-Mishnayot* explains the basis for the disqualification without any detail beyond the number of birds from Leah's nest sacrificed and without any reference to Rachel's nest. The explanations of both the standard interpretation and Raavad provide more elaborate justification, referencing Rachel's nest in detail. According to Rambam, disqualification apparently results entirely from the number of birds sacrificed on behalf of Leah from the birds currently in her nest; apparently, only birds originally from Leah's nest can be counted. Though hardly provable, it is entirely plausible that disqualification simply results from my inability to benefit from your donation unless there is some shared, reciprocal (or even equal) benefit to you as well. In this case, Rachel receives no benefit from Leah's sacrifice of her bird. **Since Rachel receives no benefit, neither can Leah.**⁴⁶ That simple principle, may explain an element of Rambam's approach. If your bird enters my nest, unless our sacrifices are somehow (completely) linked, I am allowed no benefit from your bird. In fact, Leah must also add a bird to become the mate to the bird for which she receives no credit. Were Leah to sacrifice only 5 birds 3 as say *olot* and 2 as *hatta'ot*, we must consider the possibility that it was Rachel's bird that was sacrificed as 1 of the 2 *hatta'ot*. By adding a bird and sacrificing 3 birds as

⁴⁶ R. Heller makes an almost identical argument in his introduction to the second chapter. He argues that I cannot receive credit for a bird that you designated for sacrifice. It appears that I cannot gift that right to you as well; other commentators who do not raise this issue in their approaches to the entire tractate, must assume that in this case everyone relinquishes/gifts their rights or it is assigned by the power of the *Beit Din*.

hatta'ot and 3 birds as *olot*, we guarantee that at least 2 of the *olot* and 2 of the *hatta'ot* came from Leah's 5 birds, the 4 original and 1 new bird that she added.⁴⁷

Undoubtedly, Rambam could have also used the same basis for disqualification as Raavad; a result that is disallowed even after the fact, would certainly apply even with initial consultation. Even denying the principle or the applicability of implicit designation, Rambam clearly maintains that however a *hovah* of size $2*N$ is sacrificed (here by two women) it cannot provide more than N valid *olot* or *hatta'ot*. In our case of $(2*2=)$ 4 birds in each nest, assume that Rachel sacrifices the 3 remaining birds in her nest as 2 *olot* and 1 *hattat* with the added bird sacrificed as a *hattat*. Leah may separately also sacrifice the bird that arrived from Rachel's nest as an *olah* which would be a third *olah* from Rachel's original nest. This circumstance cannot be excluded and therefore despite sacrificing 6 birds only 4 are valid, excluding the bird from Rachel's nest and its mate. Thus, even without use of the standard notion of implicit designation, in addition to the principle Rambam employed, Rambam could support the principle used by Raavad for disqualification. As we will see below, Rambam likely used a principle of this sort in a case where birds flew many times between two nests.

In summary:

- the **circumstance** assumes consultation,

⁴⁷ R. Heller apparently goes one step further; not just can I not get credit for a bird I do not own, but I cannot even sacrifice the entire nest. As a result, he asserts that Rambam (also) assumes no prior consultation, something that I do not believe is necessary.

- the **principle** that supports the ruling is some formulation of the concept that without any shared or (perhaps balanced) reciprocal benefit with the owner, I cannot use another person's donation to *hekdesh* to fulfill my obligation and
- the **halakhic ruling** is a loss of one bird by both Leah and Rachel.

Note all three views differ on the principle involved. Rambam agrees with Raavad on the *halakhic* decision and with the standard interpretation on the assumed circumstance. A simple reading of the opening *mishnayot* in the second chapter (given the opening of the third chapter) would assume a **circumstance** of prior consultation, and a **halakhic ruling** that each woman loses one bird. **Only Rambam's approach comports with such a reading on both points.** However, to abandon the principle of implicit designation, as Rambam apparently does, may have been easier for Rambam not having the benefit of 800 years of commentary to the contrary.

To complete these two *mishnayot*, we must also consider, according to each of the three approaches, the consequences of one of the five birds in Leah's nest entering Rachel's nest. If the same bird returned, then all four birds could be sacrificed. Of course, we do not know that; therefore, we must decide based on the assumption that each nest now contains one bird from the other nest.

According to the **standard interpretation**, the returning bird causes 2 of Leah's birds to be disqualified. The principle is identical; one must prevent implicit designation of a bird in Rachel's nest that would occur if more than 2 birds in Leah's nest are sacrificed. As a result, each woman sacrifices

- 2 birds from their current nest, one each as an *olah* and *hattat*, and
- then sacrifices separately an additional *ken*.

According to **Raavad**, we must again construct the worst case. Like before, we assume Rachel's bird in Leah's nest and Leah's bird in Rachel's nest are sacrificed identically to the way 2 birds in their original nest are sacrificed. That can happen in two ways, with the 3 birds all sacrificed as either *hatta'ot* or *olot*. For example, Rachel sacrifices Leah's bird and 1 of her 3 original birds as *olot*, and the other 2 birds originally in her nest as *hatta'ot*. Leah sacrifices Rachel's bird and 1 of her 3 original birds as *hatta'ot*, and her other 2 original birds as *olot*. Alternately, one can construct a parallel worst case with *hatta'ot* and *olot* reversed. As 3 of the 4 birds from both original nests could have been identically sacrificed as either *hatta'ot* or *olot*, each woman must sacrifice 2 additional birds, unable to determine whether the bird potentially sacrificed incorrectly was a *hattat* or an *olah*.

For **Rambam**, matters are more complex. Rambam explicitly discusses the **case** of 4 birds in each nest with a bird flying from one nest to the other and then a bird returning in the opposite direction. Rambam then discusses the **general** case with a bird successively fly between the two equal sized nests multiple times, where the Mishnah explicitly limits the disqualification to half of the birds in the two nests. The Mishnah clearly states if birds continue to fly between the two nests, half of the birds are always valid even though in the case of two nests, each with 4 birds, after 8 flights, all the birds in both nests may have originally belonged to the other nest. Note that after any even number of flights, were the two nests

merged and half of the combined nest sacrificed, the benefit both Leah and Rachel derive from each other is completely reciprocal; Rambam's assumed approach disallowing benefit in cases where the benefit is not reciprocal, would not apply. Nonetheless, even without a notion of implicit disqualification, when jointly sacrificing to fulfill a *hovah*, Rambam would certainly restrict deriving more than N *olot* or *hatta'ot* from a *hovah* of size $2*N$, using a principle like Raavad's (and one universally accepted) as the basis for disqualification. While Rambam does not explain his basis for disqualification, Rambam does refer to his explanations in the previous chapter where this fundamental notion is discussed. How Rambam circumscribes use of this principle is addressed in the next section.

In any case, Rambam does not deal with the case of a bird flying from Rachel's to Leah's nest and a bird flying back from Leah's to Rachel's nest the first time in the same way he does the **general** case of an arbitrary even number of flights between two nests. Instead, Rambam first explains the **case** of just one round trip between nests of 4 birds as he does the first flight from Rachel's nest. The reason may just result from the way the Mishnah in *masekhet Kinnim* is formulated; however, I would propose it indicates Rambam's preference for clear ownership. Consider what would occur after many flights in the **general** case. The two nests are now intermingled; the Kohen takes half the birds from both nests, perhaps even without regard to how many he takes from each. Those birds are sacrificed jointly, on behalf of both women. No method to sacrifice separately for either woman exists; how one chooses birds from either nest is inconsequential. In addition, each woman then brings an additional nest, half the size of her original nests. Those two nests are sacrificed separately for Rachel and

Leah. After receiving credit jointly for half of the birds in the first two original nests, each then receives credit separately from the additional nest that each brings.

On the other hand, unlike the **general** case, in the **case** after only two flights between nests of four birds, there is no reason for sharing. Rambam uses the same basis for disqualification – I cannot benefit from your bird. Note that the result is not the same. Each of the birds in the original nests, **all four birds in both of those two nests are sacrificed**, and each sacrifice is associated with only one woman. Of the 4 birds sacrificed on behalf of both Rachel and Leah separately, each receives credit for the provably valid pair of birds that unquestionably belong to her and are still resident in her nest. Potentially, this may again indicate Rambam's preference for explicit ownership, wherever possible, a point we return to in the next part.

Part 3: Chapter 1 - Second pass

It would have been fortuitous if Rambam's unique interpretation of the second chapter would explain his strongly disputed positions in the first chapter. In any case, Rambam's approach to the second chapter does offer some insight into his approach. The following case will be used to illustrate Rambam's approach:

- A group of 4 *olot* becomes intermingled with an undesignated nest of 6 birds. The standard ruling is that 3 birds are sacrificed as *olot*, and 7 birds are lost. Rambam suggest that all 10 birds be sacrificed as *olot*, with the owners getting credit for 7 *olot*.

The primary question to address is why Rambam does not apply implicit designation⁴⁸ to prevent sacrificing more than 3 *olot*? The potential rationale proposed in the second chapter that implicit designation cannot apply across distinct nests is not applicable, as the nests are now intermingled. Perhaps, one can assume that implicit designation only applies if one is attempting to sacrifice *olot* and *hatta'ot* in an undesignated nest or in a group of intermingled undesignated nests. It is the intent to sacrifice in mated pairs that is critical. If one is sacrificing *olot* or *hatta'ot* alone, and that intent to sacrifice in pairs is absent, then implicit designation is inapplicable as well. Since, no birds have been designated, sacrificing all as either *hatta'ot* or *olot* is allowed, but clearly does not fulfill one's obligation to bring mated pairs of *hatta'ot* and *olot*. Nonetheless, half of one's obligation is fulfilled.

How this rationale is designated and defended in detail is still unclear. Rambam clearly maintained a more restricted view of implicit designation. In fact, he may have gone even further: Rambam's language clearly specifies that **the sacrifice of a bird**, but by implication **not of its mate, is the point of designation**. Assume that Rambam is to be taken literally, he completely denies the principle of implicit designation. One can then turn to a question in the other direction and ask what principle Rambam might use in place of implicit designation to explain the

⁴⁸ Evaluating an incisive comment by R. Shabtai Rappaport in his approbation to Prof. Koppel's book stimulated this approach. Assuming the position of the standard interpretation that Prof. Koppel was following, R. Rappaport noted that initial designation of a bird by the owner equates with implicit designation by the sacrifice of a bird's theoretical mate. The difference is merely the method of designation. While agreeing with R. Rappaport's observation with respect to the standard interpretation, it is clearly not axiomatic; as hypothesized Rambam would not agree at all.

Mishnah's view that would limit the sacrifice of *olot* to 3 in the two examples above. Clearly Rambam agrees that when sacrificing mated pairs from a *hovah* of size $2*N$ one cannot derive more than N valid *olot* or *hatta'ot*. Rambam may then argue that sacrificing more than half of an undesignated nest as say *olot* has no utility; Rambam might then simply **disallow any sacrifice that has no utility**.

Thus, sacrificing more than half of an undesignated nest as say *olot* is not allowed. Again, consider the simplest case of an undesignated nest of 2 birds, where one bird has been already sacrificed as an *olah*. The standard interpretation would declare the second bird a *hattat* and thereby restrict its sacrifice. Rambam might well disagree that the bird is already declared a *hattat*, but nonetheless disallow its sacrifice as an *olah* because such a sacrifice would have no utility.

Independent of any notion of implicit designation, a useless sacrifice is disallowed. However, bringing all the birds as say *olot*, is not viewed by Rambam as useless; it guarantees that every bird that is credited was sacrificed by its owner.⁴⁹

⁴⁹ To summarize, there are at least two ways to formulate Rambam's position:

- 1) Rambam completely rejects implicit designation and uses only a disqualification based on bringing a sacrifice with no utility.
- 2) Rambam limits implicit designation to a case where the intent is to fulfill requirements for the sacrifice of an undesignated nest. When that is not the purpose, there is no implicit designation.

The first alternative, one that I prefer, was explained in detail above and will be restated in the summary.

There is still another subtler detail that sheds light on Rambam's approach. Rambam suggests sacrificing 10 birds as *olot* while credit is received only for 7. Assume instead of 10 sacrifices being brought, only 7 *olot* were sacrificed. Could they not agree to grant each other credit with each receiving credit for their required *olot*, regardless of the ownership of the 7 birds sacrificed? Why the additional, and perhaps unnecessary, sacrifice of 3 birds?

Again, some of Rabbi Heller's comments on Rambam's approach in the second chapter are helpful. The answer may be based on the ability to ensure that a woman receives credit for a bird that she clearly owns outright, without the need to rely on reciprocal sharing. While sharing is allowed,⁵⁰ Rambam perhaps feels that it is not optimal and when it can be avoided it ought to be, even at the expense of additional sacrifices. This observation strengthens the assumption that Rambam places a significant premium on clear ownership. Note as well that in the **case** in the second chapter of 4 birds in each nest, after two flights, it appears that all the birds in each nest were sacrificed, rather than getting the identical number of valid sacrifices by combining the nests and sacrificing only half of the birds on behalf of both women jointly.

Rambam's positions limiting or denying implicit designation, replacing it with a more general principle of required utility, and requiring or preferring ownership are not logically linked. One could certainly agree with Rambam on either issue without the need to agree on the other. Even if one were to accept all Rambam's positions, the overwhelming question, to which we cannot even speculate about

⁵⁰ It is possible that in this case and others Rambam may disallow sharing since a better alternative is available or because the benefit is not exactly shared equally.

what Rambam might answer, is why the *mishnayot* did not provide or even hint at Rambam's practical advice in the first chapter?⁵¹

One additional critical issue in Rambam's position must be addressed. In the case of N *olot* mixing with an undesignated nest of $2 * M$ birds, Rambam suggests sacrificing all $(N + 2 * M)$ birds as *olot* and receiving credit for $N + M$ *olot*. As noted, any reduced number of sacrifices **cannot guarantee** that credit is given only to bird's owner without any need for sharing. However, when 2 undesignated nests (particularly of unequal size, $2 * N$ and $2 * M$, $N > M$) are intermingled, Rambam suggests the sacrifice of all $2 * N + 2 * M$ as opposed to only $2 * N + M$ of the birds. It is easy to demonstrate that once only $2 * N + M$ birds are sacrificed, at least N belonged to the woman who brought $2 * N$ birds and M belonged to the woman who brought $2 * M$ birds have been sacrificed. Sacrifice of the remaining M birds has no additional utility. If as we have claimed that this case is after the fact, the question is not compelling, since the women could not be assumed to have described the situation to the Kohen. Furthermore, lacking any obligation by either woman to sacrifice only *olot*, it is unlikely that such a sacrifice can be created *ab initio*; absent an obligation by either woman to sacrifice only *olot*, such an occurrence cannot be (artificially) created.

Part 4: Chapter 2 - Second pass

The third Mishnah in the second chapter concerns birds that fly in a precise pattern between undesignated nests. There are seven nests in the Mishnah,

⁵¹ Yet more troubling is why this option is only raised in the 3rd chapter where no prior consultation is assumed. Rambam might maintain that the Mishnah restricted itself to the sacrifice of *hovot*.

arranged in sequence, with 2, 4, 6, 8, 10, 12 and 14 birds respectively.⁵² Unlike the previous case of two nests of Rachel and Leah, where the *halakhic* ruling was disputed by the standard interpretation versus Rambam and Raavad, the *halakhic* ruling in this more involved case is stated unambiguously in the Mishnah. All agree on what the *halakhic* rules stipulate. Before turning to the circumstance and principles, where disagreement reappears, the *halakhic* rule must be examined in detail.

The birds fly three or six times, depending how you choose to count, three round-trips or six one-way trips.⁵³ One bird flies from the nest of 2 birds to the nest of 4 birds, then a bird (not necessarily the same bird, but always any bird present in the nest at that point) flies from the nest of 4 birds (actually 5 birds at that point) to the nest of 6 birds, and so on until a bird flies from the nest of 12 birds (13 birds at that point) to the nest of 14 birds. Each such sequence represents a one-way trip. At that point, having reached this last nest, the birds begin to fly in the other direction. A bird flies from the nest of 14 (15 birds at that point) to the nest of 12, and so on until a bird finally returns and restores 2 birds to the first nest. The Mishnah declares that after this first round-trip there are 0, 0, 2, 4, 6, 8 and 12 valid birds within their respective nests. The first and last nest, that had 2 and 14 birds originally, lost 2 birds, the five nests in between lost 4 birds. Note that in

⁵² The nests are referred to as the first through the seventh nest counting from the smallest to the largest.

⁵³ One minor question for which again I cannot even speculate, is why Rambam who normally counts roundtrips, chose once in *Peirush Ha-Mishnayot* to count one-way trips, instead.

each of the six flights, the birds fly sequentially from nest to adjacent nest; a bird leaves from a nest only after a bird has arrived.⁵⁴

This pattern of birds flying from the smallest (active) nest to the seventh nest and then back, reoccurs two more times. As well, this *halakhic* ruling that the two nests at the two ends lose 2 birds, while those in between lose 4 birds, also reoccurs. However, the standard interpretation assumes that each time two additional nests, having no valid sacrifices, remain dormant; their birds no longer fly. As it simplifies the case, we will use this assumption throughout.⁵⁵ Raavad's (as well as likely Rambam's) assumption that birds from dormant nests continue to fly has no impact on the *halakhic* rule.

After the second round-trip, the *halakhic* ruling is the same: the first valid nest, the third nest of the original set and the last nest each lose 2 birds, and the three nests in between, nest 4, 5, and 6 of the original nests, lose 4 birds each. Starting with the previous sequence of valid birds in each nest, there are now (0, 0, 2-2, 4-4, 6-4, 8-4, 12-2) or 0, 0, 0, 0, 2, 4, and 10 valid birds within their respective nests. Now birds fly between the three remaining nests; after this third round-trip, only the largest nest has not been entirely disqualified and it still has 8 valid birds. An alternate opinion stated in the Mishnah allows not just 8 birds, but all 14 birds in

⁵⁴ An alternative case where all flights occur simultaneously within each of the six trips is not considered. See footnote 69 for a consequence of such a case.

⁵⁵ This assumption is not critical to any of the interpretations and is explicitly rejected by Raavad. Raavad states that all nests participate in each of the three rounds of flights. As will be shown, in the construction proving the correctness of his rule, it is not necessary to assume that any birds from nests with no valid birds continue to fly. Rambam's assumption as to whether nests without valid sacrifices remain dormant or continue to fly is not entirely clear from his commentary.

the largest nest to be sacrificed, ostensibly, but not necessarily, as a result of all other nests have no valid birds remaining.

The circumstances, principles and *halakhic* rules for the simple case of two undesignated nests of 4 birds where a single bird flies from one nest to the other leaving 3 and 5 birds in the two nests and a second flight restoring 4 birds to each nest were explained in detail in a previous section. It may not be obvious, but the three approaches, the standard approach as well as those of Raavad and Rambam, all generalize from that simple case to this complex case, without introducing any fundamentally new principles. While the complexity of the case requires concentration,⁵⁶ no fundamentally new concepts are introduced.

Commentators struggle with the accuracy of the standard interpretation's explanation of this Mishnah; no complete solution has been proposed. Unlike the standard interpretation, the accuracy of both Rambam and Raavad will be proven.⁵⁷ Rambam's approach generalizes beyond the cases considered, providing a simple, but more general, halakhic *rule* that is proven mathematically.⁵⁸

One caution is critical; the Mishnah is difficult enough without again revisiting the rationale for the principles that have been identified. Going forward, only the circumstance, principles and *halakhic* rules are considered. Unlike the simpler

⁵⁶ And perhaps pen and paper to help keep track of 58 birds in 7 nests.

⁵⁷ In contrast, the minor inaccuracy of the standard approach is demonstrated as well.

⁵⁸ Raavad's approach does not (seem to) lend itself to a fundamentally more insightful general theorem; see footnote 62 for a theorem that adds generality but not any additional insight.

case in the second Mishnah of the chapter, in which the *halakhic* ruling is in dispute, in this third Mishnah the three opinions must all agree; the *halakhic* rulings of the Mishnah are expressed unambiguously.

First, the standard approach disqualifies 2 birds from the nest that a bird left and none from the nest to which a bird arrived. Ostensibly, this *halakhic* ruling comports with the Mishnah's ruling. In their first round-trip, the first and last nest each had one only bird departing during the round-trip; no bird departs from the last nest in one direction to go to a larger nest, and no bird departs the first nest in the other direction to go to smaller nest. Thus, the nests with 2 and 14 birds lose 2 birds each, exactly as specified. Each of the intermediate nests has a bird fly out twice, once on each one-way trip, and again lose 4 birds each, as the Mishnah specifies. The *halakhic* ruling seems to be exactly as the Mishnah specifies.

However, the problem is illustrated by carefully examining the return trip when a bird leaves the nest that originally had 6 birds going to the nest that originally had 4 birds. Prior to that bird departing, the nest had 4 valid birds (of the 7 in the nest.) When this second bird departs, 4 valid birds are now reduced by 2, the bird that flew away and one other.⁵⁹ The bird that left clearly has no value to its previous nest and we demand the disqualification of a second bird still in that nest. If we do not disqualify a second bird, its mate that just flew away may be improperly sacrificed in the nest (or a subsequent one) into which it flies. But in this case, that possibility does not exist since no birds in the two smallest nests

⁵⁹ Only 2 of the 6 birds remain valid.

are ever sacrificed. Why should that third nest lose a bird to protect against a completely non-existent event? This dilemma has no resolution and represents an unrelieved problem with the standard interpretation.

The proposed justification that the Mishnah is trying to teach a general rule that has an occasional inaccuracy would be more plausible if these were common events for which *halakhic* rulings that are normally accurate are required. Clearly these cases are theoretical and are hardly common occurrences; one can speculate if anything remotely like what the Mishnah addresses ever occurred. It is therefore reasonable that both Rambam and Raavad would reject such an explanation as an unsupportable rationalization.

Ironically, a logical equivalent to the very example identified that creates the dilemma for the standard interpretation is already referenced in the Mishnah as the alternate opinion. That opinion suggests that since all the other nests are now invalidated the large nest can utilize all 14 of its birds, not just 8. That alternative makes complete sense; regardless of where the 3 birds that left the nest of 14 birds (and, in the (highly probable) worst case, did not return) currently reside, they will not be sacrificed. Note the similarity between the last nest of 14 birds addressed by the alternate opinion and the similar case identified above that ought to raise that same alternative, which is not mentioned in the Mishnah.⁶⁰

⁶⁰ Again, one could argue that the alternate opinion is raised only once. Troubling as well, the alternate opinion appears so logical under the standard interpretation; perhaps again, the primary opinion is the result of the desire for a simple rule.

Let us turn to the views of Rambam and Raavad. Again, they assume that as a bird flies between nests, one bird is lost from each nest. Note again that the ruling of the Mishnah appears accurate. The nests with 2 and 14 birds had one bird fly in and one bird fly out and hence each lost 2 birds. The intermediate nests had a bird fly in and out twice, and hence must lose 4 birds. As in the standard interpretation, at this point the *halakhic ruling* itself corresponds to the Mishnah. What we now prove separately for Raavad and Rambam is that their principles substantiate the *halakhic* ruling precisely, with or without consultation, according to Rambam and Raavad respectively.⁶¹

Let us look carefully at Raavad. Under his assumption of no prior consultation, we must prove that we have derived the worst-case scenario. Note that this case is in one respect more elaborate than the case addressed in *Kinnim* (3:2) where all the nests are intermingled, and we are looking only for a single number, the (even) number of birds correctly sacrificed even under the worst-case scenario. Here we must prove a worst-case scenario not for one large intermingled nest, but for each of 7 separate nests into which a few birds have flown in and out several times.

The total number of birds is $2 + 4 + 6 + 8 + 10 + 12 + 14 = 56$. We must look at the configuration at the end of each of the three round-trip flights and the results given in the Mishnah for each individual nest. The cases get progressively harder. In every case, to prove that we have derived the worst-case scenario, we must:

⁶¹ In the case of Rambam it turns out that it does not matter whether one assumes prior consultation or not.

1. Prove that no other scenario is worse than the proposed worst case, disqualifying more birds.
2. Construct that worst case⁶² using the Raavad's principles exactly matching the Mishnah's ruling while maintaining the assumption that each nest is sacrificed half as *hatta'ot* and half as *olot*.

Proving 1) is straightforward. Birds in their original nest cannot be the source of disqualification, since each nest is assumed to be sacrificed correctly. A bird both outside its original nest and from a previously active nest contributes 2 disqualifications, since one does not know if it was sacrificed as a *hattat* or *olah*.⁶³ In the first round of flights there are $(1+2+2+2+2+2+1 =)$ 12 such birds. In the second round there are $(1+2+2+2+1 =)$ 8 such birds. In the final round there are $(1+2+1 =)$ 4 such birds. Thus, there can be at most $(2 * (12 + 8 + 4) =)$ 48 disqualifications, exactly as the Mishnah stipulates at every stage; no worst scenario can exist.

To demonstrate item 2) above, a configuration disqualifying the requisite number of birds at each stage must be constructed. After the first round-trip, there are at most 12 birds outside their original nest and we must construct a configuration

⁶² One can demonstrate the worst case either by a theoretical existence proof or actual construction; in this case we chose the latter. One could generalize the case in the Mishnah from $2*1$ through $2*7$ birds to $2*1$ through $2*K$ birds, where K is odd with $(K-1)/2$ round trips, to create a theorem. While this creates work mathematically, it adds no insight into the *halakhot* of *kinnim*. A general construction, versus a formal proof, is also more likely the way Raavad conceived of the Mishnah.

⁶³ It is sacrificed incorrectly if it is sacrificed the same way as its original mate, both as *hatta'ot* or as *olot*.

that disqualifies $(2 + 4 + 4 + 4 + 4 + 4 + 2 =)$ 24 birds from the seven nests respectively. After the second round-trip, another $(2 + 4 + 4 + 4 + 2 =)$ 16 birds must be disqualified. Only 3 nests are still active and after the third round-trip another $(2 + 4 + 2 =)$ 8 birds must be disqualified. Note of the 56 original birds, exactly 8 are left; $2 * 24 = 48$ have been disqualified.

Note that after the third and final set of flights, exactly half of each nest except the last is potentially comprised of birds from an outside nest. At that point, all birds outside the largest nest can be disqualified. We will construct a case where all foreign birds are disqualified and since each of those foreign birds could have been sacrificed as either a *hattat* or an *olah*, one can always construct a parallel case interchanging *hattat* and *olah*. As a result, twice that number of birds are disqualified. In the final nest there are potentially 3 birds from outside the nest; again, disqualifying twice their number, leaving 8 valid birds. We construct all 3 configurations below. The intuition⁶⁴ in constructing each worst-case scenario is:

- have no bird fly more than once, and
- assign the sacrifices for maximal disqualification.

By having each bird fly exactly once, the number of birds outside their original nest is maximized. Assigning sacrifices for maximal disqualification is done by sacrificing every bird originally from nest X the same way, either all *hatta'ot* or all *olot*, regardless of where a bird currently resides, and assigning all birds in a nest

⁶⁴ Understanding the intuition behind the construction of a worst case is important but strictly speaking unnecessary. The construction itself is what proves the assertion.

that come from other nests the same way.⁶⁵ To maintain an equal number of birds sacrificed as *hatta'ot* and *olot*, alternate a bias towards *hatta'ot* and *olot*. Each bird outside its original nest and its original mate can be seen to be sacrificed identically and hence incorrectly, while each nest overall is sacrificed correctly. Take for example *ken* 4 after the first round-trip and examine the construction below. The *ken* has a *hatta'ot* bias while *kinnim* 3 and 5 have an *olot* bias. It has 2 stray birds, one each from *ken* 3 and *ken* 5 that are sacrificed the same way as their mates in *ken* 3 and *ken* 5 and hence disqualified. Those 2 birds happen to be *olot*; constructing a parallel case where those two birds are *hatta'ot* requires that we disqualify 2 other birds (their mates) in *ken* 4. Only the 4 birds where **both they and their mates** were originally from *ken* 4 are correctly sacrificed.

In general, it should be obvious that in addition to the case constructed, one can construct a parallel case by substituting *hattat* and *olah* uniformly throughout. Doing so creates two worst cases, each of which may have occurred. Unable to rule out either worst case, doubles the number of birds disqualified.⁶⁶

After the first round-trip:⁶⁷

⁶⁵ This avoids 2 members outside their nest being mates which reduces the number of disqualifications by 2.

⁶⁶ Note that the construction does not need to allow birds from dormant nests to fly, as excluded by many commentators, even though Raavad includes such flights. See footnote 55. To formally comply with Raavad, contrary to the assumption that each bird flies exactly once, imagine that in the second and third round, a bird flies from the first nest to the first non-dormant nest and then that same bird returns.

⁶⁷ The reader should convince himself that the sequence of flights can produce the configuration noted. Note that each bird flies as most once; it either remains in its original or

Ken 1: O: 1. H: 2. This *ken* has no valid birds. (Note that while in this scenario the first *ken* can claim credit for an *olah*, as indicated one can reverse *olot* and *hatta'ot*, throughout the construction, in which case a *hattat* not an *olah* would be valid. As we are establishing the worst case, we disqualify both. This argument applies throughout.)

Ken 2: H: 2, 2. O: 1, 3. This *ken* has no valid birds.

Ken 3: O: 3, 3, 3. H: 3, 2, 4. This *ken* has 2 valid birds.

Ken 4: H 4, 4, 4, 4. O: 4, 4, 3, 5. This *ken* has 4 valid birds.

Ken 5: O: 5, 5, 5, 5, 5. H: 5, 5, 5, 4, 6. This *ken* has 6 valid birds.

Ken 6: H 6, 6, 6, 6, 6, 6. O: 6, 6, 6, 6, 5, 7. This *ken* has 8 valid birds.

Ken 7: O: 7, 7, 7, 7, 7, 7, 7. H: 7, 7, 7, 7, 7, 7, 6. This *ken* has 12 valid birds.

After the second round-trip:

Ken 1: O: 1. H: 2. This *ken* has no valid birds.

Ken 2: H: 2, 2. O: 1, 3. This *ken* has no valid birds.

Ken 3: O: 3, 3, 3. H: 2, 4, 4. This *ken* has no valid birds.

Ken 4: H 4, 4, 4, 4. O: 3, 3, 5, 5. This *ken* has no valid birds.

Ken 5: O: 5, 5, 5, 5, 5. H: 5, 4, 4, 6, 6. This *ken* has 2 valid birds.

enters and then stays in an adjacent nest. The numbers listed below are the original nest of each bird. Note again how all birds originating in a nest are sacrificed the same way to maximize disqualification.

Ken 6: H 6, 6, 6, 6, 6, 6. O: 6, 6, 5, 5, 7, 7. This *ken* has 4 valid birds.

Ken 7: O: 7, 7, 7, 7, 7, 7. H: 7, 7, 7, 7, 7, 6, 6. This *ken* has 10 valid birds.

After the third round-trip:

Ken 1: O: 1. H: 2. This *ken* has no valid birds.

Ken 2: H: 2, 2. O: 1, 3. This *ken* has no valid birds.

Ken 3: O: 3, 3, 3. H: 2, 4, 4. This *ken* has no valid birds.

Ken 4: H 4, 4, 4, 4. O: 3, 3, 5, 5. This *ken* has no valid birds.

Ken 5: O: 5, 5, 5, 5, 5. H: 4, 4, 6, 6, 6. This *ken* has no valid birds.

Ken 6: H 6, 6, 6, 6. 6. 6. O: 5, 5, 5, 7, 7, 7. This *ken* has no valid birds.

Ken 7: O: 7, 7, 7, 7, 7, 7. H: 7, 7, 7, 7, 6, 6, 6. This *ken* has 8 valid birds.

This demonstrates that Raavad's approach is precise.⁶⁸ The alternate opinion allowing 14 valid birds is not so much a conceptual disagreement as a practical one. If each of the owners of first 6 *kinnim* desists from sacrificing, then the *ken* with 14 birds should be entirely valid. Of course, if we assume that all owners bring their nests *anyway* without consultation, then the alternate opinion would be rejected.⁶⁹

⁶⁸ A reader who remains unconvinced, can painstakingly construct those scenarios (and the parallel scenario switching *olot* and *hatta'ot*) using the method described – one flight maximum, sacrifice as many as possible of the original birds from the nest identically and alternate the bias of the nests to maximize disqualification.

⁶⁹ Interestingly, the Raavad's approach raises the need to stress one detail about the flights of the birds. During each of the three round trips, all commentators assume that birds fly

Now we (finally) come to Rambam.⁷⁰ Interestingly, Rambam's point of view has one critical feature that, once recognized, makes proving a more general theorem elementary. Given the principle that no credit is received for a bird that you do not own, the configuration(s) constructed for Raavad, demonstrates Rambam's *halakhic* result as well. For Rambam as well we must choose the case with the maximum number of birds disqualified, which in his case need only meet two simple criteria:⁷¹

- no bird flies more than once and
- all foreign birds in a nest are sacrificed the same way.

successively not simultaneously. Only when a bird flies to an adjacent nest, can a bird currently in that nest, potentially the bird that just arrived, fly. Were all birds to fly simultaneously but only to an adjacent nest, a bird can only be one nest away from its original nest in each round, and at most 3 nests away after all three rounds. On the other hand, if they fly successively, then a bird from any nest can end up in any other nest. A single bird can fly from the first to the last nest, if it is the only bird flying successively. If we modify the normal interpretation and make an assumption of simultaneity and we further assume we can restrict owners from sacrifice, then according to Raavad's principles, the alternative opinion could propose a yet stronger example: Allow 20 valid sacrifices by letting the owner of both the 3rd and 7th ken bring their sacrifice while everyone else desists. Note that in three flights a bird from the 3rd ken can be anywhere from the 1st to the 6th ken; it cannot make it to the 7th ken in three steps. Similarly, a bird from the 7th ken can be anywhere from the 4th to the 7th ken, but also cannot reach the 3rd ken. By the Raavad's principles, **both** the 3rd and the 7th *kinnim* should be valid according to the alternative opinion. This further proves that the Mishnah must be interpreted where the flights within each of the six rounds (three roundtrips) are successive.

⁷⁰ Note that the explanation and proof that follows does not depend on whether Rambam interpreted these *mishnayot* with or without prior consultation. R. Y. Heller assumed the latter. While, the principles and the rule are critical; the circumstance is not.

⁷¹ For Raavad the construction dealt with three rules: one flight maximum, sacrifice all original birds from the same nest identically and alternate the bias of the nest. Note that for Rambam there is one fewer principle that a construction must follow, allowing many more examples. (The first two principles are identical and the third is not relevant.) For Raavad, the example is essentially unique.

The reason for disqualification does not depend on the state of any other nest. If a nest of 10 birds has 4 foreign birds, then those 4 birds, assuming they are all sacrificed the same way, invalidate 4 other birds in that nest, not knowing which way they were sacrificed, and only 2 birds in the nest remain valid. Note how complex Raavad's approach is compared to that of Rambam. For Raavad, there is a 2-step process. First bird X has a mate in an adjacent nest from which it flew that was sacrificed the same way; either it or its original mate must be disqualified. For reasons of fairness we choose the bird in the improper nest, and then we disqualify a second bird in that nest based on the parallel worst case. To demonstrate the Mishnah according to Rambam, there is no need to examine other nests as was required in demonstrating the view of Raavad. According to Rambam bird X is foreign; both it and its mate in the nest in which it currently resides are disqualified, not knowing whether the foreign bird or its mate was sacrificed as an *olah* or *hattat*.⁷²

This creates a simple theorem that generalizes Rambam's approach. Assume $2 * K$ original birds, J birds join the nest and L birds leave, of course assuming we cannot identify the birds that arrived, but know that there were J such birds. Rambam would require $2K + 2J$ birds to be sacrificed to fulfill the original obligation. Think of the process in two stages: first a known number of birds enter and leave. We have $2 * K - L + J$ birds in the nest.

⁷² It should be obvious that for Rambam there are many more examples than the one given for Raavad.

Theorem: Assume K , J , and L are defined as above and $2*K - L + J$ birds are currently in the nest. $2*K - L + J + L + J = 2*K + 2*J$ birds always correctly fulfill the original requirement.

Proof: First note that if $J + L$ is equal to or exceeds $2*K$, we simply start over; too many birds have come and gone and there is no point in using the original nest. Informally, we assume all foreign birds are sacrificed the same way, but we do not know if they are *olot* or *hatta'ot*; as a result, we add a mate. Since there are (at most) J foreign birds we add J birds as mates. (If some of the foreign birds are sacrificed as both are *olot* or *hatta'ot* we would require fewer birds to be added.) We also replace the L birds that flew away restoring $2K$ birds to the nest. We also do not know if a bird leaving was an original member of the nest or one of the foreign birds who entered. If we knew the bird was a foreign bird, then we could simply reduce J by 1, reducing the number of birds that must be added (by 2). As we do not know that we assume every bird that left was a part of the original nest. This argument provides both the intuition behind the theorem and an informal proof. A formal proof follows in the next paragraph.

Assume $L+J$ is less than $2*K$ and use a simple proof by induction. Let $L=1$ and $J=0$. Clearly a bird leaving is simply replaced: $2*K - 1 + 1 = 2*K + 2*0 = 2*K$. If $L=0$ and $J=1$, then one bird entered the nest and we add one additional bird⁷³ and sacrifice $2*K + 2*1 = 2*K + 2$ birds. In the general case, consider first a bird leaving. Before we assume $2*K + 2*J$ birds sufficed and adding back a bird restores $2*K + (L+1) -$

⁷³ Sacrificing those $2*K + 2$ birds yields exactly $2*K$ valid sacrifices, the additional bird and its mate are disqualified, being unable to know whether the foreign bird was sacrificed as an *olah* or a *hattat*. To be formally correct, note that at every step in the proof, adding a bird always preserves $2*K$ valid sacrifices under any circumstance.

$(L+1) + 2*J = 2*K + 2*J$ birds. (If we knew that it was a foreign bird that left, then we need not add a bird. Since, we do not know that, a bird must be added.) If a bird joins, then J becomes $J+ 1$ and adding a bird results in $2*K + 2J + 1 + 1 = 2*K + 2(J+1)$. If $2*K+2*J$ was sufficient before, adding a bird guarantees that however sacrificed the foreign bird is not counted as a valid sacrifice. This shows that $2*J + 2*K$ is sufficient. Again, if we could guarantee that the foreign bird that arrived can be mated with a foreign bird already in the nest, another bird would not have to be added. Since, we cannot guarantee that, adding another bird to be its disqualified mate is required.

Note that Rambam demands that we be conservative to create maximal disqualification. Two assumptions ensure that conservative worst case. First, as noted the bird that arrives is not one that subsequently leaves; this maximizes foreign birds in a nest. Second, if 2 foreign birds were sacrificed as a mated pair, more valid sacrifices would result; as a result, all foreign birds are assumed to be sacrificed the same manner.

There is one major difficulty with Rambam's position; the alternative proposed in the Mishnah that all 14 birds in the last nest are valid. Since that nest may contain 3 foreign birds that position is difficult to reconcile with Rambam's approach. The fact that Rambam rejects that option still requires a plausible hypothesis why it might be permitted. Perhaps, (unconvincingly in my opinion) the alternate opinion allows use of another's birds in a *hovah* when the original owner is completely removed from the picture and would be expected to relinquish ownership.

Part 5: Section 1 – Summary

Many who study *Kinnim* are unaware of the novel approaches of Rambam and Raavad. Rambam and Raavad both avoid several issues with the standard interpretation of the 3rd Mishnah in the second chapter:

- the less than intuitive attribution of the loss of 2 birds from the nest from which a bird departs as opposed to the loss of one bird from each of the nests,
- the unavoidable imprecision of the *halakhic* rule, and
- the seemingly overwhelming logic of the alternative opinion.

However, each of their views also has their unique challenges. Raavad is challenged by

- the introduction of a case of no prior consultation in the second chapter.

Rambam is challenged by

- the lack of a clear /compelling argument in favor of that Mishnah's alternate opinion, and
- the fact that the compelling options he develops in the first chapter receive no mention anywhere in the entire tractate.⁷⁴

The challenges to Raavad's position are easiest to address. As well, Raavad's innovations are technical; conceptually he is consistent with the standard

⁷⁴ Even in the third chapter that considers cases with no prior consultation.

interpretation. However, Rambam's position is a radical departure from the standard interpretation. Both

- replacing implicit designation with a notion requiring that sacrifices have utility, and
- insistence on either ownership or reciprocal benefit

upends the classical approach to the entire tractate.

Again, to summarize Rambam's position:

- The assignment of a bird in a *ken* as an *olah* or *hattot* is based **only** on an initial specification by the owner at the time they are designated for sacrifice or by the specific bird's actual sacrifice by the Kohen, excluding designation that results from the prior sacrifice of any other birds.
- There exists a strong preference for direct ownership of sacrificed birds even when that preference requires additional sacrifices that do not increase the number of valid birds sacrificed; this does not exclude mutual ownership.
- As usually assumed, an entirely useless sacrifice is always prohibited; however, establishing clear ownership renders a sacrifice useful.
- Having initially been brought as a *hovah*, sacrificing the entire *ken* as *olot* or as *hatta'ot* is not *a priori* excluded.

The following 2 sections are being reviewed and will eventually be added:

Section 2: The term *merubah*

Section 3: The approaches of Raavad, the Ba'al *Ha-Maor* and R. Yehoshua Heller to *Kinnim* (3:4).